

Physics-informed Neural Networks based on Non-overlapping Domain Decomposition Methods for Axisymmetric Magnetostatic Problems

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Physics-informed neural network (PINN) is a method for training neural networks by constructing a loss function based on the error in the initial-boundary value problems for partial differential equations. This study focuses on the application of PINN to solve the axisymmetric magnetostatic field problems. PINN is learned using a point set but does not use connection information between points like the Yee's grid or edge elements. Therefore, interface conditions for electromagnetic fields are not expected to be satisfied naturally. To solve this issue, this study proposes a PINN algorithm based on domain decomposition methods that divides the analysis domain into conductor and non-conductor and satisfies the continuity of the magnetic flux density and magnetic field strength at the boundaries between subdomains by an iterative method.

Index Terms—Domain decomposition method, Interface boundary conditions, magnetostatic analysis, physics-informed neural networks.

I. INTRODUCTION

PHYSICS-INFORMED NEURAL NETWORKS (PINN) [1]-[2] is a machine learning method for predicting the behavior of physical phenomena described by partial differential equations (PDEs). Moreover, PINN is being studied as a numerical method for solving boundary value problems of PDEs, such as the finite element method (FEM) and the finite difference method. In the field of computational electromagnetics, there are applications to two-dimensional problems [3] and axisymmetric problems [4] in magnetostatic fields. Here, PINN evaluates the prediction error for the PDE and boundary conditions at points in the analysis domain and on the boundary and learns to minimize the mean-squared error at a given point set. In other words, it does not consider the connectivity between nodes, such as edge elements in the FEM or the Yee's grid in the FDTD method. Therefore, the interface conditions for electromagnetic fields are not expected to be satisfied naturally. To solve this issue, this study proposes a PINN algorithm based on non-overlapping domain decomposition methods (DDM-PINN). The DDM-PINN divides the analysis domain into two subdomains, conductor and non-conductor, and solves the continuity problem between the subdomains by an iterative method, in reference to DDM, which has been widely used in the FEM.

II. AXISYMMETRIC MAGNETOSTATIC PROBLEMS

A. Governing Equations

This study targets the linear magnetostatic field problem and deals with the axisymmetric problem shown in Fig. 1 (left). When the analysis domain Ω includes core, coil, and air, and homogeneous Dirichlet boundary conditions are imposed on the boundary $\partial\Omega$, the governing equations are expressed as follows [4]:

$$-\frac{\partial}{\partial r}\left(\frac{1}{\mu r}\frac{\partial(\bar{r}\bar{A}_\theta)}{\partial r}\right) - \frac{\partial}{\partial z}\left(\frac{1}{\mu}\frac{\partial\bar{A}_\theta}{\partial z}\right) = \bar{J}_\theta \quad \text{in } \Omega, \quad (1a)$$

$$\bar{A}_\theta = 0 \quad \text{on } \partial\Omega, \quad (1b)$$

where r is radius (m), z height (m), μ permeability (H/m), A_θ and J_θ circumferential component of magnetic vector potential (Wb/m) and current density (A/m²), respectively. The permeability of core is μ^{core} and that of air and coil is μ^{air} . Moreover, $\bar{x} = x^*x$ means nondimensionalization of a physical quantity x and nondimensionalization constants x^* in this study are $l^* = \max(r_l, z_l)$, $A_\theta^* = 10^{-1}$, $\mu^* = \mu^{\text{air}}$, $J_\theta^* = A_\theta^*/((l^*)^2\mu^*)$.

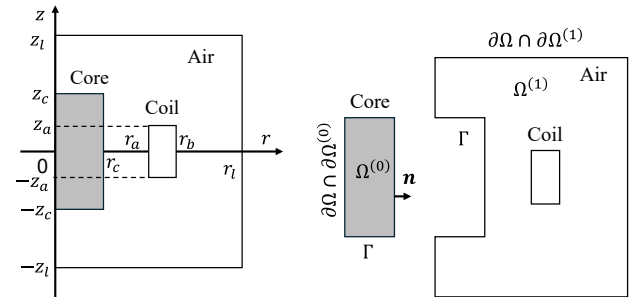


Fig. 1. An axisymmetric magnetostatic problem (left) and conductor-nonconductor domain decomposition model (right).

B. Domain Decomposition Methods

At the interface between the conductor and the nonconductor, the normal components of the magnetic flux density on the conductor side and the nonconductor side are equal, and the tangential components of the magnetic field strength are also equal. For example, it is well known that the FEM using the edge element provides a solution that satisfies these conditions. However, in the author's preliminary experiments, the results predicted by PINN for Eq. (1) did not satisfy the interface conditions at the conductor's surface. This is because the interface conditions are not incorporated into the physical laws

and geometric information such as connectivity between points is not used.

To solve this issue, this study adopts non-overlapping domain decomposition methods [5]. At first, the analysis domain is divided into two subdomains, a conductor $\Omega^{(0)}$ and a non-conductor $\Omega^{(1)}$ shown in Fig. 1 (right). Second, continuities of the magnetic flux density $\mathbf{B}$ (Wb/m²) and the magnetic field strength $\mathbf{H}$ (A/m) are considered explicitly as expressed by Eq. (2).

$$\left\{ \begin{array}{l} -\frac{\partial}{\partial \bar{r}} \left(\frac{1}{\bar{\mu} \bar{r}} \frac{\partial}{\partial \bar{r}} (\bar{r} \bar{A}_{\theta}^{(0)}) \right) - \frac{\partial}{\partial \bar{z}} \left(\frac{1}{\bar{\mu}} \frac{\partial \bar{A}_{\theta}^{(0)}}{\partial \bar{z}} \right) = \bar{J}_{\theta}^{(0)} \text{ in } \Omega^{(0)} \\ \bar{A}_{\theta}^{(0)} = 0 \text{ on } \partial \Omega \cap \partial \Omega^{(0)} \\ \mathbf{n} \cdot (\bar{\mathbf{B}}^{(0)} - \bar{\mathbf{B}}^{(1)}) = 0 \text{ on } \Gamma \\ \mathbf{n} \times (\bar{\mathbf{H}}^{(0)} - \bar{\mathbf{H}}^{(1)}) = 0 \text{ on } \Gamma \\ -\frac{\partial}{\partial \bar{r}} \left(\frac{1}{\bar{\mu} \bar{r}} \frac{\partial}{\partial \bar{r}} (\bar{r} \bar{A}_{\theta}^{(1)}) \right) - \frac{\partial}{\partial \bar{z}} \left(\frac{1}{\bar{\mu}} \frac{\partial \bar{A}_{\theta}^{(1)}}{\partial \bar{z}} \right) = \bar{J}_{\theta}^{(1)} \text{ in } \Omega^{(1)} \\ \bar{A}_{\theta}^{(1)} = 0 \text{ on } \partial \Omega \cap \partial \Omega^{(1)} \end{array} \right. , \quad (2)$$

where $\mathbf{n}$ is the unit normal vector on Γ . In Eq. (2), derivatives of the unknown function are added as boundary conditions, so this study develops a DDM-PINN method based on the Dirichlet-Dirichlet algorithm.

Let λ_B and λ_H be Lagrange multiplier for $\mathbf{B}$ and $\mathbf{H}$ on Γ , respectively. The initial value are set to $\lambda_{B_0} = \lambda_{H_0} = 0$. Then repeat steps 1 through 3 for $n = 0, 1, 2, \dots$ based on the modified Richardson iteration.

Step 1: Solving subdomain problems considering the continuity of $\mathbf{B}$ and $\mathbf{H}$ on Γ :

$$\left\{ \begin{array}{l} -\frac{\partial}{\partial \bar{r}} \left(\frac{1}{\bar{\mu} \bar{r}} \frac{\partial}{\partial \bar{r}} (\bar{r} \bar{A}_{\theta}^{(i)}) \right) - \frac{\partial}{\partial \bar{z}} \left(\frac{1}{\bar{\mu}} \frac{\partial \bar{A}_{\theta}^{(i)}}{\partial \bar{z}} \right) = \bar{J}_{\theta}^{(i)} \text{ in } \Omega^{(i)} \\ \bar{A}_{\theta}^{(i)} = 0 \text{ on } \partial \Omega \cap \partial \Omega^{(i)} \\ -\frac{\partial \bar{A}_{\theta}^{(i)}}{\partial \bar{z}} \cdot n_r + \frac{1}{\bar{r}} \frac{\partial (\bar{r} \bar{A}_{\theta}^{(i)})}{\partial \bar{r}} \cdot n_z = \lambda_{B_n} \text{ on } \Gamma \\ -\frac{1}{\bar{\mu}} \frac{\partial \bar{A}_{\theta}^{(i)}}{\partial \bar{z}} \cdot n_z - \frac{1}{\bar{\mu} \bar{r}} \frac{\partial (\bar{r} \bar{A}_{\theta}^{(i)})}{\partial \bar{r}} \cdot n_r = \lambda_{H_n} \text{ on } \Gamma \end{array} \right. . \quad (3)$$

Step 2: Solving subdomain problems with Dirichlet boundary conditions imposed on Γ :

$$\left\{ \begin{array}{l} \left(\frac{1}{\bar{\mu}} \frac{\partial}{\partial \bar{r}} \left(\frac{1}{\bar{r}} \frac{\partial (\bar{r} \bar{\psi}_{n+1}^{(i)})}{\partial \bar{r}} \right) + \frac{\partial}{\partial \bar{z}} \left(\frac{1}{\bar{\mu}} \frac{\partial \bar{\psi}_{n+1}^{(i)}}{\partial \bar{z}} \right) \right) = 0 \text{ in } \Omega^{(i)} \\ \bar{\psi}_{n+1}^{(i)} = 0 \text{ on } \partial \Omega \cap \partial \Omega^{(i)} \\ \bar{\psi}_{n+1}^{(i)} = \bar{A}_{\theta}^{(0)} + \bar{A}_{\theta}^{(1)} \text{ on } \Gamma \end{array} \right. . \quad (4)$$

Step 3: Updating Lagrange multipliers with $\omega = 0.5$:

$$\left\{ \begin{array}{l} \lambda_{B_{n+1}} = \lambda_{B_n} + \omega \left\{ \begin{array}{l} \left(-\frac{\partial \bar{\psi}_{n+1}^{(0)}}{\partial \bar{z}} n_r + \frac{1}{\bar{r}} \frac{\partial (\bar{r} \bar{\psi}_{n+1}^{(0)})}{\partial \bar{r}} n_z \right) \\ + \left(-\frac{\partial \bar{\psi}_{n+1}^{(1)}}{\partial \bar{z}} n_r + \frac{1}{\bar{r}} \frac{\partial (\bar{r} \bar{\psi}_{n+1}^{(1)})}{\partial \bar{r}} n_z \right) \end{array} \right\} \\ \lambda_{H_{n+1}} = \lambda_{H_n} + \omega \left\{ \begin{array}{l} \left(-\frac{1}{\bar{\mu}} \frac{\partial \bar{\psi}_{n+1}^{(0)}}{\partial \bar{z}} n_z - \frac{1}{\bar{\mu} \bar{r}} \frac{\partial (\bar{r} \bar{\psi}_{n+1}^{(0)})}{\partial \bar{r}} n_r \right) \\ + \left(-\frac{1}{\bar{\mu}} \frac{\partial \bar{\psi}_{n+1}^{(1)}}{\partial \bar{z}} n_z - \frac{1}{\bar{\mu} \bar{r}} \frac{\partial (\bar{r} \bar{\psi}_{n+1}^{(1)})}{\partial \bar{r}} n_r \right) \end{array} \right\} \end{array} \right. . \quad (5)$$

III. PHYSICS-INFORMED NEURAL NETWORKS

The DDM-PINN constructs approximation functions $\hat{A}_{\theta}^{(i)}$ for $\bar{A}_{\theta}^{(i)}$ in Step 1 and $\hat{\psi}_{n+1}^{(i)}$ for $\bar{\psi}_{n+1}^{(i)}$ in Step 2 by PINN. For the PINN, the multilayer perceptron neural network, whose number of layers is 8 and number of units in each layer is 16, is adopted and the hyperbolic tangent is used for the activation function. The Dirichlet boundary condition in Eq. (1b) is satisfied by setting $\hat{A}_{\theta} = D(\bar{r}, \bar{z}) \cdot \bar{A}_{\theta}$ using a distance function D whose value is zero on the boundary. The mean squared error is adopted for the loss function and the limited-memory BFGS method is used as the optimizer. Halton sequences are used as the training point set.

IV. NUMERICAL EXAMPLES

Numerical examples are demonstrated. The program used in the experiments was developed in PyTorch. The axisymmetric problem shown in Fig. 1 was trained using ordinary PINN and proposed DDM-PINN, and the magnetic flux density vector was predicted by automatically differentiating the function $\hat{A}_{\theta}$. Figure 2 illustrates the predicted results and the FEM result for comparison, and shows that the inference by DDM-PINN is closer to FEM than by PINN.

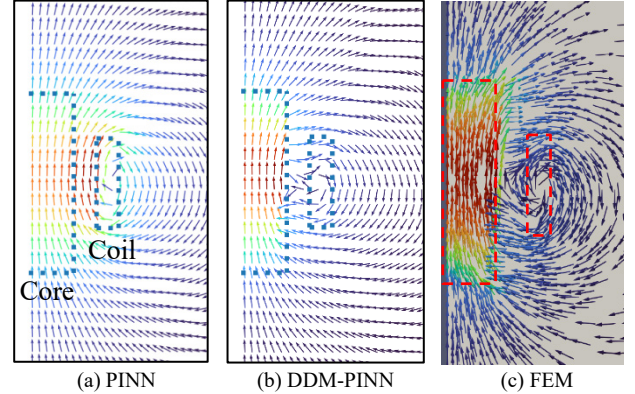


Fig. 2. Magnetic flux density vector.

V. CONCLUSION

To consider interface conditions between conductor and non-conductor in PINN, DDM-PINN based on the Dirichlet-Dirichlet algorithm was proposed.

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