



A Survey of French Research Activities on Computational Electromagnetics in the Framework of COMPUMAG Society

Abstract — This article gives a short historical survey of research activities on computational electromagnetics in France. The works cited in this article are mostly related to the conferences COMPUMAG and CEFC and published in the IEEE Transactions on Magnetics. This survey does not intent to provide an exhaustive review of rich contributions of French community but to show its active participation in the field of computational electromagnetics.

I. INTRODUCTION

The research activities on the numerical computation of electromagnetic fields in France can be traced back to the 70s of last century. The majority of works at that time were motivated by the needs in the modeling and the design of electrical machines and apparatus. Later on, the research was quickly extended to various low and high frequency formulations and numerical methods, coupled problems, material modeling, design and optimization, and so on, for different applications.

Over the past 40 years, the France remains one of the most active communities worldwide in the domain of computational electromagnetics and participates actively the international conferences COMPUMAG and CEFC. In 1979, three years after the creation of the COMPUMAG conference in Oxford, the second edition of COMPUMAG was held in Grenoble France and chaired by J.C. Sabonnadière from the laboratory LEG (today G2Elab). In 1994, the first CEFC conference held outside the USA took place in Aix-les-Bains and was again chaired by J. C. Sabonnadière. In 2001 and 2014, respectively, the 13th COMPUMAG was organized in Evian by A. Nicolas from the laboratory CEGELY (today Ampère) and the 16th CEFC organized in Annecy by Y. Marechal from the G2Elab. Recently, the 22nd edition of COMPUMAG (2019) is granted to Paris and will be co-organized by GeePs (former LGEP) / L2E of Paris and L2EP of Lille, chaired by the authors of this article. The French community is also actively involved in the organization of other european or international conferences on the computation of electromagnetic fields, such as, the NUMELEC and the EMF, among others.

The research activities of French community range from the theory of numerical methods and formulations, in low frequency as well as in high frequency, to a wide range of applications. This article by no means intends to give a complete review of the rich activities in France but tries to summarize some prominent research topics mostly related to the COMPUMAG society and the IEEE Transactions on Magnetics. Obviously, many pioneer works conducted by French colleagues, especially outside the framework of COMPUMAG Society, are not included in this survey.

II. NUMERICAL METHODS AND FORMULATIONS

France is among the first countries devoted its research interests in the development of electromagnetic formulations since the late 1970s. During the past 40 years, diverse generic and specific formulations and methods have been developed for various electromagnetic problems.

A. Formulations

The early research focused mostly on the formulations and numerical methods of low frequency problems. We can cite for example the early work on the formulations based on the finite element method (FEM) [1], the boundary integral method (BIM) [2], and the mixed FEM and BIM [3]. They are, respectively, theoretical bases of well-known electromagnetic computation software Flux3D [4], PHI3D [5] and TRIFOU [6]. In particular, TRIFOU is the world's first code that uses the edge elements to interpolate vector fields. The formulation of TRIFOU is in terms of magnetic field. Later on, this formulation is extended to its dual version in terms of electric field [7]. Since then, it has been seen appearance of various dual formulations in terms of fields and potentials. A code exploring the dual formulations - code_Carmel is latterly developed [8]. It has been also seen the development of a mixed formulation in terms of both electric and magnetic fields [9].

Under the framework of the differential geometry, A. Bossavit unified the nodal elements, edge elements, facets elements and volume elements within the Whitney complex and showed that, beyond their property of inclusion, they are naturally adapted, respectively, for the discretization of scalar and vector fields with required continuities [10]. They are widely known today under the name of Whitney elements.

The gauge condition is an important issue in the formulations in terms of magnetic vector potential. It is shown in [11] that when using an iterative solver such as the conjugate gradient method, the vector potential is implicitly gauged and the convergence is tightly related to the discretization of the source currents which must be divergence free. Further study on the discretization of the source field is carried out. Based on the tree and co-tree technique in the facet elements graph, the discretization of the current density guaranteeing the divergence free condition is developed [12].

Error estimates play an important role in the mesh based computational electromagnetism. In order to distinguish the discretization error of each potential formulation, a residual-type estimator that adapts specifically to the A-v formulation in harmonic eddy current problems, with pertinent proofs regarding reliability and efficiency, is developed [13]. The

computational cost for this estimator is cheapest among the existing estimators, which can be well used in adaptive mesh refinement.

Some attentions are also paid for the analytical method like for example in [14], the method based on the space harmonic decomposition which is very suited for the rotating machines.

The meshless formulation is an interesting alternative as regard to mesh based formulations. A thorough comparison of several meshless methods is given in [15]. A meshless Galerkin approach tackling some specific implementation issues is developed. The study showed that the hierarchical approach is more attractive than the standard approach as regard to the robustness and the cost.

Recent study has shown the interest of the volume integral formulation. The main advantage of this formulation is that there is no need to mesh the air region which is very beneficial for geometrically complex devices. It is proposed in [16] a volume integral approach combining the magnetic vector potential \mathbf{A} for magnetic region and the electric vector potential \mathbf{T} for electric region. The formulation is based on the circuit representation by using the Whitney face interpolation for unknown current and flux.

B. Methods for Specific Problems

One of the issues in the finite element method is the open boundary problem. To avoid meshing far away from the domain of interest, specific techniques have to be introduced. One of the techniques is to couple the finite element method with the boundary integral method like in [3] and [7]. Other methods with less computational cost have also been developed. Among them, a rigorous and easy technology based on the parallelepiped shell transformation is presented [17]. Its efficiency for 3D problems is demonstrated. Fig. 1 shows its application in the 3D modeling of the magnetic signature of an aircraft carrier which is a typical open boundary problem.

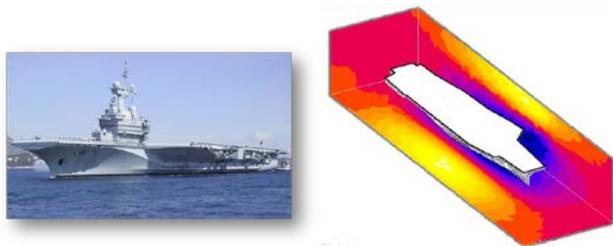


Fig. 1. Modeling of the magnetic signature of an aircraft carrier.

Geometrically thin structures are often encountered in engineering problems. The modelling of the thin structures is difficult because of the disproportion of their thickness with respect to other dimensions which leads to undesired deformed elements in the finite element modeling. In the context of computing eddy currents in the thin shells, reference [18] proposed thin shell models to take into account the field variation due to the skin effects within the shells. The models are firstly developed in the boundary integral formulation and extended later on in the finite element formulation [19] for the computation of eddy current losses in transformers. In the framework of Whitney elements, a series of nodal and edge shell elements is derived through the degeneration of prism elements [20]. The scalar and vector variables are approximated by their averages and jumps across the thin structure.

The geometry and material characteristics of electromagnetic problems are not always well known or are, by nature, variable (variability of the process of fabrication for example). These characteristics, not deterministic anymore, can be considered as random variables and the finite element problem to be solved becomes stochastic. In [21] for instance, a spectral stochastic finite element method has been developed and tested in electrostatics to handle the random permittivity.

C. Computational Performance

As the numerical methods are widely applied for modeling of complex systems, the computational performance of field solvers becomes prominent. The immediate support comes from the parallel computing. A survey of parallel solvers for the finite element method in computational electromagnetics is reported in [22]. The benefits, limits and trends of parallel methods, such as the standard iterative and direct solver, the domain decomposition methods and the multilevel methods, under the prevailing multi-computer environment MPI and PVM are summarized. In reference [23], by using the discontinuous Galerkin method, a domain decomposition strategy based on optimized Schwarz methods for high frequency problems is proposed. Its efficiency in parallel computing is demonstrated.

Another strategy for computational performance enhancement is through the diminution of the problem complexity using the model order reduction (MOR) techniques. It has been seen recently the interests on the proper orthogonal decomposition (POD) and the proper generalized decomposition (PGD) for electromagnetic problems. To solve a nonlinear magnetostatic problem coupled with external electrical circuits, the POD is combined with the discrete empirical interpolation method [24] in the finite element modeling. Numerical examples showed significant reduction of computation time while maintaining good precision.

III. COUPLED PROBLEMS

First works on the study of coupled problems were motivated by the needs in the modeling of electrical machines. The research includes the modeling of mechanical motion, the coupling with the electrical circuits and the computation of magnetic forces. Thereafter, the research concerned with various multiphysics coupling for various applications.

A. Modeling of Motion

The mechanical motion is an issue in the study of mechanical behavior of electrical machines. As the geometry configuration changes with the motion, to avoid the distortion of elements or the re-meshing at each time step in the finite element modeling, special treatment is necessary. Various techniques are developed since the 1980s. The main techniques include the macro-element method [25] where a uniform part of the airgap modeled with an analytical formula is introduced to bridge the stator and the rotor; the moving band method [26] which consists in changing the mesh connectivity of a layer of elements in the airgap according to the movement position; the mesh connection method [27] which is based on the interpolation of unknowns of the moving and fixed parts on their interface; and the mortar element method [28] where the variables of the moving part are expressed with those of the fixed part through a projection on the 3D sliding interface.

B. Coupling with Electrical Circuits

In an electromagnetic system supplied by an external circuit, the source current of the magnetic system is usually unknown and the electromagnetic problem must be solved together with the electrical circuit. Multiple works have been published on this subject, with different entering points and focuses. Reference [29] presents a coupled model considering, respectively, the solid conductors with eddy current effect and the stranded conductors without eddy currents for 2D and axisymmetric transient problems. Indirect and direct coupling models for 2D and 3D problems are presented in [30]. An electronic switch is included in the external circuit. The work presented in [31] simulates the electrical motor fed by current inverter, where the rotation of the machine is taken into account by the moving band technique.

Further work on the coupling with circuits is closely related to the advance of electromagnetic formulations in 3D as well as to the specificity of the problem to be solved. As an example, we cite the work in [32], a coupled model for magnetic problem with solid conductors using the magnetic scalar formulation was proposed. The model takes into account the multiply connected conductors and the non-linearity and was applied for harmonic and transient problems.

C. Computation of Magnetic Forces

In a coupled electromagnetic – mechanical system, the computation of electromagnetic forces has the primordial importance. It has been shown in [33] the limitation in terms of the implementation and the accuracy of the traditional Maxwell stress tensor method in the finite elements environment, and proposed a force computation method based on the virtual work principle. The implementation was realized through the local jacobian derivative. In association with the Whitney's edge elements, the local jacobian derivative method was later on applied for the local force computation [34], which is essential for mechanical problems involving the deformation.

In the context of electrical machines, various methods are compared in [35] for the computation of torque with the consideration of rotor's rotation. A correlation between the rotation step, the size of the mesh and the force accuracy of different methods is given. A comparison of the force computation methods in 3D, in particular the local force computation methods, is given in [36].

D. Multiphysics Coupling

Various works on multiphysics coupling are published since the 1990s. Below are few examples of coupling models developed in different applications.

A strong magnetic-mechanical coupling model for the calculation of the deformation in magnetostrictive materials is presented in [37], where the local forces are computed with the help of local application of virtual work principle and the magnetic and mechanical equations are solved simultaneously.

In the study of an electromagnetic pump, the computation of the rotational forces in liquid metal is achieved through Hodge-Helmholtz decomposition, allowing the coupling with a fluid flow model [38] (Fig. 2). A coupling model for electromagnetic forming of metal sheets is presented in [39]. The modeling involves the circuit, the electromagnetic field and the plastic deformation and the system equations are solved sequentially with an indirect coupling scheme.

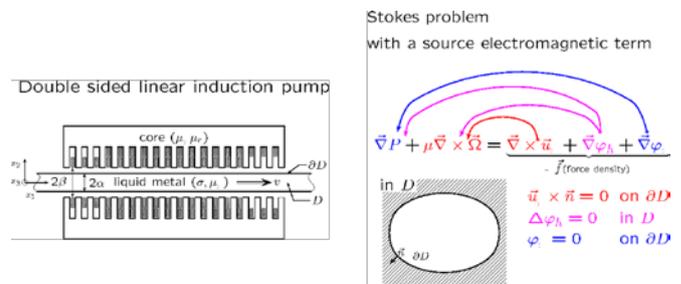


Fig. 2. Hodge-Helmholtz decomposition applied to fluid flow in linear induction pump

Electromagnetic and thermal coupling models are developed for the study of induction heating devices and for the analysis of losses and heating in electrical machines. A finite element formulation based on the direct solution of a system of dynamic, non-linear and coupled magneto-thermal equations is developed for ferromagnetic materials in induction heating devices [40]. This formulation insures a real coupling between the multiphysics equations. A 2D electromagnetic field model is coupled with an equivalent thermal network in [41] to compute the distributed losses and temperatures in induction motors. In reference [42], a 2D electromagnetic model is coupled with a 3D thermal model to study a high speed synchronous reluctance machine.

IV. MATERIAL MODELLING

The accuracy of a finite element model is based not only on the quality of the mesh (error of discretization) but also on the model describing mathematically the behavior of the materials. Some materials like ferromagnetic materials or superconductors, which play a key role in electrical devices, exhibit high non linearities and non univoc behavior. This behavior depends on the excitation history which needs the development of complex models and dedicated identification procedures. Moreover, some phenomena like piezoelectricity, magnetostriction or magnetocaloric effect require the development of finite element models coupling fields of different physical natures.

A. Ferromagnetic Materials

Ferromagnetic materials are widely used in electrical machines either as sources of magnetic flux (permanent magnets) or as magnetic conductors (soft materials). An accurate representation of the behavior of these materials is indispensable in order to design such devices more compact, robust and efficient. Numerous works have been carried out on the modelling of hard and soft ferromagnetic materials especially in order to model the hysteresis effect for soft magnetic materials [43][44][45]. Since at least one hysteresis model has to be associated to one element, this model should be fast and light. Numerous models derived from the Jiles Atherton model or the Preisach model have been proposed in order to represent the hysteresis effect and extension has been also proposed to take into account the effect of the temperature or of the stress. Experimental setups have been designed in order to characterize the material under different supply conditions and in order to collect data to identify and validate the models [46].

B. Superconductors

Dedicated behavior models of superconductors have been developed based, for example, on extension of the Beam

model [47] or on nonlinear resistivity characteristics in 3-D, characterized by a power-law E-J relation [48][49]. These behavior laws have been implemented into a finite element code in order to compute the current distribution with typical 3-D geometries, such as corner-shaped and twisted superconductors.

C. Smart Materials

The use of rigorous numerical modeling such as the finite element method is essential for the design and optimization of innovative devices involving several physical phenomena of different natures. This is particularly the case of devices involving smart materials. The electromagnetic equations have to be coupled with the equations of other physics like thermal or mechanical ones. This coupling is made by means of the behavior laws linking fields of different physical natures. In [50], the giant magnetostriction in thin films has been modeled by the finite element method where the magnetic and mechanical equations are successively solved by an iterative process. In [51], a model of the magnetic refrigeration based on the giant magnetocaloric effect of the gadolinium have been developed and validated. In [52][53], magnetolectric transducers (position sensing, energy harvesting) made of laminated piezoelectric and magnetostrictive layers have been studied using strongly coupled magnetoelastic and piezoelectric finite element models. Fig. 3 illustrates the modeling example of a magnetolectric displacement sensor.

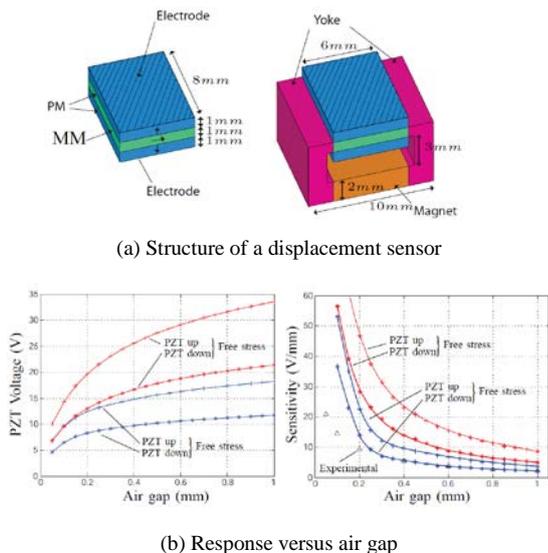


Fig. 3. Modeling of a magnetolectric displacement sensor

D. Complex Materials and Homogenization

Often the electromagnetic phenomena are at the scale of milli or micrometers (FeSi lamination stack, composite materials...) whereas the size of the studied device is of two or three orders higher. The mesh of the whole device accounting for phenomena at the microscale requires a huge number of elements leading to an intractable problem. Homogenization technics, which avoid the meshing at the microscale, is a popular approach to construct solvable FE model of the device. In [54], a new homogenized model based on an effective permeability and conductivity is proposed in order to take into account not only the effect of the eddy currents but also of the net current in a lamination stack. In [55], a multiscale homogenization method is introduced to calculate the field and the current distribution inside carbon fiber reinforced polymer composites submitted to an external electromagnetic field. The modeling of the behavior of

biological tissues have been addressed which is nowadays an important challenge for the community of computational electromagnetics [56].

V. HIGH FREQUENCY PROBLEMS

In the first editions of the conferences, the COMPUMAG community addressed mostly static and low frequency problems. Modeling activities relevant to high frequency fields and wave propagation problems have rapidly grown so that technical sessions devoted to wave problems are included in the conference for more than 20 years. In this domain the French community performed some noticeable works whose examples are listed below.

A. Models

The work of Yee in 1966 made the FDTD (Finite Difference Time Domain) method the most widely used numerical method for antenna and microwave problems. In the French community, the finite element method was progressively adapted from low frequency to high frequency and successively applied to boundary value problems in bounded regions and unbounded regions. In 1990s, solving Maxwell's equations in waveguides and closed cavities with the finite element method is known to face the problem of "spurious modes", unwanted computed numerical solutions that do not correspond to physical fields when using nodal finite elements. The pioneer work of A. Bossavit [57] demonstrated that edge elements are the straightforward remedy to this critical aspect.

For wave propagation in unbounded regions and scattering problems, boundary integral methods are natural approaches [58]. With finite elements, vector absorbing boundary conditions and hybrid approaches (combining finite elements and boundary elements) were proposed before the advent of the well-known PML (Perfectly Matched Layer). Now, in the case of large problems implying for example high ratios between the wavelength and the scattered objects parallel computing is implemented [22].

In the framework of differential forms, it has been shown that numerical schemes for the full Maxwell equations which use two dual meshes (not necessarily tetrahedral), generalize the FDTD Yee-scheme [59]. This step established the theoretical link between the finite integration technique (FIT) and the finite element method and contributed to a better understanding of the underlying concepts relevant to discrete electromagnetism.

More recently discontinuous Galerkin approaches in the case of unstructured tetrahedral meshes were also developed for three dimensional time harmonic problems and suitable domain decompositions allow now to treat large electromagnetic problems [23].

B. Applications

The significant increase of the frequency range concerning different kind of applications in electrical and electronic engineering (communication systems, microwave devices, interconnects, ...) gave rise to a growing impact of numerical modeling relevant to wave propagation. Once edge elements appeared as the solution for avoiding "spurious modes", three dimensional models were shown to provide powerful tools to compute fields in resonating cavities or microwave junctions: design of microwave ovens [60] or microwave filters [61]. The

case of radiating devices in open regions addresses a wide range of applications: ground penetrating radar, design of antennas, electromagnetic compatibility ...

In optics, the study of photonic crystal structures improved the design of new optical fibers and dedicated finite element models were proposed. Reference [62] presents finite element magnetic and electric field models for determining the propagation modes in dielectric waveguides having periodic arrangement of fibers. More generally the introduction of a new class of materials, metamaterials, whose electromagnetic properties depend on their internal structure, and the theoretical properties relevant to invisibility cloaks have driven corresponding modeling techniques. Reference [63] shows, with a finite element model the invisibility of a conducting target in a hollow cylinder coated with a heterogeneous anisotropic material.

C. Electromagnetic Compatibility

Among the various application domains asking for high frequency models, electromagnetic compatibility covers the entire spectrum from statics to microwaves. Prediction of radiated fields from power circuit boards in the near field and determination of mutual coupling between distant components in a complex system cannot be performed easily with standard finite elements approaches. For this reason, specific techniques have been proposed. Reference [64] describes the synthesis of the work achieved during 10 years in the international french-brazilian laboratory Maxwell (LIA in French) about analytical multipolar expansions. This international relationship was clearly strengthened by the Compumag community since 1980. Reference [65] presents a new volume integral formulation to consider resistive, inductive, and capacitive effects in the presence of conductors and dielectrics. This extends the capabilities of the classical partial element equivalent circuit (PEEC) used for several decades in this domain. In parallel with the development of models, efficient tools were also proposed in order to evaluate the sensitivity of the response of a system from the variations of its parameters. In [66] the authors describe a systematic approach of Linear Fractional Transformation derivation for an efficient worst-case analysis of electrical circuits (such as EMC filters). The electrical circuit is assumed to be linear but containing uncertain components parameters of which (nominal and parasites) belong to specified intervals. The main goal is to evaluate a worst-case frequency response envelop of the circuit for all possible parameters combination. The proposed approach is quite original since it allows to apply, in rather straightforward manner, the well-known, in Control community, mu-analysis method based on the efficient, polynomial time LMI optimization (Fig. 4).

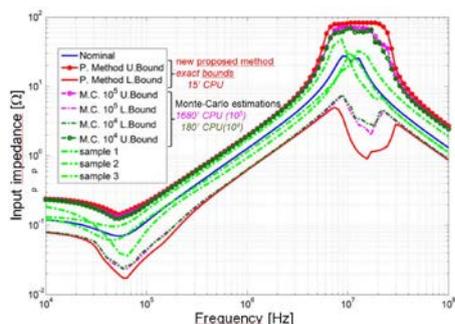


Fig. 4. Bounds computation results for the input impedance of an EMC filter in the range 10kHz-100MHz (cell phone audio amplifier, RLC linear circuit, 26 uncertain parameters)

VI. DESIGN AND OPTIMIZATION

In the domain of the design and optimization of electromagnetic low frequency structures many works have been carried out in France during the last three decades. In this section, we propose a focus on optimization techniques and on the analysis and improvement of electrical structures.

A. Design

Numerical models and optimization techniques have been widely used and combined to design electrical actuators.

An electromagnetic actuator which involves linear and/or rotary movements is designed to reach high performances in terms of dynamic behavior, efficiency and reliability [67]. The actuator is made of two stators and a cylindrical rotor, as shown in Fig. 5.

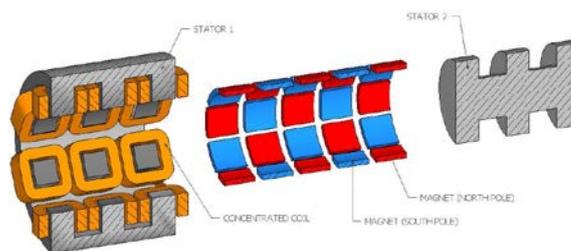


Fig. 5. Linear and/or rotary actuator

One of the issues to deal with in the design of permanent magnet synchronous motors is the torque ripple minimization. One way to achieve this goal is by means of the control of the supply current. This can be done by either changing the instantaneous current waveshape [68] or by adding current harmonics [69]. The finite element model was used to predetermine the current waveforms through the computed electromotive force or torque. The other way to minimize the torque ripple is through the optimization of the air-gap flux density by means of the permanent magnet design [70]. The key idea is to segment the magnet pole into several elementary magnet blocks and to choose either the appropriate elementary block span or the optimal position of the blocks.

The finite element analysis has been applied to evaluate the flux control capability of new double excitation synchronous machines in [71]. The double excitation combines an excitation by a winding and an excitation by permanent-magnets and provides a true field control capability.

Acoustic comfort is an important factor when designing an electrical transport system. In [72], the characterization and reduction of audible magnetic noise due to PWM supply in induction machines is carried out. Experiments are run to measure the sound power level radiated showing the validity of the proposed rule for the reduction of noise due to PWM.

B. Optimization and Inverse Methods

For the optimization, Genetic Algorithms (GA's) are commonly used today, especially niching methods for real problems having multiple optima. The sharing method is probably the best known and also used among niching techniques. In [73] an overview is given on the multiple niching GA's and points out some important issues of multimodal learning such as selection, recombination, restrictive mating, and fitness scaling.

Integrating the finite element method early in the development cycle is the very best way to succeed however it is too time consuming. The surrogate-assisted optimization strategy appears as a promising tradeoff, allowing the integration of high fidelity models, such as FE method, at a reasonable computational cost. Research has been carried out on methods like Efficient Global Optimization (EGO) and their extension to multiple objective optimization [74].

In practical applications of electrical devices, discrete design variables occur naturally in the problem formulation (e.g., the integer variables such as the number of slots; the discrete variables such as structures or materials). Discrete optimization procedures, like the branch-and-bound (BB) method, are generally very costly. This method has been combined with the space mapping technique in order to solve mixed optimization problems [75].

Besides the finite element model, reluctance networks are an interesting alternative for the sizing because they are easy to interpret and provide very short computing times. However, the construction of the model can be a tiresome task and often leads to errors. In [76], this task has been automatized. Starting from a simple and intuitive graphic interface, the model and its partial derivatives are automatically generated. Thus, optimization sizing and sensibility computation can be carried out.

More and more applications nowadays deal with inverse problems to reconstruct quantities of interest not directly measurable. In paper [77] a new approach is proposed to solve the magnetostatic inverse problem. The goal of the work is, from measurements of the magnetic field in the air, to rebuild a model for the magnetization of a ferromagnetic shell structure. This method has been tested on a mock-up with real measurements and led to very promising results.

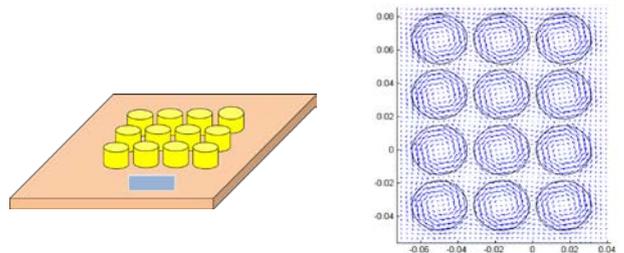
VII. APPLICATIONS

One of the objectives of the COMPUMAG is to discuss developments and practical applications in the numerical computation of electromagnetic fields for engineers and physicists engaged in the design and analysis of applied electromagnetic systems. So that the COMPUMAG decided to promote the topic of novel computational methods for machines and devices. Three kinds of illustrative examples of particular applications are described below.

A. Non-Destructive Testing

The detection and localization of thin defects or cracks in industrial complex structures is the target in many non-destructive experiments. Specific numerical developments need to be performed in order to avoid the difficulties of conventional meshes when using the finite element method.

In the case of conducting materials, eddy currents are widely used in non-destructive testing (NDT). Generally, the ideal crack model assumes that the thickness of the crack is negligible and that it is impenetrable to the current. In such situation reference [78] shows how this ideal model can be generalized to an arrayed eddy current sensor (fig. 6). Generalizing the reciprocity theorem and using axi-symmetry, the impedance variation of each coil of the arrayed sensor is expressed, taking account of the interaction between the coils. In practical NDT such sensor can bring a high speed inspection and a better space resolution by the miniaturization of its coils.



(a). Structure.

(b). Eddy current distribution

Fig. 6. An arrayed eddy current sensor above a target with a crack.

With the induction thermography the effect of the disturbance of eddy current and heat flow due to the presence of a flaw is studied. This technique provides an efficient approach for the analysis of laminated carbon-fiber-reinforced polymer (CFRP) composites. These materials are a stacking of anisotropic unidirectional plies. In [79] degenerated hexahedral Whitney elements are used to model such composites avoiding a meshing the thin regions and consider the anisotropic multilayer characteristic of the materials and also the flaws inside their volume.

B. Devices Involving Motion

Coupled phenomena can be used efficiently to provide motion and propulsion in various electromagnetic systems.

Micro-actuators for example can be realized by magnetostrictive materials in thin films. The thin film is sensitive to the magnetic field. It can be remotely controlled and bend cantilevers or membranes. It may concern micro-pumps or valves for example. In [80] strong coupling formulations for the finite element modeling of magnetostrictive materials have been proposed. They allow accurate modeling of nonlinearity and magneto-mechanical coupling. Two microdevices have been designed and realized demonstrating the validity of the theoretical models.

The propulsion of a small amount of conductive liquid can be obtained thanks to the Lorentz force using a rotating current density and a fixed magnetic field. Reference [81] presents a three dimensional model of a cylindrical tank containing an electrolyte leading to a multi-physics computation: electrical problem governed by Biot and Savart formula and mechanical problem governed by Stokes equation. The modeling of a blob of dye is performed. Such mixing processes are of interest for biomedical applications.

C. Bioelectromagnetics and Biomedical Applications

One of the most exciting studies in applications is the interaction of electromagnetic fields with biological tissues [56]. A first aspect concerns the evaluation of the level of exposure (numerical dosimetry) with 3D modeling approaches in order to comply with international standards. For example, a particular attention is paid for more than twenty years at the determination of the specific absorption rate (SAR) in the human head due to mobile phones and adequate electromagnetic analysis are performed with the well-known FDTD method [82]. The second aspect deals with the use of electromagnetic fields for the benefit of human health. In electroencephalography (EEG) a finite elements method allows a realistic description of a human head and allows to take into account the skull anisotropy on scalp potential distribution [83]. Also hyperthermia with microwaves can be used in oncology treatment to treat localized cancerous

tumours. Now progresses in material modeling and human body description (realistic phantoms) allow addressing crucial challenges for new problems in medicine like wireless charging of implanted devices for example.

VIII. CONCLUSION

The present survey is written in the occasion of the organization of 22nd COMPUMAG conference in Paris, in 2019. The papers cited in this survey are mostly the past work done by French teams who participate actively the COMPUMAG and the CEFC conferences. It is not the authors' intention to give an exhaustive review of the rich activities of French researchers in the domain, nor to predict the future trends. Readers interested in the finite element modeling of electromagnetic problems in France can also refer to the book edited by G. Meunier [84], which collects essentially the contributions of French and Belgian colleagues.

VI. REFERENCES

- [1] J. L. Coulomb, "Finite elements three dimensional magnetic field computation", IEEE Transactions on Magnetics, Vol.17, No.6, 1981, pp. 3241-3246.
- [2] L. Krahenbuhl and A. Nicolas, "Efficient techniques for boundary integral equation methods", IEEE Transactions on magnetics, Vol.19, No.6, 1983, pp.2667-2669.
- [3] A. Bossavit and J. C. Verité, "A mixed fem-biem method to solve 3-D eddy-current problems", IEEE Transactions on Magnetics, Vol.18, No.2, 1982, pp.431-435.
- [4] J.L. Coulomb, Y. Du Terrail and G. Meunier, "FLUX3D: A finite element package for magnetic computation", IEEE Transactions on magnetics, Vol.21, No.6, 1985, pp.2499-2502.
- [5] L. Krahenbuhl, A. Nicolas and L. Nicolas, "A graphic interactive package for boundary integral equations", IEEE Transactions on Magnetics, Vol.21, No.6, 1985, pp.2555 – 2558.
- [6] A. Bossavit and J.C. Verite, "The "TRIFOU" Code: Solving the 3-D eddy-currents problem by using H as state variable", IEEE Transactions on Magnetics, Vol.19, No.6, 1983, pp. 2465-2470.
- [7] Z. Ren, F. Bouillault, A. Razek, A. Bossavit, J.C. Vérité, "A new hybrid model using electric field formulation for 3-D eddy current problems", IEEE Transactions on magnetics, Vol.26, No.2, 1990, pp.470-473.
- [8] T. Henneron, Y. Le Menach, F. Piriou, O. Moreau, S. Clénet, J.-P. Ducreux and J.-C. Vérité, "Source field computation in NDT applications", IEEE transactions on magnetics, Vol.43, No.4, 2007, pp.1785-1788.
- [9] B. Bandelier and F. Rioux-Damidaou, "Mixed finite element method for electromagnetostatics in R^3 ", IEEE transactions on magnetics, Vol.34, No.5, 1998, pp.2473-2476.
- [10] A. Bossavit, "A rationale for 'edge-elements' in 3-D fields computations", IEEE Transactions on Magnetics, Vol. 24, No.1, 1988, pp.74-79.
- [11] Z. Ren, "Influence of the RHS on the convergence behaviour of the curl-curl equation", IEEE transactions on magnetics, Vol.32, No.3, 1996, pp.655-658.
- [12] Y. Le Menach, S. Clénet and F. Piriou, "Determination and utilization of the source field in 3D magnetostatic problems", IEEE transactions on magnetics, Vol.34, No.5, 1998, pp.2509-2512.
- [13] E. Creusé, S. Nicaise, Z Tang, Y. Le Menach, N. Nemitz and F. Piriou, "Residual-based a posteriori estimators for the A- ϕ magnetodynamic harmonic formulation of the Maxwell system", Mathematical Models and Methods in Applied Sciences, Vol.22, Issue 5, 2012, 1150028.
- [14] G. Vinsard, B. Laporte, "A new formulation for induction machine computation", IEEE Transactions on Magnetics, Vol.30, No.5, 1994, pp.3693-3696.
- [15] Y. Marechal, "Some meshless methods for electromagnetic field computations", IEEE Transactions on Electromagnetics, Vol.34, No.5, 1998, pp.3351-3354.
- [16] G. Meunier, O. Chadebec, J.M. Guichon, V. Le-Van, J. Siau, B. Bannwarth and F. Sirois, "A-T Volume Integral Formulations for Solving Electromagnetic Problems in the Frequency Domain", IEEE Transactions on Magnetics, Vol.52, No.3, 2016, 7001404.
- [17] X. Brunotte, G. Meunier and J.F. Imhoff, "Finite element modeling of unbounded problems using transformations: a rigorous, powerful and easy solution", IEEE Transactions on Magnetics, Vol.28, No.2, 1992, pp.1663-1666.
- [18] L. Krähenbühl and D. Muller, "Thin layers in electrical engineering. Example of shell models in analysing eddy-currents by boundary and finite element methods", IEEE Transactions on Magnetics, Vol.29, No.2, 1993, pp.1450-1455.
- [19] C. Guérin, G. Tanneau, G. Meunier, P. Labie, T. Nngueueu and M. Sacotte, "A shell element for computing 3D eddy currents-application to transformers", IEEE Transactions on Magnetics, Vol.31, No. 3, 1994, pp.1360-1363.
- [20] Z. Ren, "Degenerated Whitney prism elements-general nodal and edge shell elements for field computation in thin structures", IEEE Transactions on Magnetics, Vol.34, No.5, 1998, pp. 2547-2550.
- [21] R. Gaignaire, S. Clenet, B. Sudret and O. Moreau, "3-D spectral stochastic finite element method in electromagnetism", IEEE Transactions on Magnetics Vol.43, No.4, 2007, pp 1209-1212.
- [22] B. Butrylo, F. Musy, L. Nicolas, R. Perrussel, R. Scorretti and C. Vollaie, "A survey of parallel solvers for the finite element method in computational electromagnetics", COMPEL: The International Journal for Computation and Mathematics in Electrical and Electronic Engineering, Emerald, 2004, 23 (2), pp.531-546.
- [23] V. Dolean, S. Lanteri and R. Perrussel, "Optimized Schwarz algorithms for solving time-harmonic Maxwell's equations discretized by a discontinuous Galerkin method", IEEE Transactions on Magnetics, Vol.44, No.6, 2008, pp.954-957.
- [24] T. Henneron and S. Clénet, "Model order reduction of non-linear magnetostatic problems based on POD and DEI methods", IEEE Transactions on Magnetics, Vol.50, No.2, 2014, 7000604.
- [25] A. Abdel-Razek, J. Coulomb, M. Feliachi and J. Sabonnadiere, "Conception of an air-gap element for the dynamic analysis of the electromagnetic field in electric machines", IEEE Transactions on Magnetics, Vol.18, No.2, 1982, pp.655-659.
- [26] B. Davat, Z. Ren and M. Lajoie-Mazenc, "The movement in field modeling", IEEE transactions on magnetics, Vol.21, No.6, 1985, pp.2296-2298.
- [27] R. Perrin-Bit and J.L. Coulomb, "A three dimensional finite element mesh connection for problems involving movement", IEEE Transactions on magnetics, Vol.31, No.3, 1995, pp.1920-1923.
- [28] F. Rapetti, F. Bouillault, L. Santandrea, A. Buffa, Y. Maday and A. Razek, "Calculation of eddy currents with edge elements on non-matching grids in moving

- structures”, IEEE transactions on magnetics, Vol.36, No.4, 2000, pp. 1351-1355.
- [29] P. Lombard and G. Meunier, “A general purpose method for electric and magnetic combined problems for 2D, axisymmetric and transient systems”, IEEE Transactions on Magnetism, Vol. 29, No.2, 1993, pp. 1737-1740.
- [30] F. Piriou and A. Razek, “Finite element analysis in electromagnetic systems-accounting for electric circuits”, IEEE Transactions on Magnetism, Vol.29, No.2, 1993, pp.1669-1675.
- [31] N. Sadowski, B. Carly, Y. Lefevre, M. Lajoie-Mazenc and S. Astier, “Finite element simulation of electrical motors fed by current Inverters”, IEEE Transactions on Magnetism, Vol.29, No.2, 1993, pp.1683-1688.
- [32] G. Meunier, Y. Le Floch and C. Guérin, “A nonlinear circuit coupled t-t0-phi formulation for solid conductors”, IEEE Transactions on Magnetism, Vol. 39, No.3, 2003, pp.1729-1732.
- [33] J.L. Coulomb, "A methodology for the determination of global electromechanical quantities from finite analysis and its application to the evaluation of magnetic forces, torques and stiffness", IEEE Transactions on Magnetism, Vol.19, No.6, 1983, pp. 2514-2519.
- [34] Z. Ren and A. Razek, “Local force computation in deformable bodies using edge elements”, IEEE Transactions on Magnetism, Vol.28, No.2, 1992, pp.1212-1215.
- [35] N. Sadowski, Y. Lefevre, M. Lajoie-Mazenc and J. Cros, “Finite element torque calculation in electrical machines while considering the movement”, IEEE Transactions on Magnetism, Vol.28, No.2, 1992, pp.1410-1413.
- [36] Z. Ren, “Comparison of different force calculation methods in 3D finite element modelling”, IEEE Transactions on Magnetism, Vol.30, No.5, 1994, pp.3471-3474
- [37] M. Besbes, Z. Ren and A. Razek, “Finite element analysis of magneto-mechanical coupled phenomena in magnetostrictive materials”, IEEE Transactions on Magnetism, Vol.32, No.3, 1996, pp.1058-1061.
- [38] G. Vinsard, B. Laporte, N. Takorabet and J. P. Brancher, "An analysis of the rotational forces in the secondary of an electromagnetic pump", IEEE Transactions on Magnetism, Vol.34, No.5, 1998, pp.3552-3555.
- [39] A. Meriched, M. Feliachi and H. Mohellebi, "Electromagnetic forming modeling of thin metal sheets", IEEE Transactions on Magnetism, Vol.36, No.4, 2000, pp. 1808-1811.
- [40] M. Feliachi and G. Develey, “Magneto-thermal behavior finite element analysis for ferromagnetic materials in induction heating devices”. IEEE Transactions on Magnetism, Vol.27, No.6, 1991, pp.5235-5237.
- [41] S. Mezani, N. Takorabet and B. Laporte, “A combined electromagnetic and thermal analysis of induction motors”, IEEE Transactions on Magnetism, Vol.41, No.5, 2005, pp.1572-1575.
- [42] M.I. Lamghari-Jamal, E.H. Zaim, J. Fouladgar and D. Trichet, “A magneto-thermal study of a high speed synchronous reluctance machine”, IEEE Transactions on Magnetism, Vol.42, No.4, 2006, pp.1271-1274.
- [43] F. Ossart and G. Meunier, “Comparison between various hysteresis models and experimental data”, IEEE Transactions on Magnetism, Vol.26, No.5, 1990, pp.2837-2839.
- [44] Y. Bernard, E Mendes and F Bouillault, “Dynamic hysteresis modeling based on Preisach model”, IEEE Transactions on Magnetism Vol.38, No.2, 2002, pp.885-888.
- [45] A. Benabou, S. Clénet and F. Piriou, “Comparison of Preisach and Jiles–Atherton models to take into account hysteresis phenomenon for finite element analysis”, Journal of magnetism and magnetic materials 261 (1), 2003, pp.139-160.
- [46] R. Marion, R. Scorretti, N. Siauve, M. A. Raulet and L. Krähenbühl, “Identification of Jiles–Atherton model parameters using particle swarm optimization”, IEEE Transactions on Magnetism, Vol.44, No.6, 2008, pp.894-897.
- [47] M. Maslouh, F. Bouillault, A. Bossavit and J.C. Vérité, “From Bean's model to the HM characteristic of a superconductor: some numerical experiments”, IEEE transactions on applied superconductivity, Vol.7, No.3, 1997, pp.3797-3801.
- [48] F. Grilli, S. Stavrev, Y. Le Floch, M. Costa, E. Vinot, I. Klutsch, G. Meunier, P. Tixador and B. Dutoit, "Finite element method modelling of superconductors: from 2D to 3D", IEEE Transactions on Applied Superconductivity, Vol.15, No.1, 2005, pp. 17-25.
- [49] K. Berger, J. Leveque, D. Netter, B. Douine and A. Rezzoug, “Influence of temperature and / or field dependences of the E–J power law on trapped magnetic field in bulk YBaCuO”, IEEE Transactions on Applied Superconductivity, Vol.17, No.2, 2007, pp.3028-3031.
- [50] M.E.H. Benbouzid, C. Body, G. Reyne and G. Meunier, “Finite element modelling of giant magnetostriction in thin films”, IEEE Transactions on Magnetism, Vol.31, No.6, 1995, pp.3563-3565.
- [51] P. Clot, D. Viallet, F. Allab, A. Kedous-Lebouc, J.M. Fournier, J.P. Yonnet, “A magnet-based device for active magnetic regenerative refrigeration”, IEEE Transactions on Magnetism, Vol.39, No.5, 2003, pp.3349-3351.
- [52] N. Galopin, X. Mininger, F. Bouillault and L. Daniel, “Finite element modeling of magnetoelectric sensors”, IEEE transactions on Magnetism, Vol.44, No.6, 2008, pp.834-837.
- [53] H. Talleb and Z. Ren, “Finite element modeling of magnetoelectric laminate composites in considering nonlinear and load effects for energy harvesting”, Journal of Alloys and Compounds, Vol.615, 2014, pp.65-74.
- [54] L. Krähenbühl, P. Dular, T. Zeidan and F. Buret, “Homogenization of lamination stacks in linear magnetodynamics”, IEEE Transactions on Magnetism, Vol.40, No.2, 2004, pp.912-915.
- [55] G. Wasselynck, D. Trichet, B. Ramdane and J. Fouladgar, “Interaction between electromagnetic field and CFRP materials: a new multi-scale homogenization approach”, IEEE Transactions on Magnetism, vol. 46, No.8, 2010, pp.3277-3280.
- [56] L. Nicolas, N. Burais, F. Buret, O. Fabrègue, L. Krähenbühl, A. Nicolas, R. Scoretti, N. Siauve and C. Vollaire, “Interactions between electromagnetic fields and biological tissues : questions, some answers and future trends”, International COMPUMAG Society Newsletter, 04/09/2003.
- [57] A Bossavit, Computational electromagnetism: variational formulations, complementarity, edge elements, Academic Press, 1998.
- [58] L. Nicolas, “An integral-type approach for the computation of the far field radiated by microwave devices”, IEEE transactions on magnetism, Vol.30, No.5, 1994, pp. 3347-3350.
- [59] A. Bossavit; L. Kettunen, “Yee-like schemes on staggered cellular grids: a synthesis between FIT and FEM approaches”, IEEE Transactions on Magnetism, Vol.36, No.4, 2000, pp.861-867.

- [60] K. Sekkak, L. Pichon and A. Razek, "3-D FEM magneto-thermal analysis in microwave ovens", IEEE transactions on magnetics, Vol.30, No.5, 1994, pp.3347-3350.
- [61] S. Bila, D. Baillargeat, M. Aubourg, S. Verdeyme, F. Seyfert, L. Baratchart, C. Boichon, F. Thevenon, J. Puech, C. Zanchi, L. Lapierre, and J. Sombrin "Finite-element modeling for the design optimization of microwave", IEEE transactions on magnetics, Vol.40, No.2, 2004, pp.1472-1475.
- [62] S. Guenneau, A. Nicolet, F. Zolla, and S. Lasquellec, "Modeling of Photonic Crystal Optical Fibers With Finite Elements", IEEE transactions on magnetics, Vol.38, No.2, 2002, pp. 1261-1264.
- [63] A. Nicolet, F. Zolla and S. Guenneau, "Finite-Element Analysis of Cylindrical Invisibility Cloaks of Elliptical Cross Section", IEEE Transactions on Magnetism, Vol.44, No.6, 2008, pp.1150-1153.
- [64] A. Bréard, O. Chadebec, L. Krähenbühl, C.A.F. Sartori, O. Fabrègue, Z. Li, R.P.B. Muylaert, F. Tavernier, C. Vollaire and D. Voyer, "Overview on the evolution of near magnetic field coupling prediction using equivalent multipole spherical harmonic sources", IEEE Transactions on EMC, Vol.59, No.2, 2017, pp.584-592.
- [65] J. Siau, G. Meunier, O. Chadebec, J.M. Guichon, and R. Perrin-Bit, "Volume integral formulation using face elements for electromagnetic problem considering conductors and dielectrics", IEEE Transactions on EMC, Vol.58, No.5, 2016, pp.1587-1594.
- [66] M. Ferber, A. Korniienko, G. Scorletti, C. Vollaire, F. Morel, L. Krähenbühl and J.A. Vasconcelos, "Systematic LFT derivation of uncertain electrical circuits for worst-case tolerance analysis", IEEE Transactions on EMC, Vol.57, No.5, 2015, pp.937-946.
- [67] G. Krebs, A. Tounzi, B. Pauwels, D. Willemot and F. Piriou, "Modeling of a linear and rotary permanent magnet actuator", IEEE Transactions on Magnetism, Vol.44, No.11, 2008, pp. 4357-4360.
- [68] S. Clenet, Y. Lefevre, N. Sadowski, S. Astier and M. Lajoie-Mazenc, "Compensation of permanent magnet motors torque ripple by means of current supply waveshapes control determined by finite element method", IEEE Transactions on Magnetism, Vol.29, No.2, 1993, pp. 2019-2023.
- [69] F. Colamartino, C. Marchand and A. Razek, "Torque ripple minimization in permanent magnet synchronous servodrive", IEEE Transactions on Energy Conversion, Vol.14, No.3, 1999, pp. 616-621.
- [70] R. Lateb, N. Takorabet and F. Meibody-Tabar, "Effect of magnet segmentation on the cogging torque in surface-mounted permanent-magnet motors", IEEE Transactions on Magnetism, Vol.42, No.3, 2006, pp.442-445.
- [71] Y Amara, S Hlioui, R Belfkira, G Barakat and M Gabsi, "Comparison of open circuit flux control capability of a series double excitation machine and a parallel double excitation machine", IEEE Transactions on Vehicular Technology, Vol.60, No.3, 4194-4207
- [72] J. Le Besnerais, V. Lanfranchi, M. Hecquet and P. Brochet, "Characterization and reduction of audible magnetic noise due to PWM supply in induction machines", IEEE Transactions on Industrial Electronics, Vol.57, No.4, 2010, pp.1288-1295.
- [73] B. Sareni and L. Krähenbühl, "Fitness sharing and niching methods revisited", IEEE Transactions on Evolutionary Computation, Vol.2, No.3, 1998, pp.97-106.
- [74] A. C. Berbecea, S. Kreuawan and F. Gillon, "A parallel multiobjective efficient global optimization: The finite element method in optimal design and model development", IEEE Transactions on Magnetism, Vol.46, No.8, 2010, pp.2868-2871.
- [75] T.V. Tran, S. Brisset and P. Brochet, "A new efficient method for global discrete multilevel optimization combining branch-and-bound and space-mapping", IEEE Transactions on Magnetism, Vol.45, No.3, 2009, pp.1590-1593.
- [76] B. du Peloux, L. Gerbaud, F. Wurtz, V. Leconte and F. Dorschner, "Automatic generation of sizing static models based on reluctance networks for the optimization of electromagnetic devices", IEEE Transactions on Magnetism, Vol. 42, No. 4, 2006, pp.715-718.
- [77] O. Chadebec, J.L. Coulomb, J.P. Bongiraud, G. Cauffet and P. Le Thiec, "Recent improvements for solving inverse magnetostatic problem applied to thin shells", IEEE transactions on magnetics, Vol.38, No.2, 2002, pp.1005-1008.
- [78] A. Zaoui, H. Menana, M. Feliachi and M. Abdellah, "Generalization of the ideal crack model for an arrayed eddy current sensor", IEEE Transactions on Magnetism, Vol.44, No.6, 2008, pp.1638-1641.
- [79] H.K. Bui, G. Wasselynck, D. Trichet and G. Berthiau, "Application of degenerated hexahedral Whitney elements in the modeling of NDT induction thermography of laminated CFRP composite", IEEE Transactions on Magnetism, Vol.52, No.3, 2016, 6200404.
- [80] Ch. Body, G. Reyne and G. Meunier, "Application of magnetostrictive thin films for microdevices", IEEE Transactions on Magnetism, Vol. 33, No.2, 1997, pp.2163-2166.
- [81] G. Vinsard, S. Dufour and E. Saadjan, "Mixing of liquids with a rotating current density", IEEE Transactions on Magnetism, Vol.50, No.2, 2014, pp.273-276.
- [82] A. Hadjem, D. Lautru, C. Dale, M. F. Wong, V. F. Hanna and J. Wiart, "Study of specific absorption rate (SAR) induced in two child head models and in adult heads using mobile phones", IEEE transactions on microwave theory and techniques, Vol.53, No.1, 2005, pp.4-11.
- [83] G Marin, C Guerin, S Baillet, L Garnero and G Meunier, "Influence of skull anisotropy for the forward and inverse problem in EEG: simulation studies using FEM on realistic head model", Human brain mapping 6 (4), 1998, 250-269.
- [84] "The finite element method for electromagnetic modeling", edited by G. Meunier, ISTE, John Wiley & Sons, 2008.

ACKNOWLEDGEMENT

The authors would like to thank the French colleagues of different teams for their help and support by providing relevant references and figures to this survey. The precious advices by F. Bouillault and F. Piriou are also acknowledged.

AUTHORS NAME AND AFFILIATION

Zhuoxiang Ren, L2E, Paris, France
zhuoxiang.ren@sorbonne-universite.fr

Stéphane Clenet, L2EP, Lille, France
stephane.clenet@ensam.eu

Lionel Pichon, GeePs (former LGEP), Paris, France
lionel.pichon@geeeps.centralesupelec.fr

The organization team of COMPUMAG-2019
<http://www.compumag2019.com>