Computational Electromagnetics in Poland: A Review

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Abstract

The paper presents a historical review, the state of the art and recent advances in the field of computational electromagnetics at leading universities and research institutes in Poland. Contributions made by Polish scientists to the development of fundamental electromagnetism, as well as to computational methods, are emphasized, and some conclusions are drawn regarding expected future developments.

1. Introduction

It could be argued that as the theory of electromagnetism was formulated over a hundred years ago by Ampere, Faraday and Maxwell, nothing significant has happened since, and the question arises about the role and direction, and indeed the very definition, of research in electromagnetics. An attempt to answer these questions leads to the following assertions:

- developments in the theory and application of electromagnetic fields are very important research areas in electrical engineering and have great impact on improvements in technology,
- engineers are interested not so much in underlying theory but in the possibility to quantify the results in practical situations,
- the consequence of field computation is an ability to control field distributions to achieve desired technological effects, for example optimal designs.

It is therefore the *computation* of fields which has been the focus of attention for scientists over the years and practising engineers and designers cannot be indifferent to advances made in this field. However, before accepting that progress in field computation can be considered a scientific achievement (rather than engineering development) we must first find the *rationale* behind research in computational methods and the *motivation* of people engaged in the search for new and better techniques. It might be helpful to recall the theory of *scientific revolutions*, defined by Thomas Kuhn [1] as "... those non-cumulative developmental episodes in which an older paradigm is replaced in whole or in part by an incompatible new one". A paradigm is understood here as "... an achievement sufficiently unprecedented to attract an enduring group of adherents away from competing modes of scientific activity. Simultaneously, it is sufficiently open-ended to leave all sorts of problems for the redefined group of practitioners to resolve". Scientific revolutions necessitate the community's rejection of one time-honoured scientific theory in favour of another incompatible with it. Each produces a consequent shift in the problems available for scientific scrutiny and in the standards by which the profession determines what should count as an admissible problem or as a legitimate problem-solution. And each transforms the scientific imagination in ways that we shall ultimately need to describe as a transformation of the world within which scientific work is done. Such changes, together with the controversies that almost inevitably accompany them, are the defining characteristics of scientific revolutions. Scientific revolutions are inaugurated by a growing sense, often restricted to a narrow subdivision of the scientific community, that an existing paradigm has ceased to function adequately in the explanation of an aspect of nature to which that paradigm itself had previously led the way.

Computational electromagnetics appeared on the international scene as a rapidly maturing engineering tool for analysis and design in the early sixties and has undergone changes as a result of emergence of new paradigms. Progress has been fuelled by advances in both hardware and software, as well as by growing demand from potential users, such as designers of electromechanical devices. The historical development of "pre-computational electromagnetics" has been described particularly interestingly by Baldomir and Hammond in their recent book [2], but here we are mainly concerned with methods which started to dominate since the computer era begun. For the last forty years or so the researchers have been searching for methods which are faster, easier to use and require less expert knowledge of the user. This means *increased intellectual input* for software developers to formulate the method and *less actual effort* for users when using it – an underlaying rationale for all further progress.

The first popular general purpose method of field computation was the *finite-difference* technique. Its success was mainly due to mathematical simplicity of formulation (Taylor series) and straightforward physical interpretation. The main drawbacks were lack of flexibility in handling irregular boundaries, cumbersome data input, inefficient meshing based on regular grids and need of special techniques to handle material and interface conditions – all leading to increased effort on the part of the user when trying to solve a practical problem. The paradigm of finite-differences was valid in the sixties and early seventies. A revolutionary change came in the mid seventies (although the first papers were published in the late sixties) with the advent of the *finite element* method, which removed most of the restrictions of the finite difference approach. However, the underlying mathematics was much more demanding and physical concepts more difficult (variational methods, functional analysis, interpolation techniques). Thus the emphasis was shifted from a simple concept but inefficient implementation towards an easy to use system based on a more sophisticated formulation, in agreement with the rationale mentioned in the previous paragraph. In the early eighties this paradigm underwent some modification when the *boundary element* method found its first commercial realization: a further simplification of usage (no need to mesh the interior) at the expense of even more difficult formulation. The two methods increasingly tend to co-exist nowadays and a number of hybrid formulations have been proposed (e.g. finite elements in non-linear media and boundary elements elsewhere). All methods are subject to continuing development and improvement leading to faster and more accurate solutions (e.g. edge elements, hierarchical formulations, adaptive meshing, etc.). All these changes occur in the context of specific industrial and teaching requirements – an in-depth discussion of this environment was recently given by Sykulski and Trowbridge [3].

A new paradigm may be on the horizon as a result of growing interest in *geometry of fields* rather than geometry of objects. This new approach is based on energy methods and uses differential forms as a mathematical tool. The advantages of such description, especially when compared with vector algebra formulation, are discussed in the works by Hammond [2,4]. The best known computational method based on dual energy bound approach is the method of *tubes and slices* [5]. However, a move away from solving partial differential or integral equations towards interrogating directly the field geometry is quite a culture shock, so it may take some time before these new methods are accepted, despite the fact that basic formulations are already in existence.

Before concluding these introductory remarks let us make sure that the term *modelling* is properly understood. We shall adopt the following definition: 'A mathematical model is a set of mathematical relations which describes a physical phenomenon governed by the fundamental and constitutive physical laws, the description being made in a unique, coherent and stable way' [6]. This sets the scene for our review.

2. The state of the art in Computational Electromagnetics

2.1 The world scene

Field computation has undergone enormous developments in the last thirty years, from rather modest beginnings of fairly crude command-driven programs modelling geometrically simple linear problems under static conditions (Laplace's and Poisson's equations) to contemporary general purpose software packages solving 2D and 3D non-linear problems for static, a.c. and transient conditions. Those early programs were mainly written at universities and research institutes, but creation of software has now evolved into an independent branch of industry and is essentially a commercial operation. There is in fact little difference between hardware and software in this respect: both require development, maintenance and support.

The rapid progress in field computation has been propelled by advances in hardware and the ever growing speed and power of computers, but it has also been influenced by significant achievements in basic computational methods. The community of researchers involved in computational electromagnetics is increasing and is now a clearly identifiable group. The *International Compumag Society* was formed in 1993 and is an active forum for collaboration and exchange of information between scientists and engineers working in this area. The regular *Newsletter* [7] brings up-to-date information and publishes review articles about 'hot' technical topics. The Society organizes regular bi-annual conferences *COMPUMAG*, attended typically by 400 participants. COMPUMAG started in Oxford in 1976 and has since travelled the world – most recent meetings were held in Sorrento (Italy, 1991), Miami (USA, 1993), Berlin (Germany, 1995), Rio de Janeiro (Brasil, 1997). The next two meetings are planned in Sapporo (Japan, 1999) and Lyon-Evian (France, 2001). Other significant conferences include *CEFC* (Conference on Electromagnetic Field Computation – the eight biennial meeting will take place in Tucson, Arizona in June 1998) and *INTERMAG*. Most application conferences will usually have a section devoted to field computation, e.g. *ICEM* (International Conference on Electrical Machines). There are also a number of 'peripheral', but extremely useful, smaller conferences, symposia, seminars and workshops, which are more focussed and often address a particular type of problem, e.g. optimization, non-linear modelling, etc.. And last but not least, the activities of *TEAM Workshop* [8] and *ACES* [9] have helped to formulate benchmark problems so that comparisons and validations of new techniques can be made.

It may be useful to list some of the leading groups involved in basic research in computational electromagnetics and software development: C.W. Trowbridge, J. Simkin, C. Emson (formerly Ratherford Appleton Laboratory, now Vector Fields Ltd, Oxford), E. Freeman (Imperial College, London), K. Richter, O. Biro, K. Preis (IGTE, Graz, Austria), D. Lowther, J. Webb (McGill University and Infolytica), A. Razek (LGEP, France), D. Rodger, F. Eastham (University of Bath), Z. Cendes (Ansoft Corporation), T. Nakata, N. Takahashi (Okayama University), G. Molinari, P. Molfino (University of Genova), T. Tsiboukis (University of Thessaloniki), P. Hammond, R. Stoll, J.K. Sykulski (University of Southampton) – there are of course many more and a quick browse through COMPUMAG proceedings or ICS Newsletter will provide fairly comprehensive information. A number of commercial software systems are available: OPERA, MAGNET, MEGA, MAXWELL, FLUX3D, ANSYS, COSMOS, SLIM, BEASY, to name but the few. A short survey of available CAD systems for magnetics may be found in [10].

Achievements in computational electromagnetics to date are significant and may be summarized as follows:

- *Statics*: magnetostatic and electrostatic analysis with non-linear materials, including permanent magnets, special versions for laminated materials using a 'packing factor',
- *Steady-state eddy currents*: steady-state ac eddy-current analysis, including complex permeabilities, approximate non-linear solutions (fundamental harmonic field), background dc fields, external circuits for voltage-driven problems,

- *Transient eddy currents*: full transient eddy-current analysis, with non-linear materials, multiple drives and background dc fields,
- *Motional eddy currents*: uniform motion induced eddy-current analysis (with constant or varying topology),
- *Stress and thermal*: mechanical stress using forces, or thermal analysis using ohmic heating, calculated from electromagnetic solutions,
- 2D, 2D axisymmetric and 3D formulations.
- *Pre- and Post-Processing*: fully interactive with colour graphics, windows environment, sophisticated post-viewing facilities, comprehensive range of supported output devices, automatic and adaptive meshing,
- Implementations on a variety of hardware platforms, including 386/486/Pentium DOS and Windows machines, HP, Sun SPARC, Apollo, DEC and IBM workstations, and Cray supercomputers.

As a result of this progress CAD in magnetics has now become a mature engineering tool and is gradually reaching the design offices in various sections of industry. However, there is still some way to go before practising engineers are going to accept computational electromagnetics as their routine vehicle for designing devices. Every design process is a combination of synthesis, optimisation and experimentation and most of the time it is based on making compromises between conflicting objectives. It is also a multi disciplinary activity where electromagnetic, mechanical, thermal and economic issues all have to be addressed simultaneously. These requirements set the goal posts for researchers and current developments in magnetic CAD are moving in the following directions:

- improved fundamental formulations, such as more efficient *gauge conditions* for eddy-current problems, use of *edge elements*, hierarchical approach,
- the use of differential forms in numerical procedures and exploitation of geometry in electromagnetic systems,
- further developments of integral formulations and enhancements to *boundary element* techniques (including material non-linearity and hybrid finite element / boundary element combinations),
- automatic and adaptive meshing (*h*, *p*, mixed or anisotropic adaption, efficient error estimation),
- optimisation and inverse problem solving,
- geometric modelling,
- modelling of coupled problems, in particular when strong coupling exists between different parts of the system,
- development of models for new types of materials, e.g. high temperature superconductors,
- implementation of finite-element codes on parallel machines,
- improvements to field visualization and postprocessing, in particular for 3D problems,
- development of integrated systems for design,
- introduction of knowledge base systems,
- incorporation of new techniques which proved successful in other areas of physics, such as neural networks, genetic algorithms, fuzzy logic, etc.,

2.2 *Computational Electromagnetics in Poland*

Polish scientists and researchers have always been in the forefront of developments in theory, computation and applications of electromagnetism. The foundations for modern computational electromagnetics were laid in the late sixties and early seventies by Turowski [11], Dabrowski [12] and Sikora [13]. These three professors set themselves a mission of translating the difficult language of mathematics and physics underlying electromagnetic field theory into a practical tool to be used in everyday design tasks in industry. Generations of students in the final years of degree courses in Electrical Engineering Departments in the Universities of Lodz, Poznan and Szczecin had been exposed to quite comprehensive education in applied electromagnetism, later taking their knowledge to the industrial environment and promoting the use of field computation in design. Many former undergraduate and PhD students, who were attracted by the enthusiasm and vision of their mentors, are now university professors themselves or hold responsible positions in industry. Several 'schools of electromagnetics' were established and many of them still continue to thrive in the late nineties.

In the early days of computational electromagnetics in Poland the direction and pace of research were driven mainly by intellectual curiosity of researchers under the guidance of their supervisors. Gaining higher degrees and solving challenging problems were the two main motivating factors. Gradually industrial partners were drawn in as financial sponsors and ultimate users of the results. The research became more focussed and application oriented and at the same time the expected deliverables more tangible. In the early eighties first national programmes were established covering explicitly electromagnetic field modelling. They gave an important boost to researchers and introduced an element of national coordination to the previously rather scattered research programmes. It was almost a matter of prestige, not to mention valuable research income, to be part of these programmes. The three most relevant national initiatives were:

- sub-programme 05.5A, 'Fundamental research in electrotechnics', 1981-1985,
- programme CPBR 5.7, 'Electrical machines and devices', 1986-1987,
- programme RPBP 02.7, 'Theoretical foundations for technological advances in the construction of electrical machines, devices and systems', 1988-1990.

These government programmes were managed and monitored centrally by a team lead by Professor Tadeusz Sliwinski (from the Institute of Electrotechnics in Warsaw), and all development work in the area of electromagnetic fields was coordinated by Professor Janusz Turowski from the Technical University of Lodz. It has to be emphasised that the input made by Professor Turowski into elevating electromagnetics to the position of a nationally supported programme, as well as his personal contribution to the field through numerous books, papers and conference articles, have put him in a rather unique position amongst his contemporaries. Professor Turowski's 70th birthday was celebrated recently during the ISEF Conference in Gdansk, September 1997, as a mark of respect from his students, colleagues and friends for all his achievements for the electromagnetic community.

First seminars and conferences concentrating on the utilization of field computation techniques for industrial purposes were organized by the Universities of Lodz (1973 and 1979), Poznan (1972) and Szczecin (1975). There followed a number of national conferences in the early eighties which were gradually converted into international events and established as regular bi-annual meetings organized by Polish universities but held in various European countries. There are currently three major series of conferences: **ISEF** (International Symposium on Electromagnetic Fields in Electrical Engineering), which were held in Warsaw/Jablonna 1985, Pavia 1987, Lodz 1989, Southampton 1991, Warsaw 1993, Thessaloniki 1995, Gdansk 1997, (the next one will be in Pavia in 1999), and are organized jointly by the Technical University of Lodz and Institute of Electrotechnics in Warsaw; **ISTET** (International Symposium on Theoretical Electrical Engineering), with the following history: Bratislava 1981, Ilmenau 1983, Moscow 1985, Ilmenau 1987, Cottbus 1991, Szczecin 1993, Thessaloniki 1995 and Palermo 1997, co-organized by the University of Szczecin; and **EPNC** (Electromagnetic Phenomena in Nonlinear Circuits), organized by the Poznan University of Technology, held until recently every other year in Poland, but for the next meeting in 1998 (15th in the series) the venue has been selected as Liege, Belgium. Reports from these conferences may be found in the International Compumag Society Newsletters [7], whereas post-conference proceedings and/or selection of the enhanced papers are often published in COMPEL [14].

Polish researchers have always been visible and active at big international conferences such as COMPUMAG, CEFC and INTERMAG. At a recent COMPUMAG conference in Rio de Janeiro in November 1997, 11 Polish participants registered and presented 17 papers. There are 15 members of the International Compumag Society from Poland. In the early nineties close collaboration was established between Polish universities and the electromagnetic community in Japan, sponsored by the Polish and Japanese Societies of Applied Electromagnetism. Regular workshops are organised in both countries and exchanges of research staff take place. Formal links have been established between many Polish universities and partners in other European countries. Particularly valuable in taking computational electromagnetics 'off the ground', and placing local developments in the context of international scene, were study visits made by the younger generation of Polish researchers in the late seventies and early eighties, for example A. Krawczyk in Japan (Okayama and Kanazawa Universities) and UK (University of Southampton), S. Wiak in UK (University of Southampton) and Italy (Pavia University). Other colleagues visited Germany, France, Canada, etc.. More recently European projects such as TEMPUS and COPERNICUS have been valuable in facilitating exchanges of young research staff and promoting academic visits, not to mention the financial support for equipment and software.

A brief scan using computerised literature searching platforms (such as BIDS) reveals a significant number of high quality papers contributed by the Polish electromagnetic community to prestigious learned society and other international journals (Proceedings of IEE, Transactions of IEEE, COMPEL, IJNM, etc.). Indeed, in some areas Polish scientists are in the forefront of worldwide developments. The following brief review is not intended to be an exhaustive list but merely an example: simulation of motion in FE analysis (Demenko and Nowak) [15-17], methods of force calculation (Szymanski) [18], transient fields in ferromagnetics (Wiak) [19-21], leakage field modelling in transformer systems (Zakrzewski) [22, 23], reluctance network methods (Turowski) [24], field synthesis and inverse problems (Sikora and Palka) [25-29], boundary element method (Krawczyk) [30], electrobiological interaction (Krawczyk, Skoczkowski) [31], gauge conditions (Krakowski) [32]. Book co-authorship includes analysis of eddy currents (Krawczyk) [6] and electromagnetic optimisation (Rudnicki) [33]. Several Polish authors (Turowski, Pawluk, Sikora and Zakrzewski) contributed to the recently published monograph reviewing the state of the art in computational magnetics [10].

It would probably be fair to say that during the seventies and eighties a great intellectual effort was made in Poland by the various scientists mentioned above to lay solid foundations for future work in applied electromagnetism. The theoretical and conceptual advancements were of the highest quality. Where progress was much slower was in the area of development of electromagnetic software, which was the result of insufficient funding, lack of necessary computer equipment, but also a reflection of little interest from potential customers. Nevertheless, four particular programs are worth mentioning here. The first two use the finite element formulation: SONMAP written at the Technical University of Szczecin and FAT available from the Technical University of Warsaw. The third suite of programs, called MSR100, 200, etc., is based on the method of reluctance networks and is a product of work done at the Technical University of Lodz. Finally, a program called BEMALL, which uses the boundary element method, was developed by researchers at the Institute of Electrotechnics in Warsaw. There exist also specialised programs for solving particular engineering or design problems (e.g. leakage fields in transformers), but overall there is lack of 'home-made' software to compete with general purpose packages mentioned in Section 2.1.

Economic and political changes in Poland in the late eighties and early nineties have resulted in new organizational structures and new mechanisms for funding government sponsored research. Instead of centrally managed programmes, a system of *research grants* was introduced which are awarded to individual researchers by the *Council for Scientific Research*. Under this new system, which has been in operation since 1991 – and which in essence is similar, for example, to the UK system – individuals compete against each other for a share in limited resources, with decisions made by a specially selected panel. The work currently in progress, as well as grants under consideration, provide a good indicator regarding the direction computational electromagnetics in Poland is taking at the moment. Most current grants (e.g. those undertaken by Professors Turowski, Dabrowski, Pawluk, Wincenciak and Gramz) include modelling of 3-dimensional fields. It appears therefore that there is an intention amongst the most active scientists to 'catch up' in this area where previously there was some deficiency. One of the limitations of more widespread implementation of 3D computation is insufficient access to unix workstations – a vast majority of computer hardware in Poland is PC based. The situation is improving though and at the same time the new generation PCs make the computing intensive 3D calculations quite a practical possibility under the Windows NT or Windows 95 environment. In terms of commercial software, it is the general purpose packages like OPERA, MAGNET, etc., which are becoming common tools at increasing number of Polish universities and research institutes. As mentioned before, European projects TEMPUS and COPERNICUS have made a by no means insignificant contribution towards the improvements in computer hardware and software.

In parallel with developments of software for field computation, a lot of effort is directed to implementation of existing codes to new applications. Here are some examples of new projects:

- field problems in superconducting materials (Institute of Electrotechnics),
- computer tomography (Technical University of Warsaw),
- electrostatic micromotors (Technical University of Lodz), problems in bioelectromagnetism (Technical University of Szczecin, Institute of Electrotechnics).

New programs are expected to start soon, e.g. modelling of electromagnetic fields in semiconductors. Modelling of fields in special materials, such as superconductors, semiconductors or giant or colossal magnetoresistors, is linked with very fundamental research on properties of such materials, hence increasing collaboration with physicists and chemists. Generally, we can see a move away from 'classical' devices (electrical machines and power apparatus) towards more 'peripheral' applications; this stimulates interdisciplinary research and is consistent with similar tendencies in other countries. A particularly interesting area is application of electromagnetic fields in medicine. One such research programme is currently pursued by R. Sikora and his team on impedance and eddy-current tomography, whereas A. Krawczyk and S. Wiak are involved in a project on the use of electromagnetic fields in psychiatric therapy [34].

There is a growing interest worldwide in the topics of *electromagnetic compatibility* and the computational electromagnetics community has become very active in this field, which is going well beyond the 'traditional' areas of electrotechnics. The work has direct impact on environmental issues and is therefore very topical (one obvious benefit of getting involved in such issues is access to significant funding opportunities). Electromagnetic compatibility has now been included in syllabuses of Electrical Engineering courses at Polish universities and there is a growing number of conferences and seminars on this topic.

It appears therefore that the Polish electromagnetic community needs to make some fundamental decisions whether to invest significant time and effort to pursue development of software, or concentrate on implementation of readily available general purpose software packages, mainly imported, to apply modern field computational techniques in studies of new devices, materials and phenomena. These problems are not unique and scientists in many countries must be facing a similar dilemma – limited resources often require solutions based on a compromise.

3. Conclusions

It is hoped that the discussion above is a fair reflection of the historical development and state of the art of computational electromagnetics in Poland. It is not the intention of this article to provide advice or make any projections about future developments; nevertheless, in order to focus attention, the authors of this paper wish to make the following general points:

- Polish scientists have made, and continue to provide, a valuable contribution to the development of applied electromagnetism, in particular to its fundamental theory, new concepts and novel applications,
- in-house software is mainly directed at solving particular engineering or design problems, but cannot compete with well established general purpose packages available on the world market; it is encouraging, however, to see such packages installed at an increasing number of university and research institutes – it is also hoped that the next stage will see the packages in design offices in Polish industrial establishments,
- continuing support given by the Council for Scientific Research for studies in applied electromagnetism is crucial to maintain the momentum and underpin future work; the current level of support and coverage of topics appear to be appropriate and consistent with tendencies elsewhere in the world,
- it seems sensible to encourage the researchers to seek such directions where the emphasis is on intellectual effort rather than number-crunching capabilities,
- the computational electromagnetics community should be more pro-active in establishing closer collaboration with physicists and material engineers, in order to bring the language of field computation nearer the physical description of the underlying processes, and to secure research funding from outside the traditional sources, such as the electrical power industry,

- there should be more effort directed to electromagnetic compatibility, with the purpose of integrating field computation methods with environmental issues and thus benefiting from financial and other resources already available under such headings.

It would probably be appropriate to finish off this summary with an appeal to those members of the scientific community in Poland who have influence on the decision making processes to address the above issues as a matter of some urgency to see that computational electromagnetics in Poland continues to prosper.

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