Multi-objective Pareto optimization of a solenoid

(TEAM Workshop Problem 35)

1. General description

A model for assessing the quality of a magnetic field, produced by a distributed winding composed of circular turns, is proposed. In particular, a radius-oriented synthesis problem, assuming a two-dimensional control region Ω for the prescribed flux density, is considered (Fig. 1). In fact, the field distribution which is required in the control region can be synthesized by means a suitable distribution of the turn radii.

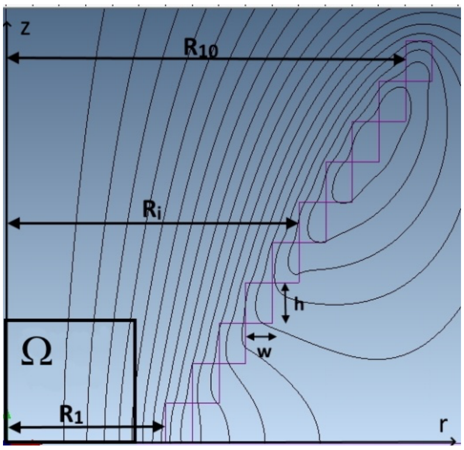

Fig. 1 - Geometry of the winding and control region

The uniformity of the field is usually of prime interest, but sensitivity cannot be overlooked. Eventually, power loss is also relevant to an efficient design; therefore, we propose a twofold formulation of the synthesis problem. Accordingly, the benchmark problem is defined in terms of two bi-objective optimization problems. To focus the attention, a small-size solenoid, as used for *in vitro* experiments involving magnetic fields, is considered here.

The winding is composed of 20 series-connected circular turns, with the width of each turn w = 1 mm and the height h = 1.5 mm, and a current of 3 A (i.e. current density 2 A mm^{-2}). Thus, assuming a symmetric distribution with respect to the plane z=0, ten unknown radii (design variables) are to be identified. A two-dimensional control region is considered: the flux density $B(r,z)$ = (0, B_0) with B_0 = 2 mT is prescribed at n_p=66 sample points, evenly spaced in a 5×5 mm^2 squared region.

2. Definition of Problem A (field uniformity and sensitivity)

2.1 Design parameters

Radii R_1,..., R_i,...,R_{10} of the ten turns (Fig. 1). Variation range $5 \le R_i \le 50$ mm.

2.2 Objective functions

$$f_1 = \sup_{j=1,n_p} \left\| \, |B_j| - B_0 \, \right\| \qquad (1)$$

with n_p=66 sample points and B_0=2 mT

$$f_2 = \sup_{j=1,n_p} \left[\left\| \, |B^+| - |B_j| \, \right\| + \left\| \, |B_j| - |B^-| \, \right\| \right] \qquad (2)$$

where B^+ and B^- are the flux density values computed after an expansion, or a contraction, of 0.5 mm of all radii with respect to the unperturbed configuration. Both objective functions must be simultaneously minimized.

2.3 Constraints

$a < R_i < b$ with a=5 mm b=50 mm i=1,n_t.

3. Definition of Problem B (field uniformity and power losses)

3.1 Design parameters

Radii R_1,..., R_i,...,R_{10} of the ten turns.

3.2 Objective functions

$$f_1 = \sup_{j=1,n_p} \left| |B_j| - B_0 \right| \quad (3)$$

with n_p=66 sample points and B_0=2 mT

$$f_2 = \sum_{i=1}^{n_t} R_i \text{ with } n_t=10 \quad (4)$$

In particular, f_2 accounts for the ohmic resistance - and so power losses - of the winding.
Both objective functions must be simultaneously minimized.

3.3 Constraints

a < R_i < b with a=5 mm b=50 mm i=1,n_t.

4. Field analysis

The field problem can be solved analytically or numerically e.g. using a finite-element axisymmetric model (or a 3D model) subject to boundary conditions:
tangential flux lines at r=0
normal flux lines at z=0.

5. Comparison of methods

In order to compare different optimization methods, the Pareto front relevant to Problem A and Problem B should be plotted in the relevant objective spaces.

References

[1] Di Barba, P., Mognaschi, M.E., Lowther, D.A., Sykulski, J.K., A benchmark TEAM problem for multi-objective pareto optimization of electromagnetic devices (2018) IEEE Transactions on Magnetics, 54 (3), art. no. 8055434.
[2] Di Barba, P., Mognaschi, M.E., Lowther, D.A., Sykulski, J.K., Improved solutions to a TEAM problem for multi-objective optimisation in magnetics, (2020) IET Science, Measurement and Technology, 14 (8), pp. 964-968.

Proposers

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