Magnetic field analysis coupled with electric circuit and motion equation

I. Introduction

Electromagnetic actuators are widely used in various electronic instruments. To determine the optimum design for actuators, it is necessary to analyze their dynamic characteristics accurately [1],[2]. The use of the 3-D finite element method to analyze the dynamic characteristics of actuators is described in this technical article. The finite element method is coupled with the electric circuit and motion equations to take into account the motion of the armature and the applied voltage equations. The method is applied to obtain the dynamic characteristics of a linear solenoid actuator, the induced voltage characteristics of a permanent magnet motor taking into account the end-coil and the dynamic characteristics of a claw-poled stepping motor. The validity of the 3-D method is confirmed using the results of experiments for each model.

II. Analysis Method

A.3-D Finite Element Analysis with Edge Elements

The fundamental equations for the magnetic field taking into account the eddy currents can be written as follows [1]:

$$\mathrm{rot}(\nu \mathrm{rot}\, \boldsymbol{A}) = \boldsymbol{J}_0 + \boldsymbol{J}_e + \nu_0 \mathrm{rot} \boldsymbol{M} \tag{1}$$

$$\boldsymbol{J}_e = -\sigma\left(\frac{\partial A}{\partial t} + \mathrm{grad}\phi\right) \tag{2}$$

$$\mathrm{div} \boldsymbol{J}_e = 0 \tag{3}$$

where ν is the reluctivity, ν_0 is the reluctivity of vacuum ($=1/\mu_0$), A is the magnetic vector potential, $\boldsymbol{J}_0$ is the exciting current density, $\boldsymbol{J}_e$ is the eddy current density, $\boldsymbol{M}$ is the magnetization of the permanent magnet, σ is the conductivity and ϕ is the electric scalar potential.

When using edge elements it is possible to solve without using the electric scalar potential, ϕ, in (2). Including ϕ causes an increase in the number of unknowns, however, it also reduces the number of ICCG iterations required [3].

In the case of tetrahedral edge elements, the vector potential $\boldsymbol{A}^{(e)}$ of the element e is denoted as follows [4]:

$$A^{(e)} = \sum_{ie=1}^{6} N_{ie} \cdot A_{ie} \tag{4}$$

where $\boldsymbol{A}_{ie}$ is the unknown variable for the ie-th edge, $\boldsymbol{N}_{ie}$ is the vector interpolation function for the ie-th edge, and can be written as follows:

$$N_{ie} = \lambda_{je} \mathrm{grad}\lambda_{ke} - \lambda_{ke} \mathrm{grad}\lambda_{je} \tag{5}$$

where λ_{je} is the barycentric coordinate of point (x,y,z) in the tetrahedron with respect to node je.

Galerkin's method is applied to obtain the system equations. The following equations are derived from (1), (2), (3) and (4).

$$\begin{aligned} G_i = &\int_\Omega \mathrm{rot} N_i \cdot (\nu \mathrm{rot} A) dv - \int_{\Gamma_{j0}} N_i \cdot J_0 dv \\ &+ \int_{\Gamma_{je}} N_i \cdot \left\{\sigma\left(\frac{\partial A}{\partial t} + \mathrm{grad}\phi\right)\right\} dv \\ &- \int_S N_i \cdot \{(\nu \mathrm{rot}\, A) \times n\} dS = 0 \end{aligned} \tag{6}$$

$$\begin{aligned} G_{di} = &\int_{\Gamma_{je}} \mathrm{grad} N_i \cdot \left\{\sigma\left(\frac{\partial A}{\partial t} + \mathrm{grad}\phi\right)\right\} dv \\ &+ \int_{S_e} N_i \left\{-\sigma\left(\frac{\partial A}{\partial t} + \mathrm{grad}\phi\right)\right\} \cdot \boldsymbol{n}\, dS = 0 \end{aligned} \tag{7}$$

where N_i is the scalar interpolation function for ϕ, Ω denotes the analyzed region. Γ_{je} and Γ_{j0} are the region of the conductors with eddy currents and that of the coil respectively. S and S_e are the boundary of analyzed region and the eddy current region, respectively. $\boldsymbol{n}$ is the unit outward normal vector on the surface S and S_e.

B. Coupled Analysis with the Electric Circuit

When a voltage source is applied to the coil of the actuator, the exciting current is unknown. To calculate the magnetic field in this case it is necessary to couple the system equation (1) to the equation for the electric circuit.

The equation of an electric circuit connected to a voltage source is given as follows [5]:

$$E = V_0 - RI_0 - \frac{d\psi}{dt} = 0 \tag{8}$$

where V_0 is the applied voltage, R is the resistance, and I_0 is the exciting current. ψ is the interlinkage flux of the exciting coil as follows:

$$\psi = \frac{n_c}{S_c} \int_{\Gamma_{j0}} A \cdot \boldsymbol{n}_s dv \tag{9}$$

where n_c and S_c are the number of turns and the cross-sectional area of the coil respectively. $\boldsymbol{n}_S$ is the unit vector along with the direction of the exciting current and Γ_{j0} is the region of the coil.

The exciting current I_0 is represented using the exciting current density $\boldsymbol{J}_0$ as follows:

$$\boldsymbol{J}_0 = \frac{n_c}{S_c} I_0 \boldsymbol{n}_s \tag{10}$$

It is necessary to know the current density vectors in the coil. However, it is difficult to know the direction of the current exactly in case where the shape of coil is complex. Therefore, the current density vector should be obtained by solving the following equation. In this case, the region to analyze is limited to the coil.

$$\mathrm{rot}\left(\frac{1}{\sigma}\mathrm{rot}\boldsymbol{T}\right) = 0, \quad \boldsymbol{J} = \mathrm{rot}\boldsymbol{T} \tag{11}$$

where σ is the electric conductivity and $\boldsymbol{T}$ is the current vector potential.

If the current density $\boldsymbol{J}$ does not satisfy the following continuous condition in the 3-D finite element analysis with edge elements, then no exact solution can be obtained:

$$\mathrm{div}\boldsymbol{J} = 0 \tag{12}$$

The continuity of current density is always satisfied when the current distribution in the conductors is calculated by (11) as follows:

$$\mathrm{div}\boldsymbol{J} = \mathrm{div}(\mathrm{rot}\boldsymbol{T}) = 0 \tag{13}$$

Furthermore, the boundary condition of $\boldsymbol{T}$ satisfies the following equation:

$$\oint T ds = I \quad (14)$$

where I is the current through the coil.

The calculated results of $\boldsymbol{J}$ from (11), when the current I in (14) is 1.0 Ampere, is used as $\boldsymbol{n}_S/S_c$ in (10). Therefore, the current density vectors and the flux distribution excited from the voltage source can be analyzed by coupling (6), (7) and (8).

C. Coupled Analysis with the Equations of Motion

The electromagnetic force of the actuator is obtained using the Maxwell stress tensor [1,6].

The linear motion or rotation motion of an armature is obtained by (15) or (16) respectively using numerical calculation [5].

$$M\frac{d^2z}{dt^2}+D\frac{dz}{dt}\pm F_k \pm F_g = F_z \quad (15)$$

where M is the mass of the moving part, z is the displacement of the armature, D is the viscous damping coefficient, F_k is the force of the load, F_g is the force of the gravity and the friction, F_z is the electromagnetic force acting on the armature.

$$I\frac{d^2\theta}{dt^2}+D\frac{d\theta}{dt}\pm T_k \pm T_g = T_m \quad (16)$$

where I is the polar moment of inertia of the moving part, θ is the rotation angle of the armature, T_k is the torque of the load, T_g is the torque of the friction, Tm is the torque acting on the armature.

The displacement of the armature can be obtained by solving (15) or (16) using time stepping [6].

D. Automatic Mesh Modification Taking into Account Movement of the Armature

1) Linear Motion

The finite element mesh should be changed according to the movement of the armature. In our method, an initial mesh as shown in Fig. 1(a) is prepared. The initial mesh is divided into 2 areas as shown in Fig. 1(b). The armature is included in one area, i.e. the armature area, and the stator is included in the other area, i.e. the stator area. The new coordinates of each node in the armature area can be calculated by the movement of the armature as shown in Fig. 1(c). The front part of the moving area is moved to the rear of the moving area as shown in Fig. 1(d). The nearest nodes between the armature area and the stator area are connected automatically at each armature position as shown in Fig. 1(e).

2) Rotational Motion[7]

The mesh should be changed according to the rotation angle of the armature. Our method is as follows. First of all, an initial mesh as shown in Fig. 2(a) is prepared. The mesh is divided into 2 areas as shown in Fig. 2(b). One is a armature area and the other is a stator area. The new coordinates of each node in the armature area can be calculated by the rotation angle of the armature as shown in Fig. 2(c). The nodes between the armature area and the stator area are connected automatically at each rotation angle as shown in Fig. 2(d).

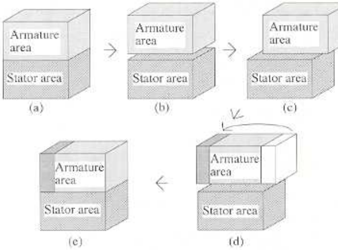

Fig. 1 Automatic connection of meshes: (a) initial mesh, (b) separating mesh, (c) moving mesh, (d) connection, (e) mesh after connection.

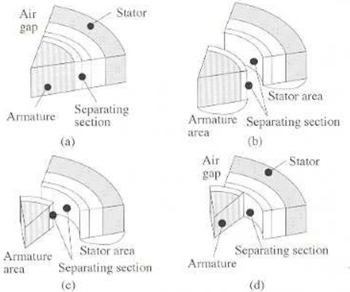

Fig. 2 Automatic rotation of the mesh: (a) initial mesh, (b) separating mesh, (c) rotating armature area, (d) mesh after connection.

III. Examples of 3-D Finite Element Analysis

A. Dynamic Analysis of Linear Solenoid

Linear electromagnetic solenoids (LES) are used in a wide range of applications because they have a simple structure and can produce a large thrust with a comparatively short stroke. A low noise-type solenoid in which the plunger goes through its frame yoke can not produce so big a thrust because of the leakage flux between the plunger and the frame yoke. Therefore, it is important to improve the efficiency when this type of solenoid is adopted as an actuator for home electric products. The 3-D finite element method is very effective for the design of such actuators.

In this section, the dynamic response analysis of a linear electromagnetic solenoid is analyzed using the 3-D finite element method.

Fig. 3 shows the basic construction of a low noise-type solenoid. This model consists mainly of the plunger, the yoke frame and the coil. The plunger goes through the frame yoke with a gap of 0.3 mm. Both materials are made of electromagnetic soft iron with an electric conductivity of 7×10^6 S/m. The number of turns, the resistance of the coil and the spring constant are 960 turns, 12 Ω and the 1 N/mm, respectively. Fig. 4 shows the 3-D finite element meshes of the analyzed model. The analytical region is reduced to 1/4 of the whole region because of the symmetry of this model.

The eddy currents which flow in the plunger and the frame yoke are taken into consideration.

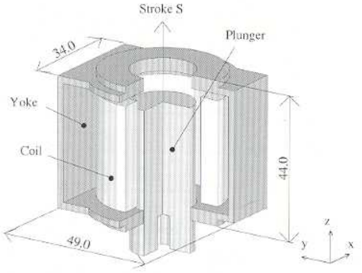

Fig. 3 Analyzed model of solenoid.

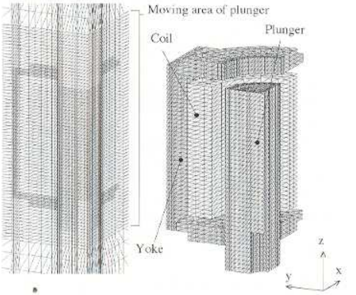

Fig. 4 3-D meshes(1/4 region).

Fig. 5 shows the time variations of the stroke when the DC voltage of 20V is applied at the stroke S=-8 mm. As shown in this figure, it is found that the computed result without friction continues to vibrate, and the computed result with friction is in a good agreement with the measured one. As mentioned above, the usefulness of this method is clarified. This method can be applied to investigate the influences of the geometry and the material of the plunger on the dynamic performance.

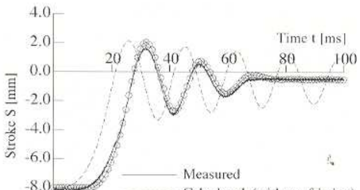

Fig. 5 Time variations of stroke.

B. Induced Voltage Analysis of Permanent Magnet Motor

In this section, the induced voltage characteristics of a permanent magnet motor taking into account the magnetization and end-coil effects are analyzed.

Fig. 6 shows the analyzed model of a permanent magnet motor. This motor has anisotropic permanent magnets. It is important to know the directions of the magnetization vectors $\boldsymbol{M}$ of the permanent magnets accurately, when the motor with anisotropic permanent magnets is analyzed. The directions of magnetization vectors $\boldsymbol{M}$ of the anisotropic permanent magnets are obtained by an analysis taking into account the magnetizing process.

Fig. 7 shows the 3-D finite element meshes of the analyzed motor. The analyzed region is reduced to 1/8 of the whole region because of the symmetry of the model. The rotor speed is 600 rpm. The induced voltage characteristics with end-coil and without end-coil are analyzed.

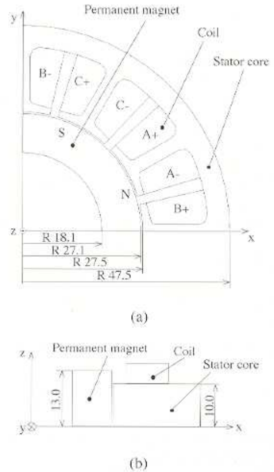

Fig. 6 Analyzed model of permanent magnet motor: (a) x-y plane; (b) x-z plane.

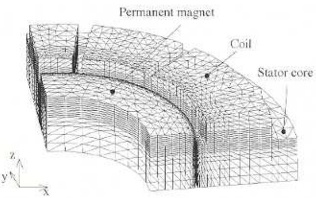

Fig. 7 3-D finite element meshes.

Fig. 8 shows the calculated results for the magnetization vectors $\boldsymbol{M}$ in the permanent magnets taking into account the magnetizing process.

Fig. 9 shows the induced line-to-line voltage waveforms. The 2-D calculated result is also shown in this figure. It is found that the calculated results with the end-coil agree better with the measured results than the calculated results without the end-coil. It is also found that the 2-D calculated result has a large discrepancy, because the 2-D model can not take into account the difference in the length of the stator and the armature. Therefore, an accurate induced voltage can be calculated by analysis taking into account the end-coil effect.

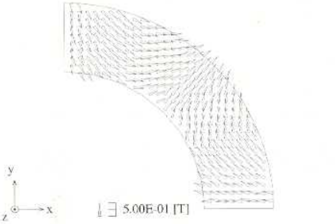

Fig. 8 Distribution of the magnetization vectors $\boldsymbol{M}$ in the permanent magnet.

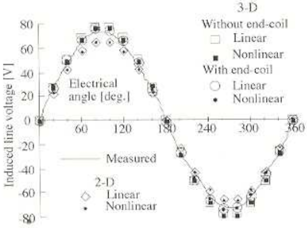

Fig. 9 Induced line-to-line voltage waveforms.

C. Dynamic Response Analysis of Claw-Poled Stepping Motors

Stepping motors are widely used in the field of OA machine tools and precision instruments such as printers or facsimiles because accurate positioning control can be made easily. Therefore, for the optimum design of these stepping motors, it becomes very important to accurately solve the dynamic step response of the motor. However, it is difficult to determine the various factors which effect the characteristics of the motor from the experimental methods, because the shapes of these stepping motors are very complicated.

In this section, the dynamic step response characteristics of a craw-poled stepping motor are analyzed using the 3-D finite element method.

Fig. 10 shows the analyzed model of a permanent magnet claw-poled stepping motor, that is used for the movement of a printer head. It consists of an armature and two stator sections. The armature consists of a shaft and two multi-polarized permanent magnets which are composed of upper and lower sections. Each stator has a bobbin-wound coil and yoke with claw poles, and both stators are sandwiched between two plates. The armature has twelve pairs of poles. Therefore, 1/12 of the whole region is analyzed. Fig. 11 shows the 3-D finite element meshes. The number of elements is 359,110.

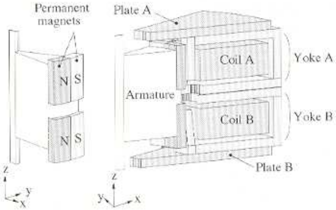

Fig. 10 Analyzed model of a claw-poled stepping motor.

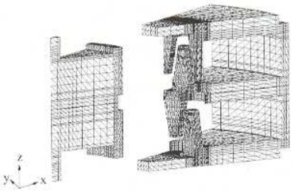

Fig. 11 3-D finite element meshes(except coils and air).

The dynamic step response characteristic is calculated when a square pulse voltage is applied to the coil at time t=5.0 ms.

Fig. 12 shows the calculated and measured time variation of the rotation angle of the armature. The calculated step response, which is 7.5 degree for one step, agrees well with the measured one.

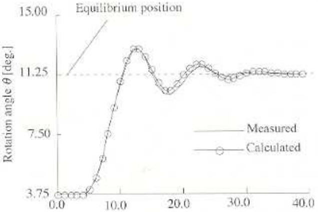

Fig. 12 Time variation of rotation angle.

IV. Conclusions

This technical article described a method that uses the 3-D finite element method to analyze the dynamic characteristics of an actuator taking account of the motion of the armature. Furthermore, some examples calculated by this method are also described. The usefulness of this method for the analysis of the dynamic characteristics of actuators is clarified.

References

[1] Y. Kawase and S. Ito, *New Practical Analysis of Electrical and Electronic Apparatus by 3-D Finite Element Method*, Morikita Publishing Co., 1997.

[2] S. Ito and Y. Kawase, *New Computer Aided Engineering of Electric and Electronic Apparatus using Finite Element Method*, Morikita Publishing Co., 2000.

[3] K.Fujiwara, T.Nakata and H.Ohashi?"Improvement of Convergence Characteristics of ICCG Method for the A-φ Method using Edge Elements", *IEEE Trans. Magn.*, Vol.32, No.3, pp. 804-807 (1996)

[4] M.L.Barton and Z.J.Cendes,"New Vector Finite Elements for Three-Dimensional Magnetic Field Computation", *IEEE J. Appl. Phys.*,Vol.61, No.8, pp. 3919-3921 (1992)

[5] T. Nakata et. al.,"3-D Finite Element Method for Analyzing Magnetic Field in Electric Machines Excited from Voltage Source", *IEEE Trans. Magn.*, Vol.24, No.6, pp. 2582-2584 (1988)

[6] Y. Kawase, H. Kikuchi and S. Ito,"3-D Nonlinear Transient Analysis of Dynamic Behavior of The Clapper Type DC Electromagnet", *IEEE Trans. Magn.*, Vol.27, No.5, pp. 4238-4241 (1991)

[7] Y. Kawase, T. Yamaguchi and Y. Hayashi,"Analysis of Cogging Torque of Permanent Magnet Motor by 3-D Finite Element Method", *IEEE Trans. Magn.*, Vol.31, No.3, pp. 2044-2047 (1995)

Yoshihiro Kawase

Fourth International Workshop on

Computational Electromagnetics in the Time-domain - TLM, FDTD and Related Techniques (CEM-TD)

Date: 17-19 September 2001
Venue: University of Nottingham, Nottingham, UK
Duration: Three days

Objectives:

- To bring together modellers and users of numerical techniques in the time-domain
- To facilitate the interchange of ideas between researchers working with different methods
- To give an impetus to the hybridization of methods and domains of application
- To offer a forum for the exchange of applications experience and for model validation

Format:

The event will be organised as a workshop with several structured interactive sessions and forums to allow for a wide discussion and interaction between participants. A number of longer tutorial presentations will also be held to give an overview of advances and to introduce complex topics to a wider audience. An applications forum will be held where results will be presented from the application of TD methods to complex problems and also canonical problems. Selected papers from the Workshop will appear in a Special Issue of the Int. Journal of Numerical Modelling. In order to maintain the character of a Workshop there will be a limit to the total number of attendees.

Preliminary list of planned sessions:

Colleagues wishing to submit papers should indicate which of the following sessions are the most appropriate:

1. Foundations of TLM/FDTD
2. Methodological advances in TLM/FDTD
3. Methodological Advances in FETD/FVTD/IETD/FITD
4. Hybridization in the time-domain
5. Sub-cell models in the time-domain
6. EM simulation coupled to other physical domains (eg circuits, thermal)
7. Canonical problems forum (eg http://www.emcs.org/tc9/)
8. Applications forum
9. Short papers (poster session)

Those wishing to present papers at the workshop should submit a three-page abstract (single-spaced, 12 points) describing the contribution the paper will make and the most appropriate sessions for it. The abstract should include results demonstrating the value of the contribution.

A full paper not exceeding 10 pages will be required for the accepted contributions.

Timetable:

Abstract deadline- 6 Nov 2000
Acceptance of abstracts confirmed- 8 Jan 2001
Full paper required by - 4 June 2001

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