Field and circuit solutions using global variables and exploiting duality

Abstract: The use of discrete formulation of electromagnetic fields allows to draw significant similarities between field and circuit analysis. The paper starts with a brief examination of the main concepts of discrete formulation and then applies these ideas to the solution of two simple electromagnetic problems, putting emphasis on equivalent lumped parameters circuit solution.

Index terms: finite formulation of electromagnetic field, equivalet network solution

1. Introduction

Field and circuit approaches to the solution of electromagnetic phenomena have gone along together for more than 150 years. They have been used by researchers with different attitudes for different purposes. Field based methods have been at the very base of understanding physical phenomena as they came out of experiments, while circuit models have been a powerful tool in simulation and quantitative analysis. In the decades, these two approaches became often tools of two separated worlds: theoreticians and engineers. Theoretical researchers, interested in understanding hidden laws and mechanisms in electromagnetic phenomena, have preferred the field instrument: mathematical tools, like differential calculus, are suitable for treating well behaved pointwise scalar and vector fields and, when their analytical solution is available, they allow explanation of interactions between different actors on the stage. Unfortunately, their application to practical cases requires highly complex solutions and thus, a part for a very limited set of cases, until the arrival of computers, they were considered a powerful instrument for knowledge, but not well suited for quantitative evaluations.

On the other hand, engineers, usually more interested in quick and approximated numbers to design something, liked very much the circuit tool able to give quantitative estimates of electromagnetic quantities. Despite their sometimes greedy objectives, electrical engineers, in their quest for the solution of practical problems, developed network theory, a powerful formalism which, under well defined hypotheses, can master in rigorous way highly complex problems.

In the times, the two environments developed different languages and way of behaving which made them almost completely separate. On the field side words like geometry, partial differential equations, material characteristics etc. are used, while on the circuit side topology, ordinary differential equations, terminal characteristics etc are part of everyday life.

Digital computers have been a major challenge for both ways of operating. Field researchers worked to translate their partial differential equations in discrete ones to be handled by numerical process: last 40 years are full of experience using different discretization schemes, both practical or deeply mathematical, testing and comparing them. At the same time, circuit specialists developed algorithms to solve automatically complex circuits which could not be handled by human beings.

This parting has been applied in education of electromagnetics too: in electrical engineering courses, electromagnetic theory is often taught in physics courses, while "thorough" electrical engineering starts with circuit theory.

Even if at first glance the two approaches are completely different, it must be kept in mind that the underlying phenomena are the same and that there are many subjects where the two approaches have to live together.

Maybe because our field of research has gained a certain maturity of judgment, in the last few years at least some researchers addressed this aspect of electromagnetics, trying to re-think a little bit about the basic analysis methods.

Without embarking on the perilous waters of "who started first" this new way of thinking we would like to cite two statements, just examples, taken from literature which address the very basic of numerical electromagnetics "*In modeling, we create an abstract structure, made of mathematical objects, that is meant to represent the part of the real structure we wish to deal with. There is no unique such structure and making it simple and orderly is our responsibility. Vector fields, in this respect, were a considerable advance, but are not the last word.*"[1] "*Is it really necessary to go from algebraic to differential formulation in order to go back to some other form of finite modeling?*" [2]. In few words, some rational simplifications in the way of thinking in electromagnetic analysis can be done by considering the problem in terms of electromagnetic quantities, which are not definitely field or circuit parameters. Maybe this process will not lead to new discoveries or to faster ways of solving electromagnetic problems, but it could build a solid bridge between field and circuit theories and, for sure, it will clarify the basics of the methods, which is a most valuable objective especially in education.

Trying to give a minimal contribute to the debate, this paper would give some space to applications of these basic concepts to the analysis of simple problems. Instead of stressing the research interest in the analysis of these phenomena, the accent will be put on the formal description of the algorithm followed to obtain the solution.

2. Basic remarks about discrete formulation and solution algorithm

This section is devoted to the definition of the basics of discrete formulation of electromagnetic fields. As it is well known, this topic has been already treated by other contributes to this Newsletter, thus we would like to address the reader interested in the theoretical aspects of these approaches to more specific papers, for instance [3], [4].

For our purposes, we will recall here only some basic concepts in way as much colloquial as possible, hoping that experienced readers will forgive some lack of strictness for the sake of explanation.

A. Local and Global variables

By *local variables* we define the ones usually involved in differential formulations and which are pointwise functions of space and time coordinates. By *global variables* we mean variables which are derived by the previous ones by some integral operation in space or time: they are usually referred to also as integral variables. The simplest example in this respect is represented by the couple electric current density/electric current. The first one is a vector field varying in time and in space, whereas electric current is the integral of the previous one on a defined oriented surface. The last global variable, being associated with a particular region of space, is not anymore depending on space coordinates, but only on time. This process can be brought on by integrating the electric current flowing through a particular surface also in time; in this case, the finite quantity of electric charge passed through the surface in a given interval of time is obtained; this process is similar to the one used in mechanics when the impulse of the force is considered. Something hinting at a "lumped parameter" solution of the problem can be guessed. In order to keep similarity with network equations, where time derivatives are present, this last integration will not be performed here.

Variables can be further subdivided in source and configuration variables. Source variables define the causes of the field and in electromagnetic problems are charges, currents and all variables which are linked to them by operations like sum, integration, product etc.. as dielectric flux, magneto-motive force etc..

Configuration variables give the pattern of a field like electric potential, e.m. force, magnetic flux, etc..

B. Space and time discretization

If local variables have to be integrated to get the global ones, some space and time elements have to be defined and this requires a domain subdivision. In addition, if an efficient computational scheme has to be built, this discretization must be able, with a suitable assignment of the global variables to the space-time entities, to well represent topological constraints.

The use of space and time elements must deal with two kinds of orientation: inner orientation defined on the element itself, for instance an arrow on an edge, a direction along the edges bounding a surface or on the surfaces bounding a volume; outer orientation relating the element to some other entity, for instance the outer orientation of a surface is defined by the inner one of a line piercing it. Incidence matrices containing in compact form connection and orientation of related instances, are very well suited to this purpose; as in description of circuits, the topological connection of two oriented entities can be represented by an integer number, which can assume 0 value if two entities are not related, +1 if they are connected with concordant orientation, -1 in the opposite case.

C. Dual meshes

The subdivision of variables into two separate sets, source and configuration, calls for a special attention on how discretization is performed and how global variables are defined. Operatively, a mesh endowed with inner orientation is firstly defined, as it is usually done for instance with finite element method, starting from a geometric description of the domain (shape, material interfaces, boundaries etc.). On this mesh configuration variables are defined (i.e. potential on primal nodes, electro-motive forces on edges, magnetic fluxes on faces etc.). The primal mesh is then processed to build a dual system of elements: duality implies that for each primal entity only one element of the dual mesh is defined (primal node↔dual volume, primal edge↔dual face, primal face↔dual edge, primal volume↔dual node). Elements of the dual mesh will have outer orientation inherited by the primal ones (for instance for each primal edge a dual face is built whose orientation is defined by that of the primal edge). Usually elements of the dual complex are identified by a ~ symbol.

D. Constraints on global variables

Topological constraints act on the same kind of variables, for instance Ampère theorem links magneto-motive forces and currents which are both of source type. On the other hand, electromagnetic induction law ties magnetic flux and electro-motive force which are both configuration variables.

Global variables are thus related by equations of algebraic form, where orientation of space elements introduces + or – signs. For instance, starting by a set of primal edges and quadrangular faces defined with their inner orientation (fig. 1).

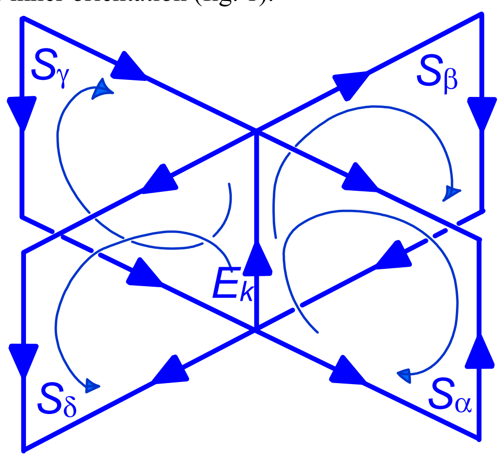


Fig. 1 Set of primal faces hinged on a primal edge. All elements are endowed with inner orientation.

A set of dual face and edges can be obtained whereas now these elements inherit their orientation by the primal one.

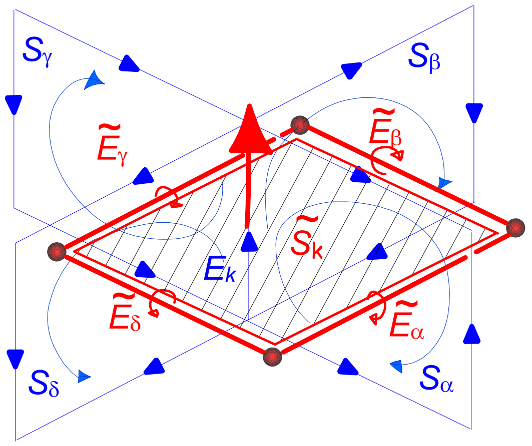


Fig. 2 Dual set of elements obtained by the primal one of fig. 1, dual elements are characterized by outer orientation.

In order to write down Ampère theorem, the mutual orientation of dual face and edges must be evaluated. By inspection of Fig. 2, it can be obtained that:

	$\tilde{E}_\alpha$	$\tilde{E}_\beta$	$\tilde{E}_\gamma$	$\tilde{E}_\delta$
$\tilde{S}_k$	+1	+1	-1	+1

thus the topological link can be written as:

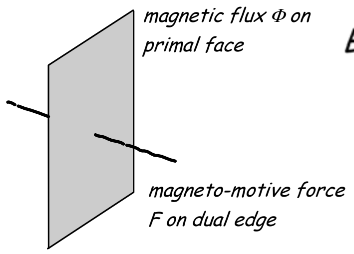
$$\{+1 \quad +1 \quad -1 \quad +1\} \begin{Bmatrix} F_\alpha \\ F_\beta \\ F_\gamma \\ F_\delta \end{Bmatrix} = i_k$$

It must be remarked that no use of right hand rule has been done to write down the constraint. This rule would have to be used when local variables are defined, for instance when a magneto-motive force must be obtained by a magnetic field. In this case the line integral operator requires that an inner orientation along line is defined. A link between outer (required by duality) and inner orientation (required by integration) of the line is provided by the right hand rule. Identical considerations can be done with the Faraday Neumann law linking configuration variables on primal edge-face set.

E. Material characteristics

Magneto-motive force and magnetic flux involved in Ampère Theorem and Faraday Neumann law are not independent but, as it is well known by field theory, are related each other by material characteristic, which has to be imposed if the problem has to be solved, otherwise the number of unknowns will overwhelm the constraints. The exploitation of this duality link on the dual mesh system is crucial for an efficient field solution as it has been extensively and efficiently shown in electromagnetic computational schemes, like for instance Finite Difference Time Domain (FDTD) [5] and Finite Integration Techniques (FIT) [4] which are based on this concept.

The choice of using two dual meshes is important, but the efficiency of the computational scheme depends on how these two meshes are linked together and thus on how material characteristic is enforced in solution scheme. The most efficient way of coupling the two is, by no doubts, to build a system of orthogonal meshes. In this way not only dual elements are univocally linked together, but they are also orthogonal in geometry. In this case, it is easy to impose material characteristics exploiting the fact that, for instance, the average normal component of magnetic flux density crossing the surface is parallel to the average tangential magnetic field component on the line (fig. 3). This assumption makes life easy, because with a simple relation recalling the usual formulas for resistance and reluctance computation, the two variables, source and configuration, are linked together. Inner and outer orientation relation ensures that these coefficients are always positive. Despite their efficiency in enforcing material characteristics, orthogonal (structured) meshes have the drawback of being very awkward in dealing with curved boundaries, as it always happens in practical cases. For a long time there has been confusion between the concepts of duality and orthogonality, perhaps thinking that dual relations could be obtained only by means of orthogonal meshes. Work of several researchers has now shown that duality between meshes can be exploited also in case of unstructured meshes, for instance made of tetrahedra [3], [6], [7].



$$B_n = \frac{\Phi}{S} \qquad H_t = \frac{F}{l} \qquad B_n = \mu H_t$$

Fig. 3 Magnetic material characteristic on an orthogonal dual mesh.

F. Relation between discrete formulation and circuit solutions

By the analysis of the previous paragraphs, the introduction of discrete formulation in the electromagnetic field has brought on the page terms like "topological constraint", "incidence matrix", "current" and "electro-motive force" which, by no doubts, are proper of the network theory. That is, in simple words, the use of discrete formulation allows to treat the field problem as if it would be a circuit one. Analyzing the way in which the problem was set, the crucial passages we had to perform were:

- definition of global variables;
- definition of dual meshes.

From then on, the usual field laws could be treated as topological bonds in a way which is equal to the usual Kirchhoff laws of circuits, where voltages are configuration variables and thus related to a primal mesh whereas currents are source variables and thus defined on a dual mesh.

The problem could thus be solved with the following flowchart:

- define system of dual mesh and global variables;
- compute lumped parameters (resistance, inductance, capacitance) linking source to configuration variables for each couple inside the mesh;
- use a network solution scheme to solve the problem.

In this way, the approach is somewhat oversimplified, because it does not take into account coupling between electromagnetic domain treated by means of field and possible external circuit connection. This fact requires to take into account interface conditions between field quantities and lumped parameters, which impose constraints between more complex sets of global variables; see [8] for an exhaustive treatment of the problem.

In the following, two different problems coming from industrial applications are approached and solved in this way.

3. Analysis of massive ferromagnetic circuits with eddy currents

The solution method outlined in the previous chapter can be applied to the solution of magnetic flux distribution inside a ferromagnetic lamination in presence of eddy currents. The problem is geometrically simple, but its solution is complicated by non uniform distribution of flux due to eddy currents. The domain of the problem can be reduced to one dimensional considering that the thickness of the lamination is much smaller than its other dimensions (*h*>>*d*) (fig. 4).

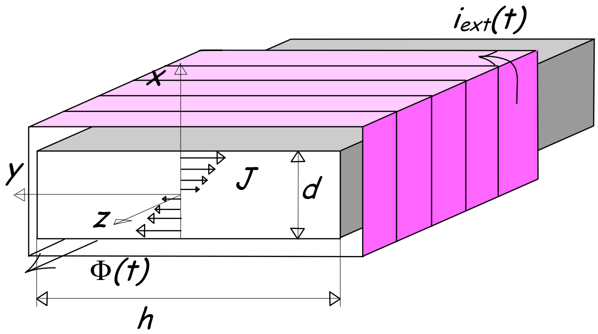


Fig. 4 Ferromagnetic lamination surrounded by an external circuit winding.

Considering that eddy currents are directed parallel to *y* axis and that all quantities are function of the *x* coordinate only and considering symmetry with respect to plane *yz,* the primal/dual mesh discretization of fig. 5 can be set where each of the cell presented is extended indefinitely along *y* and *z* directions. Magnetic fluxes are defined over primal faces, which lie in the *xy* plane, and are bounded by primal edges which are parallel to *y* axis. Magneto-motive forces are defined over dual edges along *z* axis crossing orthogonally primal faces and currents are defined on dual faces, portion of *xz* plane, bounded by dual edges.

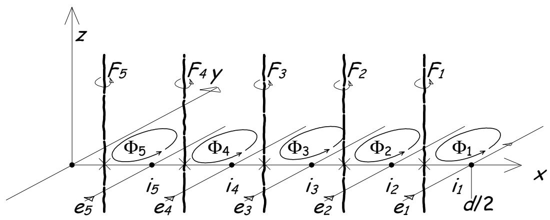


Fig. 5 Schematic view of primal-dual mesh over half lamination thickness

A. Topological constraints

Ampere law can be set on each dual face, making reference to fig. 5, for instance:

$$F_{k+1} - F_k = i_k \qquad k = 1,..,4$$

Faraday-Neumann law can be set on each primal face, for example:

$$e_k - e_{k+1} = -\frac{d\Phi_k}{dt} \qquad k = 1,..,5$$

remembering that, due to symmetry reasons, the central edge of the mesh have to have a null electromotive force.

This last relation can be re-written in a more convenient way. Considering, in fact, that a closed line made up of a primal edge and its symmetric, located at the opposite *x* position, links all fluxes on internal faces and that, due to symmetry, the two electro-motive forces on opposite edges must be equal, electromagnetic induction law for primal edges can be set as:

$$e_k = -\frac{d}{dt}\sum_{j=k}^{N}\Phi_j \qquad k = 1,..,N$$

where *N* is the maximum number of primal faces (*N*=5 in fig. 5).

B. Material characteristics

Considering orthogonality between meshes, magnetic relation can written as:

$$F_k = \frac{h}{\mu s_k}\Phi_k = R_k\Phi_k$$

where *h* is the extension of the lamination in *y* direction and s_k is the area of the *k*-th primal face and □ is material permeability.

Ohm law can be defined as:

$$i_k = \sigma\frac{\tilde{s}_k}{h}e_k = G_k e_k$$

where $\tilde{s}_k$ is the area of the dual face and □ is material conductivity.

C. Magnetic network building

The previous discretization of the problem can be efficiently translated in a magnetic network, where magnetic flux is treated as a current and magneto-motive force as a voltage. The resulting network is a ladder one where each loop is given by the Ampere law applied to a dual face. Dual currents are treated as external magneto-motive (voltage) generators (fig. 6).

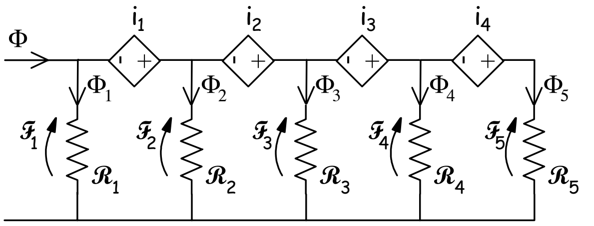


Fig. 6 Magnetic equivalent circuit considering eddy currents magneto-motive force, obtained by discrete formulation

By considering:

$$i_k = G_k e_k = G_k \left(-\frac{d}{dt} \sum_{j=k+1}^{N} \Phi_j \right)$$

it can be seen that dual currents can be expressed as function of primal fluxes. In this way, current values can be related to fluxes by components which have the same terminal relation of an inductor (please remember that here magnetic flux is treated as a current). Therefore, the circuit can be re-written as in fig. 7.

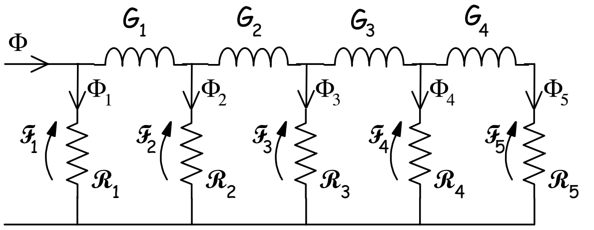


Fig. 7 Magnetic equivalent circuit with magneto-motive force generators replaced by inductances.

D. Numerical examples

The magnetic circuit of fig. 7 can be connected to an external circuit: for instance, in flux controlled conditions, total magnetic flux through the lamination Φ(t) will be assigned, otherwise a more complex circuit can be used taking into account interactions with electric ports. The proposed algorithm have been employed to study a lamination 0.5 mm thick with relative permeability of 1000 and conductivity of 1 MS/m, supplied by an external circuit known sinusoidal current at frequency of 50 Hz. Total flux pattern vs. time is reported in Fig. 8 together with results obtained by a Finite element procedure.

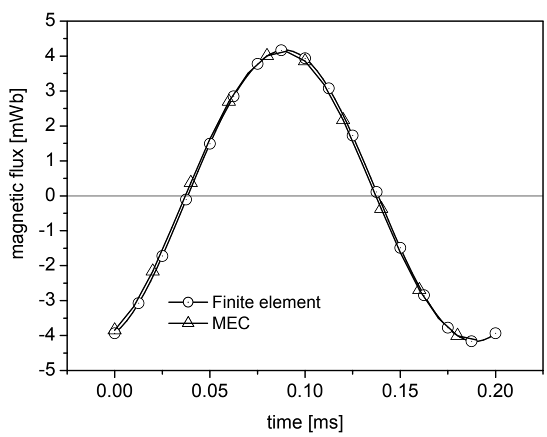


Fig. 8 Total magnetic flux flowing through a lamination 0.5 mm thick under sinusoidal current supply.

4. Study of conductive shielding of ELF magnetic fields

Shielding systems in power systems are often realized by means of thin non ferromagnetic conductive foils; eddy currents induced inside the sheet tend to cancel out source field abating thus magnetic field levels. The proper design of this system must take into account actual three dimensional fields and cope with an efficient treatment of far field conditions. To this aim, integral formulations of electromagnetic problems with special handling of the thin conducting sheets are used [9]. In this case, a discrete analysis of the shielding system is performed, computing interactions among different field entities by means of integral formulas

A. Discretization of conducting non-magnetic shields

The geometrical domain of the problem under study is made up of three regions:

- source conductors region where current density is imposed by an external circuit;
- eddy current region made of a set of thin conducting foils under the hypothesis that current density is uniformly distributed over its thickness (sheet thickness is considered to be lower than the penetration depth);
- air surrounding the two previous domains.

The source conductor region is dicretized in hexaedral volumes with an imposed current flowing through each of them and their contribute to the field solution is computed by means of the Biot-Savart law.

The eddy current region is discretized by means of orthogonal dual grids made of quadrilaterals. Since the depth of the sheet is largely smaller than its other dimensions, a surface discretization is performed, neglecting the thickness, which is anyway taken into account in the field formulation (fig. 9). Without going through details of mesh definition, the main actors involved in the computation algorithm are: the primal grid with nodes (N_N) and edges (N_E)and the dual grid with cells (by duality $\tilde{N}_V = N_N$) and faces ($\tilde{N}_F = N_E$). Faces of the mesh have a thickness □ in the dimension orthogonal to the sheet surface.

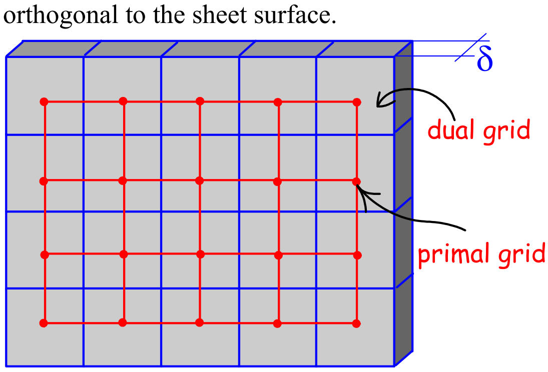


Fig. 9 Discretization of the eddy current domain

The duality between the two meshes allows to define two sets of unknowns each one linked to a specific space entity:

- electric current flowing through the dual faces ι ($\tilde{N}_F = N_E$);
- electro-motive force (emf) along primal edges ε (N_E).

By exploiting orthogonality between meshes and imposing a local uniformity of electric field and current density around the face-edge couple, the constitutive Ohm equation can be written as:

$$\varepsilon_k = \lambda_k E_{tk} ; \iota_k = \tilde{s}_k J_{nk} \Rightarrow \varepsilon_k = \rho \frac{\lambda_k}{\tilde{s}_k} \iota_k$$

where index k identifies the face-edge couple, □$_k$ is the length of the primal edge and $\tilde{s}_k$ is the area of the dual face, E_{tk} and J_{nk} are the tangential and normal component of electric field and current density on edge and face respectively.

From the discretization performed, the domain under study contains $2N_E$ unknowns and an equal number of constraints must be set. It must be remarked that, due to the definition of dual meshes of fig. 9, boundary dual faces can be neglected because they naturally have a null current value.

The use of global variables now makes the problem very similar to an electric circuit where, with a certain number of branches n, gives rise to $2n$ unknown values: n currents and n electro-motive forces. Following a way of reasoning similar to that of circuits, by imposing constitutive equations (terminal characteristics), N_E constraints can be set. The remaining constraints must be obtained by imposing the satisfaction of the field equations. Under the hypothesis of quasi-stationary magnetic field, the solution has to satisfy two set of constraints:

- current flow on closed surfaces must be null (Kirchhoff current law on nodes);
- Faraday law must be satisfied on closed loops (Kirchhoff voltage law on loops).

Thus N_N-1 Kirchhoff independent current laws can be imposed on N_N -1 dual cells, while the remaining N_E-N_V+1 unconstrained values can be obtained by Faraday laws imposed on fundamental loops once a tree has been defined along primal mesh edges.

In order to take into account magnetic coupling among "components", linked flux can be expressed by means of circulation of magnetic vector potential:

$$p_\lambda = \int_\lambda \vec{A} \cdot d\vec{l}$$

by computing off-line the contributes of the source conductors p_s, the following integral equations can be obtained:

$$\sum_{m=1}^{N_i} \rho \frac{\lambda_m}{s_m} \iota_m + \frac{d}{dt} \sum_{m=1}^{N_i} \left(\sum_{k=1}^{N_E} C_{mk}(P) \iota_k \vec{e}_k \cdot \lambda_m \vec{e}_m \right) = -\frac{d}{dt} \sum_{m=1}^{N_i} p_{Sm}$$

(3)

where N_i is the number of edges belonging to the i-th fundamental loop, $C_k(P)$ are geometric coefficients obtained by integrating Biot-Savart law on the induced currents, $\vec{e}_k$ and $\vec{e}_m$ are the versors of k-th and m-th edge and p_S is the electro-kinetic momentum created by imposed current sources.

In case of sinusoidal excitation, equation (2) can be expressed in terms of phasor quantities, and becomes:

$$\sum_{m=1}^{N_i} \rho \underline{\iota_m} \frac{\lambda_m}{s_m} + j\omega \sum_{m=1}^{N_i} \left(\sum_{k=1}^{N_E} C_{mk}(P) \underline{\iota_k} \frac{\lambda_k}{s_k} \vec{e}_k \right) \cdot \vec{e}_m \lambda_m = -j\omega \sum_{m=1}^{N_i} \underline{p_{Sm}}$$

(3)

where underlined variables are the phasor of currents and magnetic vector potential circulations.

B. Network model building

Looking in particular to the equations obtained, the shield surface can be represented as the network of components of fig. 10. Each primal edge is represented as a lumped parameter component with an ohmic resistance, inductive parameter and an e.m.f. generator. The magnetic component takes into account self and mutual inductance effects computed by means of the coefficients C_{mk} present in loop equation and one e.m.f. generator tied to the linked flux generated by current sources (p_{sm}). The result obtained is equivalent to the one presented with a finite elements method by Albanese and Rubinacci [10], but here it has been obtained in an independent way, which does not make use of differential formulation.

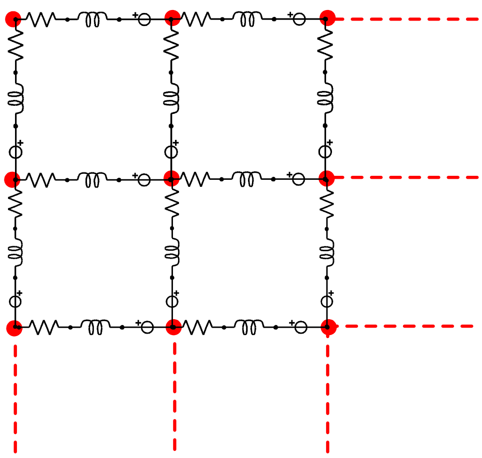


Fig. 10 Equivalent electric circuit of the conductive shield

C. Numerical results

The time-harmonic version of the proposed formulation has been implemented in a computational procedure, which has been extensively tested versus measurements and two dimensional codes, obtaining a very good level of accuracy [11]. In figure 11 the pattern of eddy currents induced in a Aluminium shield 2 mm thick over a two wire 50 Hz conductor system is shown, while in Fig. 12 the same pattern is presented for a 1kHz solenoid shielded by a closed square pipe.

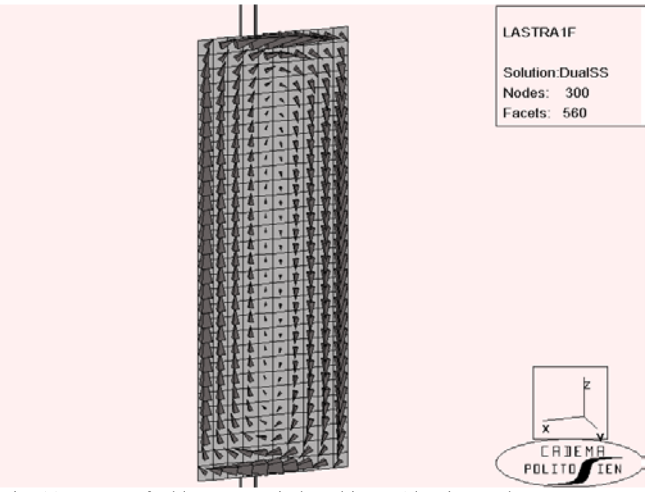


Fig. 11 Pattern of eddy currents induced in an Aluminum plate over a 50 Hz two wires line

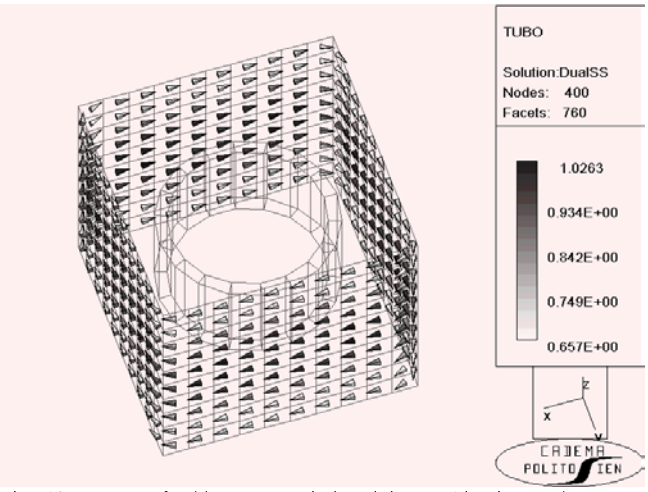


Fig. 12 Pattern of eddy currents induced in an Aluminum close square pipe around a 1 kHz solenoid (A/m^2/A)

6. Conclusions

The paper has presented the application of discrete formulation of electromagnetic field to the solution of two simple cases of time-varying electromagnetic fields. The use of the discrete formulation allows to draw important relations between this solution and an equivalent circuit representation of the problem. This approach, based on rigorous treatment of field quantities, by means of global variables, can be used in industrial problem solving and in teaching electromagnetic field analysis where the application of the circuit method can simplify the treatment of the problem.

Acknowledgment

Authors wish to thank Prof. Enzo Tonti for his careful reading of paper draft and for his valuable suggestions.

References

[1] A. Bossavit, L. Kettunen, "Yee-like schemes on staggered cellular grids: a synthesis between FIT and FEM approaches", IEEE Trans on Magnetics, Vol. 36, NO. 4, July 2000, pp. 861-867.
[2] E. Tonti, "A discrete formulation of field laws: the cell method", CMES Computer Modeling in Engineering and Science, Vol. 2, NO. 2, 2001, pp. 237-258.
[3] E. Tonti, "Finite formulation of Electromagnetic field", ICS Newsletter, Vol.8, No. 1, July 2001.
[4] M. Clemens, T. Weiland, "Discrete electromagnetics: Maxwell's equations tailored to numerical simulation", ICS Newsletter, Vol.8, No. 2, July 2001.
[5] K.S. Yee, "Numerical solution of boundary value problems involving Maxwell equations in isotropic media", IEEE Trans. Antennas Propagat., Vol. AP-14, pp. 302-307, 1966.
[6] M. Marrone, "The Equivalence Between Cell Method, FDTD and FEM", IEE Fourth International Conference on Computation in Electromagnetics, 8 - 11 April, 2002.
[7] M. Repetto, F. Trevisan, "3D Magnetostatic with the Finite Formulation", presented at 10th CEFC 2002 Conference, Perugia Italy, June 16-19, 2002, to be published on IEEE Trans. on Magnetics, 2003.
[8] L. Kettunen, "Fields and circuits in computational electromagnetism", IEEE Trans on Magnetics, Vol. 37, NO. 5, Sept. 2001, pp. 3393-3396.
[9] H. Igarashi, A. Kost, T. Honma, "A three dimensional analysis of magnetic shielding with thin layers" proceedings of 7th Int. IGTE Symposium, Graz, Austria, 1996.
[10] R. Albanese, G. Rubinacci, "Finite Element Method for the Solution of 3D Eddy Current Problems", Advances in Imaging and Electron Physics, vol.102, pp.1-86, April 1998.
[11] A. Canova, GB. Gruosso, M. Repetto, "Quasi-static integral formulation using duality and network equations", proceedings of CEM2002, Bournemouth UK, April 8-11, 2002.

Aldo Canova, Giambattista Gruosso, Maurizio Repetto
Dipartimento di Ingegneria Elettrica Industriale
Politecnico di Torino
C.so Duca degli Abruzzi 24
10129 Torino, Italy
(aldo.canova@polito.it, gruosso@elet.polimi.it, maurizio.repetto@polito.it)