Technical Article

Overview of Meshless Methods

Abstract— **This article presents an overview of the main developments of the mesh-free idea. A review of the main publications on the application of the meshless methods in Computational Electromagnetics is also given.**

I. Introduction

Several meshless methods have been proposed over the last decade. Although these methods usually all bear the generic label “meshless”, not all are truly meshless. Some, such as those based on the Collocation Point technique, have no associated mesh but others, such as those based on the Galerkin method, actually do require an auxiliary mesh or cell structure. At the time of writing, the authors are not aware of any proposed *formal* classification of these techniques. This paper is therefore not concerned with any classification of these methods, instead its objective is to present an overview of the main developments of the *mesh-free* idea, followed by a review of the main publications on the application of meshless methods to Computational Electromagnetics.

II. Meshless Methods - The History

A. The Smoothed Particle Hydrodynamics

The advent of the *mesh free* idea dates back from 1977, with Monaghan and Gingold [1] and Lucy [2] developing a Lagrangian method based on the *Kernel Estimates* method to model astrophysics problems. This method, named *Smoothed Particle Hydrodynamics* (SPH), is a *particle method* based on the idea of replacing the fluid by a set of moving particles and transforming the governing partial differential equations into the kernel estimates integrals [3].

Despite the success of the SPH in modelling astrophysics phenomena, it was only after the 90’s that the SPH was applied to model others classes of problem. This consequently exposed some inherent characteristics of the method, such as tensile stability, energy zero, lack of interpolation consistency and difficulty in enforcing essential boundary conditions [4]. The latter two are consequences of using the SPH to model bounded problems, since originally the SPH was formulated to model open problems. Over the past years many modifications and correction functions have been proposed in an attempt to restore the consistency and consequently the accuracy of the method [5]. The SPH method has been successfully applied to a wide range of problems such as free surface, impact, magnetohydrodynamics, explosion phenomena, heat conduction and many other computational mechanics problems which were presented and discussed in extensive reviews of SPH [6] [5].

B. The Diffuse Element Method (DEM)

The first *mesh-free* method based on the Galerkin technique was only introduced over a decade after Monaghan and Gingold first published the SPH method. The *Diffuse Element Method* (DEM) was introduced by Nayroles and Touzot in 1991. Many authors state that it was only after the Diffuse Element method that the idea of a *mesh-free* technique began to attract the interest of the research community. The idea behind the DEM was to replace the FEM interpolation within an element by the *Moving Least Square* (MLS) local interpolation [7].

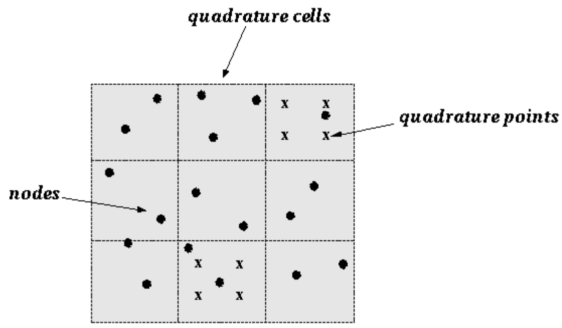

Fig. 1. EFG background cell structure.

C. The Element-Free Galerkin (EFG)

In 1994 Belytschko and colleagues introduced the *Element-Free Galerkin Method* (EFG) [8], an extended version of Nayroles’s method. The Element-Free Galerkin introduced a series of improvements over the Diffuse Element Method formulation, such as

- **Proper determination of the approximation derivatives**: In DEM the derivatives of the approximation function $\mathcal{U}^h$ are obtained by considering the coefficients $\mathbf{b}$ of the polynomial basis $\mathbf{p}$ as constants, such that
$$\frac{d\,\mathcal{U}^h(\mathbf{x})}{d\mathbf{x}} = \frac{d\,\mathbf{p}^T(\mathbf{x})}{d\,\mathbf{x}}\mathbf{b}(\mathbf{x}) \quad (1)$$
In EFG the *full* form of the derivatives is used, such that
$$\frac{d\,\mathcal{U}^h(\mathbf{x})}{d\mathbf{x}} = \frac{d\,\mathbf{p}^T(\mathbf{x})}{d\,\mathbf{x}}\mathbf{b}(\mathbf{x}) + \mathbf{p}^T(\mathbf{x})\,\frac{d\,\mathbf{b}(\mathbf{x})}{d\,\mathbf{x}} \quad (2)$$
Belytschko argued in his paper that neglecting the derivatives of $\mathbf{b}(\mathbf{x})$ detracts significantly from the accuracy of the method.
- **Imposing essential boundary conditions**: The MLS trial function does not yield an interpolation, i.e. $\mathcal{U}(\mathbf{x}) \neq \mathcal{U}^h(\mathbf{x})$, therefore the essential boundary conditions are not directly satisfied. Belytschko showed that the DEM fails to pass the patch test due to the fact that these conditions are not properly fulfilled. In the EFG formulation, Lagrange Multipliers are used in the weak form to enforce the essential boundary conditions.
- **Process for Numerical Integration**: Meshless methods based on the Galerkin technique require numerical integration of the weak form. In the Element-Free Galerkin an auxiliary cell structure, shown in Fig. 1, is used in order to create a “structure” to define the quadrature points.

The Element-Free Galerkin method was found to be more accurate than the Diffuse Element method, although the “improvements” implemented in the method increased its computational costs [8]. EFG is one of the most popular mesh-free methods and its application has been extended to different classes of problems such as fracture and crack propagation, wave propagation , acoustics and fluid flow.

Many authors state that the use of a background cell to perform the numerical integration eliminates the *mesh-free* characteristic of the EFG, therefore the method is not truly mesh-free.

D. Reproducing Kernel Particle Method

In 1995 Liu proposed the *Reproducing kernel particle method* (RKPM) [9] in an attempt to construct a procedure to correct the lack of consistency in the SPH method. The RKPM has been successfully used in multiscale techniques, vibration analysis, fluid dynamics and many other applications. Later, a combination of MLS and RKPM resulted in the *Moving Least Square Reproducing Kernel Particle method* [10] [11].

E. Finite Point Method

The *Finite Point method* was proposed by Onãte and colleagues in 1996 [12] [13]. It was originally introduced to model fluid flow problems and later applied to model many other mechanics problems such as elasticity and plate bending. The method is formulated using the Collocation Point technique and any of the following approximation techniques, *Least Square approximation* (LSQ), *Weighted Least Square approximation* (WLS) or *Moving Least Squares* (MLS) can be used to construct the trial functions.

F. Meshless Local Petrov-Galerkin

The Meshless Local Petrov-Galerkin introduced by Atluri and Zhu in 1998 [14] presents a different approach in constructing a *mesh-free* method. It is based on the idea of the *Local weak form* which eliminates the need of the background cell and, consequently, performs the numerical integration in a mesh-free sense. The MLPG uses the Petrov-Galerkin method in an attempt to simplify the integrand of the *weak form*. The method was originally formulated using the MLS technique and later Atluri extended the MLPG formulation to other meshless approximation techniques. The freedom of choice for the test function in the Petrov-Galerkin method gives rise to different MLPG schemes [15]. The MLPG and its different schemes have been applied to a wide range of problems such as Euler-Bernoulli Beam Problems, solid mechanics, vibration analysis for solids, transient heat conduction, amongst many others.

G. Radial Basis Functions

Radial basis functions (RBF) were first applied to solve partial differential equations in 1991 by Kansa, when a technique based on the direct Collocation method and the Multiquadric RBF was used to model fluid dynamics [16] [17]. The direct Collocation procedure used by Kansa is relatively simple to implement, however it results in an asymmetric system of equations due to the mix of governing equations and boundary conditions. Moreover, the use of Multiquadric RBF results in global approximation, which leads to a system of equations that is characterised by a dense stiffness matrix. The RBF Hermite-Collocation was proposed as an attempt to avoid the asymmetric system of equations. Both globally and compactly supported radial basis functions have been used to solve PDEs and results have shown that the global RBF yielded better accuracy. However the compactly supported stiffness matrix is sparse, while the global RBF results in a dense matrix that may become very expensive to solve in the case of a large number of collocation points. Recently, another approach based on the global RBFs has been proposed. The method, named *Local Multiquadric*, suggests the construction of the approximation using sub-domains, causing the Multiquadric RBF to be truncated at the "boundaries" of the sub-domains, resulting in a local approximation and a sparse stiffness matrix [18].

Radial basis functions have also been used in the Boundary Element method formulation, such as in the Dual Reciprocity Method (DRM), Method of Fundamental solution (MFS) and the RBF Boundary Knot method (BKM). These methods have been successfully applied to solve non-linear problems in Computational Mechanics.

A variational approach to solve the Boundary Value Partial (BVP) using compactly supported radial basis functions has been investigated by [19] and another approach suggested the use of radial basis functions in the Meshless Local Petrov-Galerkin formulation [15].

In the last decade the radial basis function approximation technique has undergone intensive research. However, a large number of publications on the subject concern its mathematical proof and foundations. An extensive review of the mathematical background of RBFs is presented in [20]. Some applications of the RBFs in the solution of physical problems worthy of mention are transport phenomena, heat conduction, analysis of Kirchoff Plates and Euler-Bernoulli beam problems amongst others.

H. Point Interpolation Method

The Point Interpolation method (PIM) uses the *Polynomial Interpolation* technique to construct the approximation. It was introduced by Liu in 2001 as an alternative to the Moving Least Square method [21]. The PIM, originally based on the Galerkin method, suffers from the singularity of the interpolation matrix and also from the fact that it does not guarantee the continuity of the approximation function. Several approaches have been investigated by Liu in an attempt to overcome these problems [3]. Improvements have been obtained using the *Local Petrov-Galerkin* method and Multiquadric radial basis functions. This procedure resulted in *Local Radial Point Interpolation methods* (LRPIM). The LRPIM has been applied to solid mechanics, fluid flow problems and others. These applications are referred to and examined in detail in [3].

I. Other Meshless Methods

Some of the most popular and important meshless methods have been presented in the previous subsections. However there are a great number of Meshless Methods documented in the literature and it is beyond the scope of this work to present a detailed description of all of them. Nevertheless, the following methods are also considered worthy of mention:

Duarte and Oden introduced in 1995 the *H-p Cloud* method which is a mesh-free method based on *h* and *p* enrichment of the approximation functions [22] [23].

A meshless method based on the Boundary Element method (BEM) was first introduced by Mukherjee and Mukherjee [24] and was named the *Boundary Node Method*. Later a similar approach was used by Zhu and Atluri and was named the *Local Boundary Integral Method* (LBIM) [25]. The LBIM differs from the former due to the use of the local weak form, instead of the global weak form approach.

In 2000, De and Bathe introduced the method of *Finite Spheres* [26], which can be seen as one of the MLPG schemes. The method of Finite Spheres uses the Partition of Unity [27] to construct the approximation function and therefore essential boundary conditions are satisfied a priori.

III. Meshless in Electromagnetics - An Overview

The application of meshless methods to computational electromagnetics started in the early 90's, just after Nayroles published his paper on the Diffuse Element method. However, at present, the range of application is still very modest as compared with that found in the field of Computational Mechanics. In this section the most relevant publications on the subject are covered briefly.

The application of meshless methods to model computational electromagnetics was first introduced by Yve Marèchal in 1992, when he applied the Diffuse Element method to model two-dimensional static problems [28]. Later Marèchal examined the application of DEM as a post-processing tool for CEM [29] [30]. More recently, the Diffuse Element method has been used in electromagnetic device optimisation [31] [32].

In 1998 the Moving Least Square Reproducing Kernel Particle Method (MLSRKPM) was applied to model two-dimensional static electromagnetic problems [33] [34]. This technique is a modified version of the Element-Free Galerkin where the MLS approximation is replaced by the MLSRKPM approximation. Viana also presented a comparison between both procedures and showed that the MLSRKPM yields better accuracy [34]. Results were also compared with FEM.

The Element-Free Galerkin method has been applied to model small gaps between conductors [35], static and quasi-static problems [36] and to model the detection of cracks by pulsed eddy current in Non-Destructive Testing [37].

A combined FEM and EFG technique was used in [38] and [39]. The proposed technique suggested the use of a coarse Finite Element mesh and *free-nodes* (meshless) as a refinement tool for the FEM solution. The *free-nodes* should be added in regions subjected to sharp gradients. The method was successfully used to investigate 3-D eddy current problems [38].

The Element-Free Galerkin has also been successfully applied to model Magnetohydrodynamics [40].

The Point Collocation Fast Moving Least Square Reproducing Kernel method was introduced and applied to model two-dimensional electromagnetics problems [41]. Kim proposed an alternative formulation to the MLS-RKPM that uses a variable dilation parameter, which allows a more flexible algorithm and improves the accuracy.

Several aspects of the meshless formulation have been investigated under the CEM context such as the interface between different regions, boundary conditions [42] [43] [44], and meshless nodal distribution [45].

Different meshless methods have been proposed to model a two-dimensional power transformer. In [46] the Wavelet-Element Free Galerkin was proposed. This technique used the so called Wavelet-Element Free Galerkin method combined with a single layer of Finite Element mesh along the boundary containing essential boundary conditions. In [47] the Meshless Local Petrov-Galerkin based on the MLS approximation modified by the *jump function* was used. Lagrange Multipliers were employed to enforce the essential boundary conditions. In [48] a hybrid Wavelet and Radial basis function was investigated. The radial basis functions approximation method is used along the external boundaries to enforce the essential boundary conditions in a straightforward manner.

A coupled Meshless Local Petrov-Galerkin and FEM was investigated in [49] to model a two-dimensional electrostatics problem.

Meshless Radial Basis Functions have also been applied to CEM. In [50] the authors apply the Hermite-collocation method using Wendland's RBF to model elliptical waveguides. The use of meshless techniques to model curved boundaries offers great advantages over mesh based methods, since the boundaries can be accurately represented. The results shown in [50] presented reasonable accuracy when compared with the analytical solutions.

The Meshless Local Petrov-Galerkin with Radial basis functions was applied to model 2-D magnetostatic problems in [51]. In this work a Heaviside step function was used as the test function in the RBF-MLPG formulation. The procedure reduces considerably the computational cost required in the numerical integration and the results presented good agreement with the Finite Element method. Later, Viana examined the Local Radial Point Interpolation Method to model 2-D eddy current problems [52]. The method yielded good agreement compared to the analytical solution. In both [51] and [52] Viana used the *Local Multiquadric* approach and the local weak form technique. The procedure results in a truly mesh-free method, alleviating the need for a background mesh and constraint techniques to impose the essential boundary condition.

Very recently the use of the SPH to model time-domain Maxwell equations was proposed in [53]. This procedure uses the SPH approximation function to represent the fields, $\mathbf{E}$ and $\mathbf{H}$, in the finite difference time domain scheme. The nodes, or particles, as they are normally referred to in the SPH, are arranged in a uniform grid, similar to the *Yee* grid [54]. The absorbing boundary conditions, traditionally used in the Finite Difference Time Domain (FDTD), are easily implemented in the SPH procedure. The application of SPH to model time domain electromagnetic problems may open a new range of possibilities in Computational Electromagnetics Modelling.

A combination of PIM and BEM, named *Boundary Meshless Method* (BMLM), was proposed in [55] to model two-dimensional transient electromagnetics problems. The proposed method uses the advantages of BEM in reducing the dimensionality of the problem under investigation, and then uses the PIM approximation technique in one dimension, avoiding the singularity problem of the interpolation matrix. The authors pointed out that the BMLM is elegant and efficient. Their results were compared to the analytical solution and showed very good agreement. However, applying this method to higher dimensions may lead to difficulties due to the inherent problems of PIM interpolation that leads to singularities of the interpolation matrix.

IV. Conclusions

Meshless methods theory is still in its infancy compared with that of Finite Elements and Finite Differences. However, in the last decade the pace of development of *mesh-free* theory has increased as a result of intensive research. Some claim that no proper mathematical analysis has been performed on these methods, others that there is a need for proper classification of these methods, claiming that methods based on the Galerkin formulation are not truly *mesh-free* due to the use of background cells. Collocation point methods are said to be truly mesh-free, however, the procedure is known for its instability and low accuracy. On the other hand the Galerkin procedure is stable and more accurate. The application of meshless methods to CEM has not yet made a great contribution. However, from the few publications found, one can verify that these methods offer advantages either on their own or coupled with FEM and more investigation is needed in order to take full advantage of these procedures.

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