Stochastic Design in Applied Electro-magnetics ...The Genetic Algorithms Approach and System Optimization Strategies

The need to solve electromagnetic problems accurately and efficiently for large-scale complex problems has caused the emergence of computational electromagnetics. The methods developed, such as the finite difference method, the method of moments, the finite element method, and the boundary element method, have become increasingly sophisticated with ever increasing computational power. These numerical techniques are used by researchers as indispensable tools to compute the field parameters and predict the performance of devices in all areas of electromagnetics. These applications range from two-dimensional electrostatics to nonlinear eddy current calculations in three dimensions and to high frequency applications. The tremendous advancements in computer technology make it possible to enhance these analysis tools with state of the art modelling and post-processing capabilities to offer the researcher or the engineer everything that is necessary to analyze an electromagnetic problem in a user-friendly package.

Many researchers and engineers, however, seldom need to perform analyses alone. Their major task is to come up with the best solution for a problem. In other words, engineers need a tool that can help them synthesize the best solution for the problem in their hands. The place of an analysis package in this task description is to provide the solution and the performance measure of the current design, which help the researcher navigate in search of the optimum. With the definition of this new need, computational electromagnetics adopts another field, design optimization. Using the already perfected analysis modules in coordination with a search method, design optimization in computational electromagnetics seeks the optimum solution for a specified problem. This problem has an objective function and constraints. Design optimization's task is to maximize (minimize) the objective function while not violating the constraints. The objective function $f(x_i)$, is a function of the design variables, x_i. The number of design variables defines the dimension of the search space. There is a set of constraints that govern the domain of the design variables. Each constraint can be written as an inequality; $p_j(x_i) \leq 0$. The optimization is formulated as:

$$\text{Maximize } f(x_i) \text{ (objective function)}$$
$$\text{Subject to } p_j(x_i) \leq 0 \text{ (constraints)}$$

Optimization is not a new concept. Problems dealing with the optimization of parameters are seen in every field. Mathematical optimization can be dated to as early as the mid 1700s when Euler developed the calculus of variations. Many other mathematical optimization techniques are derived from the calculus of variations. In the last 20-30 years, progress has been made in solving inverse problems largely due to the mathematical regularization theory developed by Soviet scientists [1].

In electromagnetics, numerical techniques combined with the available computing capability raise the opportunity for incorporating the analysis methods with a search scheme to locate the optimum in the search space. This very promising environment results in the addition of new techniques to the arsenal of numerical optimization methods. Stochastic search techniques, such as simulated annealing, evolution strategies, and genetic algorithms, along with artificial intelligence based methods, such as neural networks, have been introduced in the last ten years. Optimization schemes that combine one or a hybrid of several of these methods with computational electromagnetics' analysis tools are reported to have successful applications to design optimization problems of various kinds. Genetic algorithms (GAs), developed by John Holland in the 1970s, attracted the interest of more and more researchers due to their robustness and efficiency in handling search problems. In computational electromagnetics, design optimization can benefit from the use of this highly successful method. The finite element method joined with GAs to form a design optimization environment for electromagnetic devices.

One of the most difficult aspects of design optimization is the shape optimization problem. The optimization technique described here is not limited to shape optimization. Other design parameters, such as excitation and material characteristics as well as device terminal parameters may also be a part of the optimization's goal. Thus, a multidimensional search space is under consideration. The optimization method must be able to perform an effective search in a multidimensional parameter space. As a shape optimization problem may require a high number of control points to be adjusted, the number of design parameters can be quite large. This should not pose a major difficulty for the optimization scheme. In other words, the developed scheme must possess the flexibility to adapt easily to different design needs.

The multiparameter objective function for a simple design optimization problem in magnetostatics can be non-convex (local extremes), non-differentiable, and stiff [2-3]. The optimization algorithm must be able to deal with these difficulties. It is known that a search method based on deterministic methods tends to fail when the objective function is discontinuous. The local extremes also present a significant danger to the success of the search because the gradient based algorithm can be easily trapped in such a local extrema. Finally, the search process is rated by its speed of finding a solution and the accuracy of the solution, its closeness to the global optimum. Therefore, the developed optimization method, these parameters must be within acceptable margins. Besides the search algorithm, an analysis scheme is also needed. The analysis module's task is to provide field solutions for the different designs suggested by the search tool. Modelling the problem is one of the responsibilities of the analysis module. Especially in shape optimization, accurate modelling of the suggested design is very critical. The analysis section of the optimization environment, therefore, must be able to model the problem accurately and efficiently, and it must provide accurate solutions for the field parameters. The search relies on the analysis done on its suggestion. Direct application of GAs to the design optimization of electromagnetic devices is explained through an example.

1. Genetic Algorithms

John Holland laid the foundation of GAs in the publication summarizing his work on designing an artificial system that simulates the characteristics of a natural system [4]. Since then, GAs have been successfully applied to optimization problems in various areas of research ranging from chemistry to social sciences [5-8] as well as in the optimization of electromagnetic devices [9-11]. Genetic algorithms mimic the mechanics of natural genetics. The search starts from a randomly created population of strings representing the chromosomes. The optimization is based on the survival of the string structures from one generation to the next. Strings that better suit the environment, the objective function, are more likely to survive. Using the bits of information, genes, of the survivors of the previous generation, creates a new improved generation.

Genetic algorithms implement methods of nature. Natural systems show a high level of robustness. This is the result of their being able to adapt to many different environments and to operate to locate the global optimum without being attracted to the local optima [8].

GAs are different from the other common optimization methods because: 1) they operate on a population of points in the search space simultaneously, not on just one point, 2) they work with a coded string representing the parameters, not the parameters themselves, 3) they use the objective function itself and not derivatives or any other additional information, and 4) their rules for transition are probabilistic, not deterministic [8].

There are three fundamental operators involved in the search process of a genetic algorithm: 1) reproduction, 2) crossover, and 3) mutation. The reproduction operator creates a new generation giving the better strings of the previous generation a better chance of having more copies in the new population. The crossover operator accounts for the information exchange between string pairs randomly mated from the entire population. Occasionally, the mutation operator changes the value of a string position. It acts as protector against the complete loss of some important genetic information [8].

2. Application of GAs to Design Optimization

There are two fundamental functional bodies in a design optimization process; the analysis tool and the search algorithm. They continuously interact, as for every new set of design parameters the search algorithm suggests the analysis tool solves the governing equations and determines the value of a new point in the search space. The genetic algorithm performs the search operation and the finite element method carries out the analysis. A schematic diagram of the procedure is given in Figure (1). The GA comprises all the blocks on the left side of the figure. The analysis is shown as a single block, which contains all of the processes of FEM.

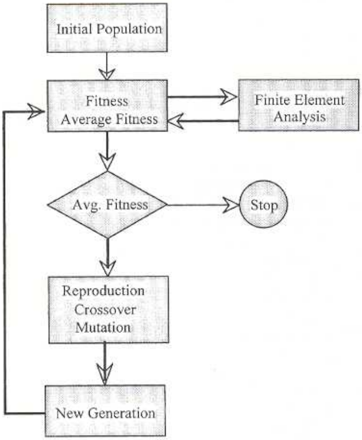

Figure 1. Genetic Algorithms in Design Optimization.

A GA starts the search from an initial population of a certain number of members. This population is randomly created within the domain of the search space. The members of the population are finite length string structures called chromosomes. Coding the design parameters using a finite length alphabet forms these structures. Generally a low cardinality alphabet is used, such as the binary system. The coding of the parameters is entirely application dependent and it is one of the critical aspect of the performance of a GA. For the binary alphabet, two of the well-known ones are binary and gray coding. The bits in the strings are called genes. As every chromosome in the population represents another design, it is necessary to determine how well fitted each design is. The FEM creates a mesh for that particular design, solves the governing equations, and informs the GA about the performance of the designed device by returning the necessary values such as magnetic flux densities at critical points.

From the results of the analysis, GA calculates the objective function value for each chromosome. This value is called the fitness of the chromosome and plays a significant role in the further steps of the search. Also, an average fitness is calculated representing the fitness of the entire population. The decision on the convergence of the search is made based on the average fitness as indicated in Figure 1. If the search has to continue, the GA creates a new generation from the old one. There are three operators, which exclusively characterize the GAs; reproduction, crossover, and mutation.

Based on the fitness of a chromosome and the average fitness of the population, the reproduction operator determines, rather randomly, whether that particular chromosome will have copies in the next generation, and how many. There are many ways of designing this operator, however, the underlying idea is to give the chromosome with a higher fitness more chance to be represented in the next generation but leave the actual decision to a random variable.

Once the reproduction is complete, the new generation is made of copies of the previous generations' members. Nevertheless, there is no new information. It is time for the chromosomes to exchange information through the crossover operator. At this point the number of chromosomes in the population has not changed.

The crossover operator forms pairs from the new generation mating chromosomes randomly. For each pair all bits from a randomly selected position on the string to the end of the string are swapped. As an example suppose the coding is binary with an alphabet A={0,1}. Assume that before crossover two strings, S_1 and S_2 are mated and are represented as:

$$S_1 = 1\,1\,0 \mid 0\,1\,0\,1\,1\,0$$
$$S_2 = 1\,0\,0 \mid 1\,1\,0\,0\,0\,1$$

Also assume that the crossover site for this pair to be 3 as indicated above. After crossover the new pair is

$$S_1 = 1\,1\,0 \mid 1\,1\,0\,0\,0\,1$$
$$S_2 = 1\,0\,0 \mid 0\,1\,0\,1\,1\,0$$

The third fundamental operator of GAs is the mutation operator. It occasionally changes the value of a gene acting as a protector against the complete loss of some important genetic information [8].

Both crossover and mutation operators have occurrence probabilities. Compared to the crossover, mutation happens much more seldom.

After all these operators perform their functions, the new generation is made of members who have gained new information through the exchange between pairs. The better traits of the "parent" chromosomes are carried along to the future generations.

3. A Magnetizer Example Problem

A magnetizer is modeled where a high current is applied to a coil, which causes a magnetic field to be set through the material that is to be magnetized. The geometry of the problem is shown in Figure 2. The region to be magnetized is assumed to be made of non-magnetic material. Permeability very close to that of air is assigned to that region. For the pole face and the outer shell, the relative permeability is $\mu_r = 2000$.

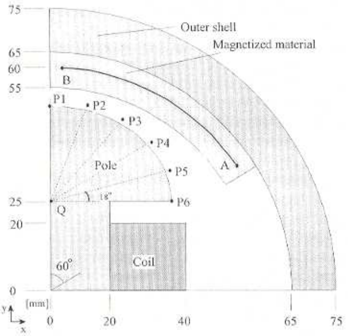

Figure 2. Geometry of the magnetizer problem.

The goal is to optimize the pole shape to achieve a sinusoidal magnetic flux density distribution along chord, AB, positioned at $r = 60.0$ mm, halfway through the width of the magnetized piece. The magnetized material is placed across a 60° angle. Points A and B are located at a 1° distance from either end of the magnetized region. Chord AB subtends a 58° angle. At point B, the flux density, $\boldsymbol{B}$, is maximum, and it is expected to follow a cosine function along AB.

There are six points that control the pole face's shape. These are indicated as P_1 through P_6. The control points are mapped on to a polar coordinate system with its origin at Q. The angular coordinate of each control point is kept constant with 18° between two consecutive points. The radial coordinates, r_1 through r_6, constitute the six design variables with mapping ranges given in Table I. Once determined, the radial and angular coordinates of a control point are mapped back to the x and y coordinates. Uniform non-rational cubic b-splines (UNBS) are used to approximate the shape of the pole face from the six control points.

The objective function in equation (1) performs a comparison of the desired and calculated magnetic flux density values, $B_{desired}$ and $B_{calculated}$, at 59 points along chord AB, separated from each other by 1°. To avoid ripples on the pole face, a penalty term, M, is included in the objective function. The value of M is calculated as a function of the slopes of the line segments connecting consecutive control points, P_1 through P_6. The curve modelling the pole face is expected to decrease monotonically. The penalty, therefore, increases with the number of line segments with a positive slope. If all segments have negative slopes, there is no penalty.

$$f = \sum_{k=1}^{59} (B_{desired,k} - B_{calculated,k})^2 + M \qquad (1)$$

$$B_{desired,k} = B_{trial,B}\sin(90^o - k) \quad \text{For } 1 \leq k \leq 59 \qquad (2)$$

A fitness for each design a is defined as $\Phi(a) = 1/f(a)$. Because $f(a)$ in equation (1) prescribes an error term for design a, maximizing the fitness translates into minimizing the objective function.

The GA is run with a population of 40 individuals. Character strings of 15 bits each are allocated to the representation of each of the 6 design variables yielding a chromosome length of 80 bits per individual. The representation scheme uses Gray code to map the real valued design variables in to binary strings. Linear fitness scaling is employed to apply selective pressure. The reproduction operator is based on the remainder stochastic sampling without replacement method. Uniform crossover is applied with an occurrence probability $p_c = 0.9$ per chromosome pair. Mutation probability $p_m = 0.005$.

4. Results of the Magnetizer Example

The evolution is stopped after 200 generations. The best of all individuals is selected as the optimal design. The optimization results of the magnetizer problem are presented in Table II. Figure 3 shows the final pole shape while Figure 4 compares the desired magnetic flux density with the flux density calculated for the optimized geometry along chord AB. The good results are attributed to the higher number of control points in modelling the pole shape and the imposed geometrical constraint.

DESIGN VARIABLES AND THEIR RANGES USED IN THE MAGNETIZER PROBLEM

Design variable	Mapping range [mm]
r_1	22.0 - 29.5
r_2	22.0 - 30.2
r_3	22.0 - 32.3
r_4	22.0 - 36.0
r_5	22.0 - 41.4
r_6	22.0 - 48.5

Table I

COORDINATES OF THE OPTIMIZED CONTROL POINTS FOR THE MAGNETIZER

Control Point	Design variable r [mm]
P_1	25.586
P_2	25.790
P_3	27.536
P_4	30.089
P_5	31.738
P_6	36.987

Table II

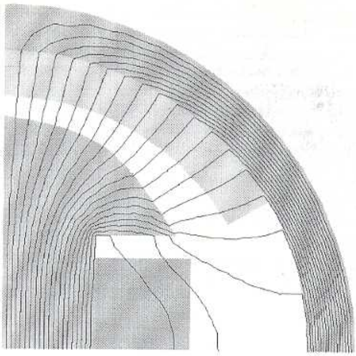

Figure 3. Optimized magnetizer and the field solution.

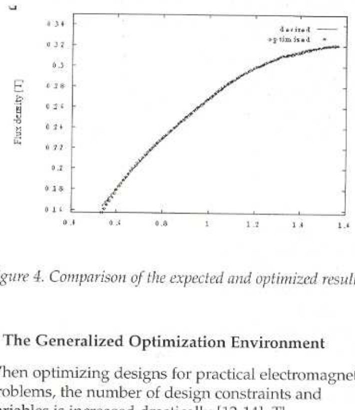

Figure 4. Comparison of the expected and optimized results.

5. The Generalized Optimization Environment

When optimizing designs for practical electromagnetic problems, the number of design constraints and variables is increased drastically [12-14]. The construction and step-by-step creation of an electrical system, in practice, is a trial and error process. The design may lead to a sub-optimal solution since the success of a design depends on the experience of the designer. It is therefore necessary to simulate the physical behaviour of the electrical system by numerical methods in a generalized fashion. To obtain an automated optimum design, numerical optimization is necessary to achieve a well-defined optimum. Optimization requires that all design goals, of a device, be connected into a single objective function with all independent variables and constraints. In applications to practical problems, effective pre-processing and post processing of data are necessary in addition to high performance computing capabilities. To achieve this process, a generalized optimization environment is suggested in Figure 5.

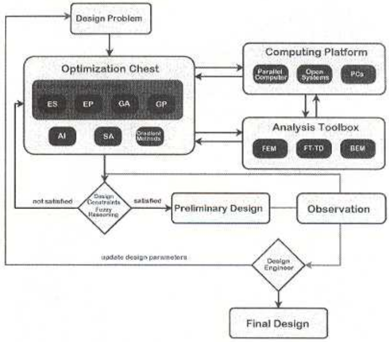

Figure 5. Generalized Optimization Architecture for Practical Design of Electrical Devices.

The system architecture for this optimization can be utilized for the computational creation of an electrical device or a system. It is intended to be a design and development toolbox system for practical applications in an industrial environment. As shown in Figure 5, the generalized optimization environment involves four main interacting blocks. These represent: 1) the computing platform, 2) the analysis toolbox, 3) an optimization chest, and 4) the design engineer's observation and decision making path. The optimization chest is a black box of various optimization techniques. The choice of an optimization procedure versus another is based on the expectation that the selected method will work faster or give best answers to a particular problem rather than the heuristic choice of a method that is application specific. The interaction between the modules is described in a shared open system computational environment which will make use of available computing facilities and utilize the experiences and tools gained from previous applications.

The Computing Platform

The current improvements and increase in capabilities of desktop computing facilities will enable the designer to utilize vast computing resources that are available in the design environment. The computing platform block in Figure 5 represents the computing facilities that may be available in the design environment. In this platform, parallel computing machines, clustered workstations and personal computers could be utilized to provide super-computing capabilities at low cost. Workstation farms connected to data switches and data communication networks could provide automated access to clusters of workstations that could be shared at distant locations or at the work place.

The Analysis Toolbox

This toolbox includes a collection of modelling tools and analysis techniques. Software packages for FEM, BEM, FDTD as well as other techniques will be accessed concurrently by the Optimization Chest to determine the objective function values, and by the design engineer to observe the performance and check the design parameters interactively. The analysis toolbox also include information about the various methods and their formulation, facilities for pre- and post processing of input and output data, mesh generation, mesh density information, adaptive meshing and additional interfacing software to access the various blocks in the overall system.

The Optimization Chest

This block includes a collection of design methods. Each is treated as a black box with an interface that accepts the appropriate problem specification. The techniques in this box include evolutionary algorithms such as evolution strategies, evolutionary programming, genetic algorithms and genetic programming. It also includes artificial intelligence algorithms such as neural networks, simulated annealing, expert systems, fuzzy reasoning and adaptive learning. In addition to these emerging methodologies, standard search algorithms ranging from gradient methods, random search, hill climbing and biased random walk will also be included in the optimization chest. As practical applications require many objectives that may not be met by one of the methods, combinations and generalizations of these algorithms have proved to be powerful. Design sensitivity techniques are also included in the optimization chest.

Parametric Design in the Optimization Chest

The optimization chest described above is a stochastic blackbox approach to design optimization. To check its performance in a practical environment, each box representing a technique should be based on the same assumptions. These may include: 1) the values for each parameter must be found, 2) each parameter has finite number of possible values, 3) combinations of parameter values may be unacceptable, 4) there is a function to give a measure for each set of parameters, and 5) the objective function is to produce an optimal set of design parameter values for an acceptable scoring measure. The choice of an optimization method should be based on which method can work faster or give best answers to the problem rather than if the method will work at all for the given application. Approximating continuous parameters with discrete values will work for most applications but may increase the problem dimensionally. Combination methods are used to reduce the size.

Problem Specification

Applying any or all of the design methods to a given problem can be successful with proper problem specification. Some suggested methods to enable the user specify the problem are discussed here. These methods could be coded in software that will execute before invoking the design process. The creation of a design policy, in the form of a rule-generating function that produces a feasible set of rules, may be developed. Each use of this function may produce a different set of rules for every application. For this reason, the feasibility of the generated rules must be checked. Formulating a function that will be used to determine the feasibility of the generated rules is an important task in specifying the problem. Such a function will take any generated set of input and determines if the values satisfy all users imposed constraints. The next step in specifying the problem is a scoring function that takes the rules as input, evaluates it and returns a measure of merit. In order to evaluate several methods, the same scoring function must be used. The performance of any method in the optimization chest could be determined based on the number of objective function evaluations, or on the time it takes to return a feasible solution or a combination of these ideas. The final item in the problem specification process is an output function, which will display the input to the optimization chest. Here, other user inputs such as relating the parameters to those in a database or empirical data from previous design or adding explanations and additional restrictions on the parameters may be added. This process can be used at the end of the search process for determining if a good design is reached. This process can also be used during the search as a checking mechanism to interrupt to see the search result at some point.

Observations and Decision Making

This part of the optimization environment includes information access in the form of query search database in an industrial or a research environment. This may include interactive features to access information on field and experience data, design aids and rules, empirical data, knowledge base, cost tables and parts list for off-the-shelf design optimization.

6. Conclusion

GAs provides a high level of robustness by simulating nature's capability of adapting to many different environments. Through the application of GAs to design optimization problems, the performance characteristics of the GAs are shown to be powerful in solving optimization problems with high dimensional objective functions containing several local minima. As they use only the fitness values, GAs does not require derivatives or any other additional information about the objective function. The performance of GAs can be improved significantly by applying selective pressure and adequately providing a shape modelling technique to control the range of variation for the shape to be optimized.

We also described a generalized optimization environment that may be valuable for current and future activities in computational electromagnetics for analysis and design of practical applications. This environment involve interacting blocks and adds the input of the design engineer to the process and giving him the final decision making on the achievement of an optimal design. The interactions between the various blocks of the system can be implemented in an industrial or a research environment.

References

[1] V. B. Glasko, *Inverse problems of mathematical physics*, New York, New York: American Institute of Physics, Translation Series, 1988.

[2] A. Gottvald, K. Preis, C. Magele, O. Biro, and A. Savini, "Global optimization methods for computational electromagnetics", *IEEE Transactions on Magnetics*, Vol. 28, no. 2, pp. 1537-1540, March 1992.

[3] K. Preis, O. Biro, M. Friedrich, A. Gottvald, and C. Magele, "Comparison of different optimization strategies in the design of electromagnetic devices", *IEEE Transaction on Magnetics*, Vol. 27, No. 5, pp. 4154-4157, September 1991.

[4] J. H. Holland, *Adaptation in Natural and Artificial Systems*, Ann Arbor: University of Michigan Press, 1975.

[5] C. B. Lucasius and G. Kateman, "Application of genetic algorithms in chemometrics", *Proceedings of the Third International Conference on Genetic Algorithms*, George Mason University, pp. 170-176, 1989.

[6] K. A. De Jong and W. M. Spears, "Using genetic algorithms to solve NP-complete problems", *Proceedings of the Third International Conference on Genetic Algorithms*, George Mason University, pp. 124-132, 1989.

[7] Y. Tanaka, A. Ishiguro, and Y. Uchikawa, "A method of estimation of current distribution using genetic algorithms with variable-length chromosomes", *Proceedings of the International ISEM Symposium on Simulation and Design of Applied Electromagnetic Systems*, Elsevier Science B.V., pp. 163-166, 1994.

[8] D. E. Goldberg, *Genetic Algorithms in Search, Optimization & Machine Learning*, Reading, Massachusetts: Addison-Wesley Publishing Company, Inc., 1989.

[9] O. A. Mohammed, F. G. Uler, S. Russenschuck, and M. Kasper, "Design Optimization of a Superferric Octupole Using Various Evolutionary and Deterministic Techniques", *IEEE Transactions on Magnetics*, Vol. 33, No. 2, pp. 1816-1821, March 1997.

[10] O. A. Mohammed, and F. G. Uler, "A Hybrid Technique for the Optimal Design of Electromagnetic Devices Using Direct Search and Genetic Algorithms", *IEEE Transactions on Magnetics*, Vol. 33, No. 2, pp. 1931-1934, March 1997.

[11] O. A. Mohammed, and F. G. Uler, "Ancillary Techniques for the Practical Implementation of GAs to the Optimal Design of Electromagnetic Devices", *IEEE Transactions on Magnetics*, Vol. 32, No. 3, pp. 1194-1197, May 1996.

[12] M. H. Lean, "Concurrent Multi-level Simulation in Computational Prototyping", *International Journal of Applied Electromagnetics in Materials*, Elsevier, pp. 9-12, 1994.

[13] T. Back, and H. P. Schwefel, "An overview of Evolutionary Algorithms for Parameter Optimization", MIT, Evolutionary Computation 1(1): 1 -23, 1993.

[14] O. A. Mohammed, R. S. Merchant, and F. G. Uler, "An Intelligent System for Design Optimization of Electromagnetic Devices", *IEEE Transactions on Magnetics*, Vol. 30, No. 5, pp. 3633-3636, September 1994.

Osama A. Mohammed