

Physics-Informed Neural Networks Based on Maxwell's Equations for Modeling Power-Bus Structures

Kazuhiro Fujita¹, *Member, IEEE*

¹ Saitama Institute of Technology, Fukaya, Saitama, Japan, kfujita@sit.ac.jp

The physics-informed neural network (PINN) method is of great interest for a wide range of applications in physics and engineering. Recently, the PINN method based on the Helmholtz equation has been successfully applied to the electromagnetic modeling of power-bus structures on printed circuit board. In the present study, a PINN method based on Maxwell's equations in the frequency domain is presented as one more possibility for its modeling. This method is verified in comparison with the analytical solution of circular power-bus structure. It is also applied to a rectangular structure with round corners and a voltage distribution is discussed.

Index Terms—Artificial neural networks, deep learning, electromagnetic fields, Maxwell equations, printed circuits.

I. INTRODUCTION

THE PHYSICS-INFORMED neural network (PINN) method [1] is of great interest in the interdisciplinary field of machine learning and scientific computing and also for a wide range of applications in physics and engineering. Recently, the PINN method has been successfully applied to the solution of electromagnetic (initial-) boundary-value problems, see e.g. [2]–[5]. More recently, the PINN method for modeling power-bus structures on printed circuit boards was introduced in the research area of electromagnetic compatibility [6].

In the present study, a PINN method based on 2-D Maxwell's equations in the frequency domain is proposed as one more possibility for full-wave modeling of power-bus structures. Unlike the previous method in [6], the proposed method embeds 2-D Maxwell's equations directly into the loss function of a deep neural network (DNN) to be trained. The main purpose of this study is to show the applicability of the proposed method to power-bus structures.

To the best of the author's knowledge, the proposed method is the first application of PINN to Maxwell's equation-based full-wave modeling of power-bus structures.

II. METHOD

Here we model a power-bus structure with power and ground planes and a dielectric substrate as in Fig.1. The planes are separated with the distance h . The substrate has the depth h and the relative permittivity ϵ_r and the loss tangent $\tan\delta$. Here we assume that h is much smaller than wavelengths of interest. In this case, variations of fields in the vertical direction to the planes are neglected. The electric and magnetic fields (E_z , H_x , H_y) in the structure are described by 2-D Maxwell's equations in a Cartesian coordinate system. Note that, due to the existence of the dielectric loss, the field solution will have both real and imaginary parts. Therefore, the output layer has six outputs for the real and imaginary parts of the fields. Although omitted for simplicity, it is necessary to transform the equations, boundary condition and excitation condition under the proper scaling formulation [4],[5].

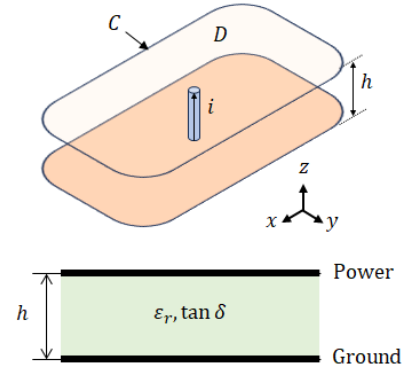


Fig. 1. Two-dimensional boundary-value problem for power-bus structure.

To represent the field components in the structure, we adapt a fully connected NN with an input layer, an output layer and $L-1$ hidden layers. This DNN can be recursively defined as:[7]

$$N^0 = \mathbf{r} \quad (1)$$

$$N^l = \sigma(\mathbf{W}^l N^{l-1} + \mathbf{b}^l), l = 1, 2, \dots, L-1 \quad (2)$$

$$N^L = \mathbf{W}^L N^{L-1} + \mathbf{b}^L \quad (3)$$

where $\mathbf{r} \equiv (x, y)$ is the position vector in the structure, N^0 is the input layer, N^L is the output layer, N^l is a hidden layer, σ is a nonlinear activation function, $\mathbf{W}^l$ is the weight matrix for the connection between layers $l-1$ and l , $\mathbf{b}^l$ is the bias vector for the layer l . Note that the output layer N^L is corresponding to the field solution.

Here we briefly summarize the proposed PINN method based on Maxwell's equations in the frequency domain for modeling power-bus structures. First we construct the above DNN as the solution surrogate to 2-D Maxwell's equations in the structure. We use the Glorot uniform initializer [7] for the weights and set the biases to zero. We set up physical constants, material and geometry parameters and excitation ports. Next, we generate (quasi-) randomly (or regularly) sampled points within the 2-D domain D , on its contour C and the excitation port. One can use the Latin-hypercube sampling and the Sobol sequence [8] for this purpose. There is no point outside the structure and inside the port. Note that field data are not generated and not required for training. Then, we need to train the constructed DNN to satisfy Maxwell's equations, the boundary and excitation conditions by embedding them into the loss function L of the

DNN. L is defined by the weighted summation of the mean square errors (MSE). For the training process, we find the best parameters $\theta = \{\mathbf{W}^l, \mathbf{b}^l\}_{1 \leq l \leq L}$ by minimizing L via a gradient-based optimizer [8], until L gets smaller than a threshold ϵ or the number of iterations gets larger than N_{max} . Note that the differential operators in Maxwell's equations and the boundary condition can be evaluated using the so-called automatic differentiation (AD) [9]. An overview of the proposed PINN method is shown in Fig.2.

After the above training process, one can predict the field components at any position inside the power-bus structure. The input impedance Z_{in} is obtained from the predicted field.

Throughout this work, we used the Swish activation function, 8 hidden layers and 60 neurons per layer, and the Adam optimizer [8]. The training process is performed with $N_{max}=100000$ or $\epsilon=10^{-5}$.

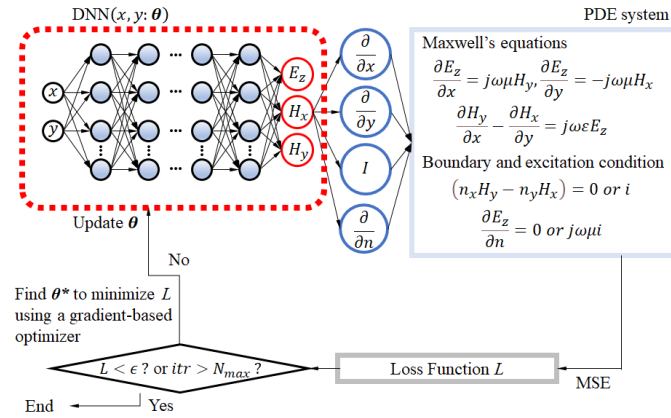


Fig. 2. Overview of physics-informed neural networks based on Maxwell's equations in the frequency domain.

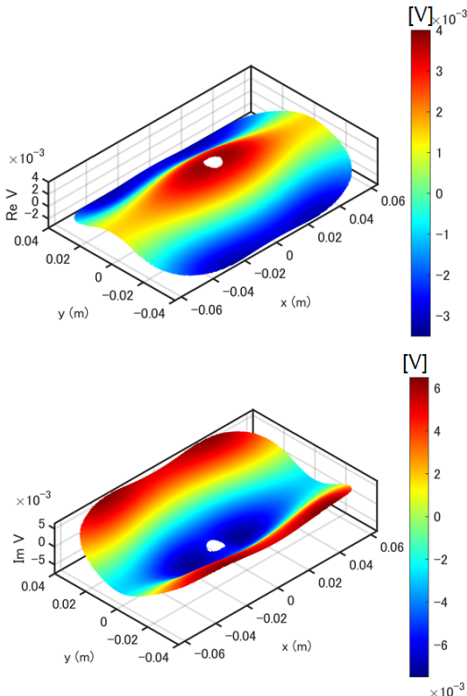


Fig. 3. PINN-predicted voltage distribution for the rectangular power-bus structure with round corners at 2GHz. (Top) real, (bottom) imaginary parts.

III. NUMERICAL RESULTS

Here we apply the proposed PINN method to two different power-bus structures to show the applicability of the PINN to various power-bus structures.

The first example is the circular power-bus structure with radius $R=4.0\text{cm}$, $h=0.48768\text{mm}$, $\epsilon_r=4.0$, $\tan\delta=0.02$. We generated 4658 sampling points for D , 1000 points for C and 100 points for the port. The PINN-predicted and analytical [10] impedances $|Z_{in}|$ at 2GHz are 4.0389 and 4.0435, respectively. The relative error is about 0.1%.

The second example is the rectangular power-bus structure with round corners and width $a=12.8\text{cm}$, length $b=8.0\text{cm}$, $h=0.48768\text{mm}$, $\epsilon_r=4.0$, $\tan\delta=0.02$ as shown in Fig.1. We used 8824 sampling points for D , 1600 points for C and 100 points for the port. Fig.3 illustrates the real and imaginary parts of the PINN-predicted voltage distribution at $f=2.0\text{GHz}$. The voltage distribution of the rectangular structure with round corners is quite different from that of the circular one presented in [6]. This result can be attributed to the difference between the two structure geometries.

IV. CONCLUSION

The PINN method based on 2-D Maxwell's equations in the frequency domain for full-wave modeling of power-bus structures has been proposed. The proposed method has been applied to the circular structure and the rectangular structure with round corners. The PINN-predicted impedance has been consistent with the analytically calculated one. An effect of structure geometry on the voltage distribution has been also demonstrated.

REFERENCES

- [1] M. Raissi, P. Perdikaris, and G. E. Karniadakis, "Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations," *J. Comput. Phys.*, vol. 378, pp. 686-707, Feb. 2019.
- [2] Y. Chen, L. Lu, G.E. Karniadakis, and L.D. Negro, "Physics-informed neural networks for inverse problems in nano-optics and metamaterials," *Opt. Exp.*, vol.28, no.8, pp.11618-11633, April, 2020.
- [3] P. Zhang, Y. Hu, Y. Jin, S. Deng, X. Wu, and J. Chen, "A Maxwell's equations based deep learning method for time domain electromagnetic simulations," *IEEE J. Multiscale Multiphys. Comput. Techn.*, vol. 6, pp. 35-40, 2021.
- [4] K. Fujita, "Physics-informed neural network method for space charge effect in particle accelerators," *IEEE Access*, vol.9, pp.164017-164025, 2021.
- [5] K. Fujita, "Physics-informed neural network method for modelling beam-wall interactions," *Electron. Lett.*, vol.58, 390-392, 2022.
- [6] K. Fujita, "Modeling power-bus structures with physics-informed neural networks" *Proc. the 2024 IEEE Joint Int. Symp. EMC Japan/APEMC Okinawa*, 552-555, May, 2024.
- [7] I. Goodfellow, Y. Bengio and A. Courville, *Deep Learning (Adaptive Computation and Machine Learning series)*, The MIT Press, 2016.
- [8] Mykel J. Kochenderfer, Tim A. Wheeler, *Algorithms for Optimization*, The MIT Press, 2019.
- [9] A. G. Baydin, B. A. Pearlmutter, A. A. Radul, and J. M. Siskind, Automatic differentiation in machine learning: A survey, *J. Mach. Learn. Res.*, vol.18, pp. 5595-5637, 2017.
- [10] M. Xu and T. H. Hubing, "The development of a closed-form expression for the input impedance of power-return plane structures," *IEEE Trans. Electromagn. Compat.*, vol. 45, no. 3, pp. 478-485, Aug. 2003.