

Computationally Efficient Sensitivity Analysis for robustness of Electric Motor considering Manufacturing Uncertainty

Dong-Hoon Ko¹, Yong-Min Lee¹, Soo-Hwan Park³ and Min-Ro Park^{1,2}

¹Department of Electrical and Communication Systems Engineering, Graduate School, Soonchunhyang University, Asan 31538, Korea

²Department of Electrical Engineering, Soonchunhyang University, Asan 31538, Korea

³Department of Mechanical, Robotics, and Energy Engineering, Dongguk University, Seoul 04620, Korea

This paper proposes a computationally efficient sensitivity analysis method to evaluate the impact of manufacturing uncertainties on Surface Permanent Magnet Synchronous Motors (SPMSMs). Key uncertainties, including geometric and material variations, are analyzed using an analytical approach, reducing the reliance on computationally intensive finite element analysis (FEA). Latin Hypercube Sampling (LHS) is employed for experimental design, with performance and robustness quantified through variance analysis. Finally, the proposed process is validated against FEA and prototype testing.

Index Terms—Analytical Method, Manufacturing Uncertainty, Robustness, Sensitivity Analysis

I. INTRODUCTION

Surface Permanent Magnet Synchronous Motors (SPMSMs) use high-energy-density magnets for superior power but face performance deviations due to manufacturing uncertainties in mass production.

To estimate performance variations caused by these uncertainties, many studies have employed the Taguchi method, which utilizes the Signal-to-Noise ratio as a measure of robust performance. Despite its utility, this approach has limitations in accurately predicting performance variance [1]. Furthermore, sensitivity analysis to account for manufacturing uncertainties often requires numerous simulations, especially for uncertainties affecting individual poles and slots. Finite Element Analysis (FEA), commonly used for this purpose, imposes significant computational costs. To address this, previous research has attempted to simplify motor geometries, including uncertainties, into equivalent shapes to reduce simulation costs [2], [3]. However, these methods still rely on FEA and fail to account for independent manufacturing uncertainties occurring at each pole and slot.

This paper proposes a computationally efficient sensitivity analysis method to address these challenges. First, an analytical approach that incorporates manufacturing uncertainties is introduced. The validity of this method is confirmed through comparisons with FEA-based simulations and actual prototypes. Second, sensitivity analysis is conducted using the proposed analytical method, coupled with analytical variance to quantify performance robustness. This process identifies uncertainties and design variables with significant impact on performance variation, which are further validated through Monte Carlo simulations.

II. EFFECTS OF MANUFACTURING UNCERTAINTY

Fig. 1 presents the no-load back electromotive force (BEMF) and cogging torque data obtained through experiments and simulations, along with their harmonic analysis. In particular, Fig. 1 (b) illustrates the harmonic analysis of the cogging torque, highlighting how manufacturing uncertainties during the motor production

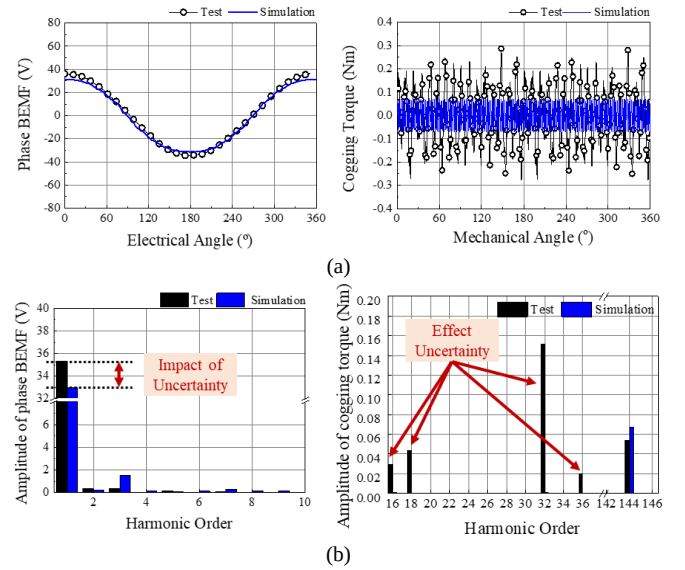


Fig. 1. No-load BEMF and cogging torque from FEM and test results. (a) waveform, (b) harmonic analysis.

process can lead to deviations from the designer's intended performance. Therefore, to analyze and ensure the robustness of the motor against manufacturing uncertainties, it is essential to identify the uncertainties that have a significant impact on motor performance and select the key design parameters necessary for robust design. In addition, to efficiently obtain large amounts of data for sensitivity analysis considering uncertainties, it is necessary to employ a computationally efficient analytical method, appropriate experimental design, and a quantitative evaluation of robustness.

Fig. 2 illustrates the process of the computationally efficient sensitivity analysis proposed in this study. The uncertainties considered in this research include geometric uncertainties such as the stator inner radius, R_{si} , slot opening width, W_{so} , permanent magnet (PM) thickness, t_{pm} , pole angle, θ_p , eccentric length, l_{ecc} , and eccentric angle, θ_{ecc} , as well as material property uncertainties such as the residual flux density, B_r of the PM. These defined uncertainties are incorporated into the proposed analytical method to calculate the cogging torque and BEMF. The experimental design is structured based on Latin Hypercube Sampling (LHS), where

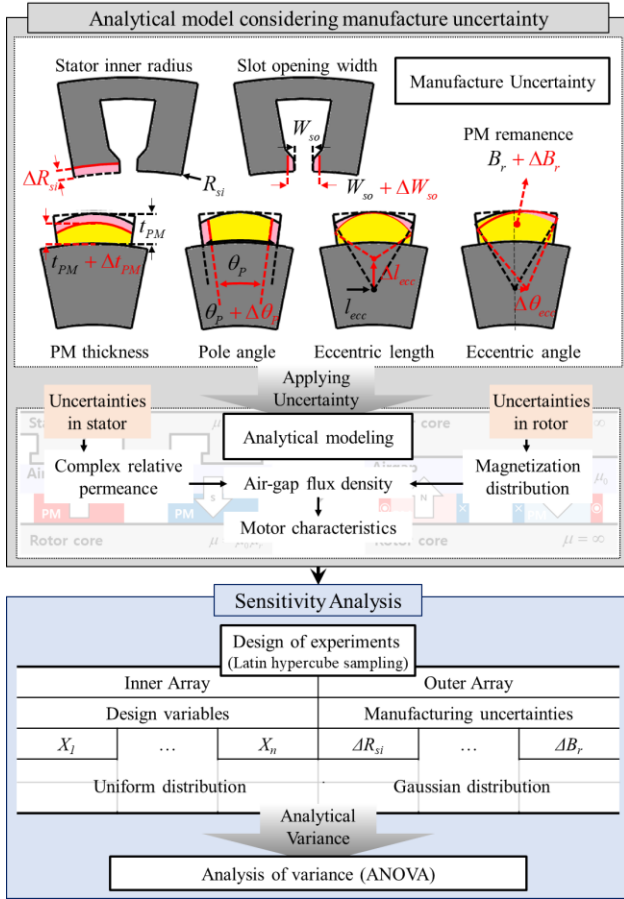


Fig. 2. Proposed sensitivity process based on analytical model considering manufacturing uncertainty.

uncertainty-free design variables are uniformly distributed in the inner array, and manufacturing uncertainty variables are normally distributed in the outer array. Subsequently, motor performance and robustness are evaluated for each experimental point, and analysis of variance (ANOVA) is performed to identify the design variables and uncertainty factors that significantly influence performance and robustness.

III. ANALYTICAL MODEL CONSIDERING UNCERTAINTY

A. Analytical model with uncertainty variables in rotor

Manufacturing uncertainties in the rotor alter the magnetization distribution of the PM, subsequently affecting the air-gap flux density. Therefore, the Laplace and Poisson equations are applied with the modified magnetization vector to account for rotor uncertainty variables, resulting in the air-gap flux density being calculated as follows [4]:

$$B_{r_nos}(r, \theta) = \sum K_B(n) \cdot f_{B_r}(r) \cdot \cos(np\theta) \quad (1)$$

$$B_{\theta_nos}(r, \theta) = \sum K_B(n) \cdot f_{B_\theta}(r) \cdot \sin(np\theta) \quad (2)$$

The changed magnetization distribution affected by uncertainties is accounted for in K_B and f_B .

B. Analytical model with uncertainty variables in stator

Manufacturing uncertainties in the stator affect the complex relative permeance (CRP) distribution, λ_a and λ_b , while considering slot effect. Thus, the air-gap flux density reflects all manufacturing uncertainties is calculated by multiplying

the air-gap flux density with rotor uncertainties by the CRP.

$$B_{r,\theta}(r, \theta) = B_{r,\theta_nos}(r, \theta) \lambda_a(r, \theta) \pm B_{\theta,r_nos}(r, \theta) \lambda_b(r, \theta) \quad (3)$$

Finally, the motor characteristics such as BEMF and cogging torque with all uncertainties reflected are calculated using the air-gap flux density obtained through this method.

IV. SENSITIVITY ANALYSIS FOR ROBUSTNESS

To perform sensitivity analysis, experimental points are efficiently allocated using the LHS. Design variables without uncertainty are placed in an inner array with a uniform distribution, while manufacturing uncertainties are assumed to follow a Gaussian distribution and are placed in an outer array. The proposed analytical method is then employed to quickly and accurately evaluate motor performance and robustness across these experimental points. Analytical variance is used to quantify the robustness against uncertainties as follows [1]:

$$\sigma_y^2 \cong \sum_{i=1}^n \left(\frac{\Delta y}{\Delta x_i} \right)^2 \sigma_{x_i}^2 \quad (4)$$

where σ_y^2 represents the analytical variance, and σ_x^2 denotes the variance of the uncertainty variables.

Based on the calculated performance and robustness for all experimental points, ANOVA is conducted to identify key uncertainty and design variables that significantly impact robustness. Finally, the proposed process and methods will be validated through Monte Carlo simulations.

V. CONCLUSION AND FUTURE WORK

This paper proposes the robust sensitivity analysis method based on the analytical approach considering manufacturing uncertainty. The proposed analytical model achieves fast analysis throughout the entire process as it does not rely on FEA. The analytical model is verified through comparisons with FEA results and prototype test results. In the sensitivity analysis stage, ANOVA is conducted using the proposed analytical method, LHS-based DOE, and analytical variance to quantify robustness while reducing computational costs and ensuring accurate evaluation. This process identifies the design and uncertainty variables that significantly impact robustness. Additional equations, analyses, and validation will be provided in the full paper.

REFERENCES

- [1] S. Lee, S. Kim, M. Park, T. H. Lee and J. Hong, "Sensitivity analysis for robust performance of electrical machines affected by manufacturing tolerance," *Tenth International Conference on Computational Electromagnetics (CEM 2019)*, Edinburgh, UK, 2019.
- [2] J. Kolb and K. Hameyer, "Sensitivity Analysis of Manufacturing Tolerances in Permanent Magnet Synchronous Machines With Stator Segmentation," *IEEE Trans. Energy Convers.*, vol. 35, no. 4, pp. 2210-2221, Dec. 2020.
- [3] S. -G. Lee, S. Kim, J. -C. Park, M. -R. Park, T. H. Lee and M. -S. Lim, "Robust Design Optimization of SPMSM for Robotic Actuator Considering Assembly Imperfection of Segmented Stator Core," *IEEE Trans. Energy Convers.*, vol. 35, no. 4, pp. 2076-2085, Dec. 2020.
- [4] M. Wang, H. Li, X. Xu, J. Zhu and Y. Shen, "Magnetic Field Analysis of Permanent Magnet Linear Synchronous Motor Based on the Improved Complex Relative Permeance Function," *2021 13th International Symposium on Linear Drives for Industry Applications (LDIA)*, Wuhan, China, 2021, pp. 1-6.