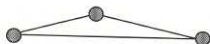


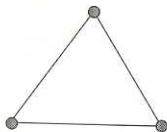
# How Flat Are Flat Elements?\*

## 1. Flat Elements and What They Can Do for You\*

Anyone who ever used the Finite Element Method (FEM) knows that a triangular element like this one



is "bad", while an element like this one



is "good". The 'flatness' of the first element should presumably result in poor accuracy of the numerical solution.

But how flat are flat elements? What is the precise criterion characterizing the element shape in FEM? This question is especially obscure in three dimensions, in particular for tetrahedral elements.

In 2D for triangular elements, one can claim that small angles *should*\* be avoided. The mathematical basis for that is given by Zlámal's minimum angle condition: if the minimum angle of elements is bounded away from zero,  $\theta_{\min} \geq \theta_0 > 0$ , then the FE interpolation error does tend to zero for the family of meshes with decreasing mesh sizes<sup>1</sup>. Geometrically equivalent to Zlámal's condition is boundedness of the ratio of the maximum element edge  $l_{\max}$  to the radius  $\rho_i$  of the inscribed circle (Fig. 1).

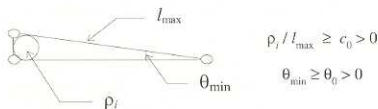


Fig. 1. The minimum angle condition and the equivalent  $\rho_i / l_{\max}$  condition.

Zlámal's condition implies that small angles *should* be avoided. But *must* they? This is not a question of semantics. In mathematical terms, one may wonder if Zlámal's condition is not only sufficient but in some sense necessary for accurate approximation.

\* Disclaimer: Any resemblance to the titles of other papers (ref. [\*, \*\*] at the end of the article) is purely coincidental.

<sup>1</sup> For simplicity and to emphasize the main ideas, throughout the paper we omit precise mathematical conditions on the smoothness of approximated functions.

Consider a right triangle with a small acute angle:



If Zlámal's condition were necessary, such an element would have to be rejected. However, on a regular mesh with right triangles the FE discretization of Laplace's equation is known to be no different than the standard Finite Difference scheme. But the FD scheme does not have any shape related approximation problems. (The accuracy is limited by the maximum mesh size but not by the aspect ratio<sup>2</sup>.) This observation suggests that Zlámal's condition could be too restrictive.

And indeed, a less restrictive shape condition for triangular elements exists. It is sufficient to require that the *maximum* angle of an element be bounded away from  $\pi$  (Fig. 2). In particular, according to this condition, right triangles, even with very small acute angles, are acceptable (what matters is that the *maximum* angle is  $\pi/2$ ).

The maximum angle condition appeared in the monograph by Syngé [4] in 1957, well before the finite element era. (He considered a piecewise-linear interpolation on triangles without calling them finite elements.) In 1976, Babuška and Aziz [1] published a more detailed analysis of FE interpolation on triangles and proved that the maximum angle condition was not only sufficient, but in a sense essential for the convergence of FEM.

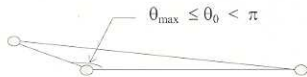
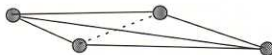
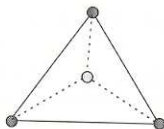


Fig. 2. Syngé-Babuška-Aziz "maximum angle" condition.

This clarifies the situation for 2D triangular elements but sheds little light on the 3D case. Of the two tetrahedral elements shown below the first one appears to be 'good' and the second one 'bad', but how can this be accurately quantified?



<sup>2</sup> The influence of the aspect ratio on the condition number of the matrix and consequently on the convergence rate of iterative solvers is a different matter.

One of the most common measures is the ratio of the element diameter (i.e. the maximum edge) to the radius of the inscribed sphere. Boundedness of this ratio is known to be sufficient for convergence of FE interpolation on a family of tetrahedral meshes with decreasing mesh sizes. However, as in the 2D case, such a condition is a little too restrictive: 'right tetrahedra' (having three mutually orthogonal edges) are rejected, even though it is intuitively felt (by analogy with right triangles) that there is really nothing wrong with them.

A precise characterization of the shape of tetrahedral elements is one of the particular results of the general analysis that follows. First, we will find an *algebraic* source of interpolation errors for arbitrary finite elements, and then will give its *geometric* interpretation for triangular and tetrahedral elements.

## 2. The Algebraic Mechanism of Interpolation Errors

Even though the approach presented in this section is general, for simplicity and to clarify the main ideas we consider the 2D case, and even more specifically – first order triangular elements.

First, we need to introduce some notation. The tilde sign ( $\sim$ ) will be used to denote FE functions in the (finite dimensional) space spanned by the FE basis functions (e.g. piecewise-linear potentials on a tetrahedral mesh). If  $\tilde{w}$  is such a function, then  $w$  (without the tilde) will denote the Euclidean vector of the degrees of freedom for  $\tilde{w}$  (e.g. the vector of nodal values). Superscript  $(e)$  will stand for quantities related or restricted to a particular finite element  $(e)$ . Interpolation errors will be measured in the energy norm (or seminorm)  $\|\cdot\|_E$  induced by a certain (semi)coercive bilinear form  $a(\cdot, \cdot)$ :

$$\|w\|_E^2 = a(w, w) \quad (1)$$

The most common example is the energy norm proportional to the field energy for the scalar potential in a computational domain  $\Omega$ :

$$\|w\|_E = \left[ \int_{\Omega} |\nabla w|^2 dS \right]^{1/2} \quad (2)$$

$$\text{with} \quad a(w, u) = \int_{\Omega} \nabla w \cdot \nabla u dS$$

Then, by the  $(e)$ -superscript convention, the energy over a particular element is denoted as

$$\|w\|_E^{(e)} = \left[ \int_{(e)} |\nabla w|^2 dS \right]^{1/2} \quad (3)$$

Let  $\tilde{u}_I$  be the standard piecewise-linear interpolating function for a smooth potential  $u_*$ . Essential in the analysis will also be the Taylor approximation  $\tilde{u}_T$  of  $u_*$  around an arbitrary point  $r_0 = (x_0, y_0)$  in an element  $(e)$ :

$$\tilde{u}_T(r) = u_*(r_0) + \nabla u_*(r_0) \cdot (r - r_0), \quad r_0 \in (e)$$

One key observation is that  $\tilde{u}_T$  approximates  $u_*$  uniformly within a (sufficiently small) circle; in other words, the approximation error is completely independent of the element shape (Fig. 3).

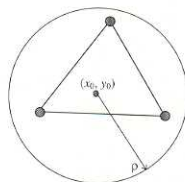


Fig. 3. The Taylor approximation is uniform in a circle.

This makes  $\tilde{u}_T$  an ideal reference for the analysis of shape-related errors. More precisely,

$$|\tilde{u}_T - u_*| \leq c\rho^2, \quad |\nabla \tilde{u}_T - \nabla u_*| \leq c\rho \quad \text{if } |r - r_0| < \rho_0$$

Measured in the energy (seminorm), the Taylor approximation error is also shape-independent:

$$\|\tilde{u}_T - u_*\|_E^{2,(e)} = \int |\nabla \tilde{u}_T - \nabla u_*|^2 dS \leq c h^{(e)2} S^{(e)} \quad (4)$$

where  $S^{(e)}$  is the element area and  $h^{(e)}$  is the maximum edge.

Another key observation: although both  $\tilde{u}_T$  and  $\tilde{u}_I$  are linear approximations of  $u_*$ , the approximation error by  $\tilde{u}_T$  does *not* depend on the element shape, whereas the approximation error by  $\tilde{u}_I$  does.

This somewhat paradoxical difference between  $\tilde{u}_T$  and  $\tilde{u}_I$  is worth a closer look. The *node values* of  $u_*$  are approximated quite well by both  $\tilde{u}_T$  and  $\tilde{u}_I$ : in fact, the node values of the interpolant  $\tilde{u}_I$  are exact by definition, while the difference between the node values of  $\tilde{u}_T$  and  $u_*$  is of the order of  $O(h^{(e)2})$  and shape-independent. This is illustrated in Fig. 4 where the 1D case is shown for clarity.

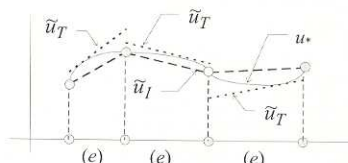


Fig. 4. The exact solution  $u_*$  (solid line) is approximated by its piecewise-linear node interpolant  $\tilde{u}_I$  (dashed line) and by element-wise Taylor approximations (dotted lines). The energy norm difference between  $\tilde{u}_I$  and  $\tilde{u}_T$  is generally much greater than the difference in the node values.

The difference between the *node values* of  $\tilde{u}_T$  and  $\tilde{u}_I$  is "small" ( $O(h^{(e)2})$ ) and shape-independent. At the same time, the difference between  $\tilde{u}_T$  and  $\tilde{u}_I$  in the *energy norm* is generally much greater: not only is the order of approximation lower ( $O(h^{(e)})$ ), but the error can be adversely affected by the element shape.

Obviously, somewhere in the transition from the node values to the energy norm the precision is lost. The node values (of a linear function over a triangular element) and the energy are connected via the element stiffness matrix  $A^{(e)}$ :

$$\|\tilde{u}_I^{(e)} - \tilde{u}_T^{(e)}\|_E^2 = (A^{(e)}(u_I^{(e)} - u_T^{(e)}), u_I^{(e)} - u_T^{(e)}) \quad (5)$$

Since

$$\lambda_{\max}(A^{(e)}) = \max_{u \neq 0} \frac{(A^{(e)}u, u)}{\|u\|_2^2}, \quad (6)$$

the maximum eigenvalue of the element stiffness matrix controls the approximation error (5):

$$\|\tilde{u}_I^{(e)} - \tilde{u}_T^{(e)}\|_E^2 \leq \lambda_{\max}(A^{(e)}) \|u_I^{(e)} - u_T^{(e)}\|_2^2 \quad (7)$$

A more detailed analysis yields the following estimate [5]:

$$\|\tilde{u}_I^{(e)} - u_s^{(e)}\|_E^2 \leq c(\lambda_{\max}(A^{(e)}) h^{(e)4} + h^{(e)2} S^{(e)}) \quad (8)$$

The global error over the whole computational domain can be obtained by simple summation of element-wise estimates (8):

$$\|\tilde{u}_I - u_s\|_E^2 \leq c \sum_{(e) \in \text{mesh}} (\lambda_{\max}(A^{(e)}) h^{(e)4} + S^{(e)} h^{(e)2}) \quad (9)$$

This can be simplified to

$$\|\tilde{u}_I - u_s\|_E \leq c(h_{\max}^2 \sqrt{\text{tr } A} + h_{\max} \sqrt{S}) \quad (10)$$

where  $S$  is the area of the whole computational domain.<sup>3</sup>

Alternatively, introducing the scaled element stiffness matrix

$$\hat{A}^{(e)} \equiv A^{(e)} / S^{(e)}, \text{ one can simplify (9) to yield} \quad (11)$$

$$\|\tilde{u}_I - u_s\|_E^2 \leq c \sum_{(e)} (\lambda_{\max}(\hat{A}^{(e)}) h^{(e)4} + h^{(e)2} S^{(e)})$$

or

$$\|\tilde{u}_I - u_s\|_E \leq c \sqrt{S} \max_{(e) \in \text{mesh}} (h^{(e)2} \sqrt{\lambda_{\max}(\hat{A}^{(e)})} + h^{(e)}) \quad (12)$$

(The maximum eigenvalue can again be replaced with the matrix trace).

So far only first order triangular elements have been considered for simplicity. However, the derivation is fairly general and can be applied to any finite elements satisfying several not too restrictive assumptions [5]. The bottom line remains the same: the maximum eigenvalue of the FE stiffness matrix governs the interpolation error. The relevant estimates are, up to higher order terms [5],

$$\|\tilde{u}_I - u_s\|_E \leq c \sqrt{S} \max_{(e) \in \text{mesh}} h^{(e)\kappa} \sqrt{\lambda_{\max}(\hat{A}^{(e)})} \quad (13)$$

or, alternatively,

$$\|\tilde{u}_I - u_s\|_E \leq c h_{\max}^{\kappa} \sqrt{\text{tr } A} \quad (14)$$

where  $\kappa$  is the order of Taylor approximation of the FE degrees of freedom. (For example, nodal values are approximated by a linear Taylor function with  $\kappa=2$ .) Obviously, in the 3D case areas should be replaced with volumes in (13) and similar formulas.

It is perhaps surprising that as simple a thing as the trace of the global stiffness matrix  $A$  is an indicator of the FE interpolation error (14).

### 3. Geometric Interpretation for Triangular and Tetrahedral Elements

The algebraic estimates of the previous section can be implemented in the overall FE procedure in a straightforward way and with negligible overhead. However, for some common types of elements, such as first order triangles or tetrahedra, it is worthwhile interpreting these estimates geometrically.

In all cases that follow, it can easily be verified that the assumptions used to derive estimates (13), (14) are valid.

#### A. $L_2$ -Approximation on Tetrahedral Node Elements

Suppose that the potential  $u$ , itself, rather than the field  $\nabla u$ , is approximated, i.e. the  $L_2$ -norm is of interest:

$$\|u\|_E = \left[ \int_{\Omega} u^2 dS \right]^{1/2} \quad (15)$$

This norm in the FE space is induced by the "mass matrix"

$$A_{ij}^{(e)} = \int_{(e)} \phi_i \phi_j dS$$

where  $\phi$ 's are the standard first order nodal basis functions.

The scaled element stiffness matrix is

$$\hat{A}_{ij}^{(e)} = \frac{1}{S^{(e)}} \int_{(e)} \phi_i \phi_j dS = \frac{1}{12} \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \quad (16)$$

The maximum eigenvalue of the scaled matrix,

$$\lambda_{\max}(\hat{A}^{(e)}) = \frac{1}{3}, \text{ does not depend on the element}$$

shape. Therefore, according to (13), approximation of the potential is also shape-independent. This known result is obtained here directly from the general eigenvalue criterion.

#### B. First Order Triangular Node Elements

If the interpolation error is measured in the standard energy (semi)norm (2), the maximum eigenvalue of the scaled stiffness matrix can be evaluated via the matrix trace as

$$\lambda_{\max}(\hat{A}^{(e)}) = O(\sin^2 \theta_{\min})$$

Zlámal's minimum angle condition follows directly from this result.

<sup>3</sup> Symbol  $c$  is a generic constant not necessarily the same in different expressions.

### C. First Order Tetrahedral Node Elements

For tetrahedral elements, the trace of the scaled nodal stiffness matrix can be interpreted geometrically (Fig. 5) and evaluated as [5]

$$\lambda_{\max}(\hat{A}^{(e)}) \leq \mu(\hat{A}^{(e)}) = \sum_{i=1}^4 1/d_i^2$$

$$\leq 2(1/h^{(e)})^2 \sum_{i \neq j} \sin^{-2} \alpha_{ij} \quad (17)$$

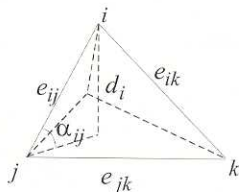


Fig. 5. Geometric parameters of a tetrahedral element.

Formulas (13), (17) lead to the *minimum-maximum angle* condition for the angles  $\alpha_{ij}$  between edges and faces:

$$0 < \alpha_{ij} \leq \alpha_{ij} < \alpha^* < \pi$$

This result is quite sensible, but we can do better. A more precise characterization of tetrahedral shape can be obtained by using *edge* elements as outlined in the following section.

### D. Triangular and Tetrahedral Edge Elements

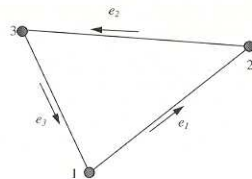
Conservative fields on triangles or tetrahedra can be represented by the first order nodal approximation of the potential or, alternatively, by Whitney-Nedelec edge elements [2, 3]. It is well known that these two representations of a conservative field are equivalent [2, 3]. However, the maximum eigenvalue condition not only is not guaranteed to give identical results for these two types of approximation, but in fact leads to a more accurate estimate for the edge element interpolation of the field than for the nodal element interpolation of the scalar potential.

For simplicity, let us focus again on triangular elements (the main ideas and results will remain valid for tetrahedra). The degrees of freedom for edge elements will be chosen as the tangential components of the field at edge midpoints (not edge circulations).

Note that the Whitney element interpolant of a conservative field  $H$  is simply constant within the element (and equal to the gradient of a linear potential within the element). This field can be represented as a Cartesian vector  $H_{xy} \in \mathbb{R}^2$  or, alternatively, as a vector  $H_{edge} \in \mathbb{R}^3$  comprising the three tangential components of  $H$  at the edge midpoints.

To relate the Cartesian and edge representations of a vector, one can introduce three unit vectors  $e_1, e_2, e_3$  directed along the element edges, and a  $2 \times 3$  'element edge shape matrix'

$$E = [e_1 \mid e_2 \mid e_3] = \begin{bmatrix} e_{1x} & e_{2x} & e_{3x} \\ e_{1y} & e_{2y} & e_{3y} \end{bmatrix} \quad (18)$$



Then

$$H_{edge} = E^T H_{xy} \quad (19)$$

According to the maximum eigenvalue criterion, the interpolation error is governed by the maximum eigenvalue of the scaled element stiffness matrix in the subspace of conservative fields<sup>4</sup>:

$$\lambda_{\max}(\hat{A}^{(e)})_{H_{xy}=\nabla \bar{u}} =$$

$$\max_{H_{xy} \in \mathbb{E}^2} \left( \hat{A}^{(e)} H_{edge}, H_{edge} \right)_{E^3} / \left( H_{edge}, H_{edge} \right)_{E^3}$$

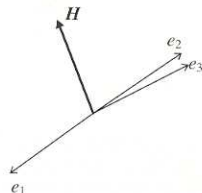
$$= \max_{H_{xy} \in \mathbb{E}^2} \frac{\frac{1}{S^{(e)}} \int_{(e)} \tilde{H}^2 dS}{\left\| H_{edge} \right\|_{E^3}^2} = \max_{H_{xy} \in \mathbb{E}^2} \frac{\left\| H_{xy} \right\|_{E^2}^2}{\left\| H_{edge} \right\|_{E^3}^2} \quad (20)$$

It follows from (19) (see [6]) that the maximum ratio in (20) is directly related to the *minimum singular value*  $\sigma_{\min}(E)$  of the edge shape matrix  $E$ :

$$\lambda_{\max}(\hat{A}^{(e)})_{H_{xy}=\nabla \bar{u}} = \sigma_{\min}^{-2}(E) \quad (21)$$

This result has the following geometric interpretation.

Suppose that the unit vectors  $e_1, e_2, e_3$  are 'almost collinear'.



On the one hand, this implies that the corresponding triangular element is 'flat'. On the other hand, it also implies that there is a vector  $H$  that is almost orthogonal to all three unit vectors  $e_1, e_2, e_3$ , i.e. has very small projections on the edges  $e_1, e_2, e_3$ . Suppose further that such an  $H$  happens to represent the interpolation error (more precisely, the

<sup>4</sup> With some abuse of notation. The 'maximum eigenvalue in the subspace' is understood as the maximum ratio (6) in the subspace rather than the parameter in  $Ax = \lambda x$ .



difference of  $\nabla \tilde{u}_T$  and  $\nabla \tilde{u}_j$ ; then the interpolation error may be large despite having very small edge components.

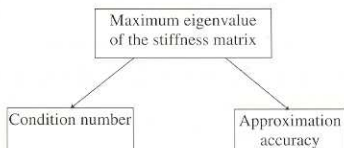
This clearly shows a connection between the 'flatness' of an element and interpolation errors. Mathematically, the minimum singular value of the edge shape matrix provides simultaneously a measure of the flatness and a measure of FE interpolation errors. The final error estimate obtained in [6] is as follows:

$$\|\tilde{u}_T - u\|_E^2 \equiv \|\tilde{H}_T - H\|_{(L_2)^3}^2 \leq \sum_{(e)} h^{(e)2} \sigma_{\min}^{-2}(E) S^{(e)} \quad (22)$$

Numerical experiments supporting the theoretical results of this section and of the previous sections are described in references [5-7].

#### 4. Approximation and the Condition Number

The interpolation error is related to the maximum eigenvalue of the global stiffness matrix [5]; therefore it is also related to the condition number of the same matrix. (Note, however, that the minimum eigenvalue has no bearing on interpolation accuracy and can be viewed as a fixed parameter associated with the size of the computational domain.) On the basis of experimental data, Zgainski *et al.* [8] proposed to use the condition number as a measure of 'mesh quality'. We can see, however, that the maximum eigenvalue or the trace of the stiffness matrix are in fact better measures, since the minimum eigenvalue is irrelevant. The following simple chart clarifies the point:



#### 5. Discussion

What has been accomplished? First, we have a general estimate of the FE interpolation error via the maximum eigenvalue of the FE stiffness matrix. When this estimate is applied to triangular and tetrahedral elements, several known results and several new results are obtained. For triangular elements in particular, Zlámal's minimum angle condition and the Sygne-Babuška-Aziz maximum angle condition are recovered.

For tetrahedral elements, the maximum eigenvalue estimate leads to a new criterion. The shape of tetrahedral elements turns out to be most accurately represented (in the FE context) by the minimum singular value of the 'edge shape matrix'. This singular value characterizes, on the one hand, the flatness of the element and, on the other hand, the

accuracy of the FE interpolation. There are several links between the minimum singular value and some geometric parameters of the tetrahedron (see [6]), but the minimum singular value remains the most precise of all measures.

Due to its generality, the maximum eigenvalue condition can be applied in cases where no other shape criteria are immediately available. For example, anisotropy of material parameters should result, intuitively, in some 'scaling' of the coordinate axes before any geometric accuracy criteria can be considered. In contrast, the maximum eigenvalue criterion takes care of the anisotropy automatically, since material parameters are built into the stiffness matrix. Of course, the new criterion can also be applied to elements other than first order triangles and tetrahedra.

The method is not without limitations, however. First, it provides *a priori* estimates only; it remains to be seen whether similar ideas can be used to enhance *a posteriori* estimates (critical for adaptive mesh refinement).

Second, the maximum eigenvalue criterion is a *sufficient* but not generally a *necessary* condition; it does not guarantee the best error estimate. This is well illustrated by two cases considered above:

- (a) for conservative fields on Whitney edge elements, the result (expressed via the minimum singular value of the edge shape matrix) does appear to be optimal;
- (b) at the same time, for triangular node elements the maximum eigenvalue criterion leads to Zlámal's minimum angle condition rather than to the more accurate Sygne-Babuška-Aziz maximum angle condition.

In summary, the theoretical results provide a general and easy-to-implement *a priori* criterion of FE accuracy. The computational overhead in the overall FE procedure is negligible. For tetrahedral elements in particular, a precise characterization of shape via the minimum singular value of the element 'edge shape matrix' can be strongly recommended for the engineering practice.

Some people believe that deriving these results was worth the time.<sup>5</sup>

#### Acknowledgment

This work would have been impossible without extensive discussions with Drs. Alain Bossavit and Pierre Asselin. I also thank Professor Ivo Babuška for a recent discussion.

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Igor Tsukerman

## 8th International IGTE Symposium on Numerical Field Calculation in Electrical Engineering and TEAM Workshop

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 Problem 27: Modelling and optimization of NDT problem  
 Problem 28: Levitation problem

Most of the descriptions of the open problems can be found on the WWW via <http://ics.ascn3.uakron.edu> or will be sent to you on request.

Submission of abstracts and communication as well as correspondence regarding the Symposium and the Workshop should be addressed to:

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#### Organising Committee:

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## IEEE CEFC'98

*The Eighth Biennial IEEE Conference on Electromagnetic Field Computation*

*June 1-3, 1998, Tucson, Arizona, USA*

Sponsored by  
 IEEE Magnetic Society  
 IEEE Antenna & Propagation Society  
 IEEE Microwave Theory and Techniques Society

The Eight Biennial IEEE Conference on Electromagnetic Field Computation (CEFC) will be held at the Westin La Paloma Hotel and Resort, Tucson, Arizona, USA, during June 1-3, 1998. The last Conference was held in Okayama, Japan in 1996.

The aims of the IEEE CEFC are to present the latest developments in modelling and simulation methodologies for analysis of electromagnetic field and wave interactions, with the application emphasis being on the computer-aided design of low and high frequency devices, components and systems. Scientists and engineers worldwide are invited to submit original contributions in the areas of static and quasi-static fields; wave propagation; material modelling; coupled problems; optimization; numerical techniques; software methodology; applications of electromagnetic CAD to electrical/electronic devices; component and system prototyping. The Conference will feature oral and poster presentations.

#### Keyword list for CEFC'98

- 1. Static and quasi-static fields**  
 (a) electrostatics, (b) magnetostatics, (c) eddy currents, (d) numerical methods, (e) others
- 2. Wave propagation**  
 (a) scattering, (b) radiation, (c) time and frequency domain, (d) microwaves, (e) antennas, (f) numerical methods, (g) others
- 3. Material modelling**  
 (a) superconducting materials, (b) composite materials, (c) hysteresis and anisotropy, (d) permanent magnets, (e) magnetostrictive or

piezoelectric materials, (f) microwave absorbing materials, (g) others

- 4. Coupled problems (electromagnetic field problems coupled to:)**  
 (a) mechanical problems, (b) electric circuits, (c) thermal problems, (d) others
- 5. Numerical techniques**  
 (a) mesh generation and adaptive meshing, (b) solving linear systems of equations, (c) eigenvalue problems, (d) nonlinear problems, (e) parallel and vector computations, (f) others
- 6. Optimization and design**  
 (a) sensitivity analysis, (b) deterministic methods, (c) stochastic methods, (d) neural networks, (e) artificial intelligence and expert systems, (f) others
- 7. Software methodology**  
 (a) software design, (b) software engineering and software quality, (c) computer graphics and data representation, (d) man-machine interfaces, (e) Computer Aided Engineering in classroom, (f) others
- 8. Devices and applications**  
 (a) electric machines and drives, (b) nondestructive testing, (c) induction heating, (d) power electronics devices, (e) wave guides, (f) microwaves resonators, (g) magnetic recording, (h) microsystems, (i) biomedical applications, (j) charged particles trajectories, (k) accelerators, (l) electromagnetic launchers, (m) fusion machines, (n) electromagnetic compatibility, (o) others