

Application of A-input Cauer Ladder Network Method to Model Order Reduction of Induction Motors

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This article presents the application of the magnetic flux input (A-input) Cauer ladder network (CLN) method to the model order reduction (MOR) of induction motors. This study develops an A-input rotor CLN, and a hybrid stator CLN having the source currents and vector potential at airgap as input. The developed motor MOR of A-input type is applied to the transient analysis of a squirrel cage induction motor. The comparison with the finite element analysis shows that the proposed method successfully reproduces the motor property under a wide range of slips as well as the conventional H-input CLN.

Index Terms—A-input CLN, Cauer ladder network (CLN), induction motor, model order reduction (MOR)

I. INTRODUCTION

TODAY, electric motors take a significant role in industrial society. The development of electric motor control systems requires electromagnetic analyses under various operating conditions. However, it is not efficient to repeatedly use the finite-element method (FEM) due to its huge computational cost. To reduce the computation time with sufficient accuracy, the model order reduction (MOR) method is one of the effective methods of which applications to motor analyses have been extensively studied (e.g., [1]).

The multi-port Cauer ladder network (CLN) method [2] is an efficient MOR method, which can be applied to induction motors [3]. In this application, the input of the CLN was the airgap magnetic field. However, to evaluate the motor performance, the airgap magnetic flux is a natural choice for the input. On the other hand, the magnetic flux input (A-input) CLN method was proposed as a new variant of the CLN method in [4]. This study aims to apply the A-input CLN to induction motors.

II. A-INPUT CLN FOR INDUCTION MOTOR

In applications of the CLN method to the 2D induction motor analysis, multi-port CLNs for the rotor domain and the stator domain are constructed separately, which are connected through the vector potential A_z and the magnetic field H_θ at the airgap interface, as illustrated in Fig. 1. In the conventional method, the inputs of the rotor and stator CLNs are H_θ . In this paper, these CLN inputs are changed into A_z . This article discusses the two cases; the case of "sHrA" changes only the rotor CLN input to A_z , whereas the case of "sArA" changes both the rotor and stator CLN inputs to A_z .

A. Connection at gap interface

To derive a boundary condition at the gap interface, it is essential to know the corresponding output to the A-input. In the case of the H-input CLN, the corresponding output is the electric field E_z for the following reason: the CLN is an electric circuit of which construction is based on the electric power, where the Poynting vector $E_z \times H_\theta$ gives the input power flow

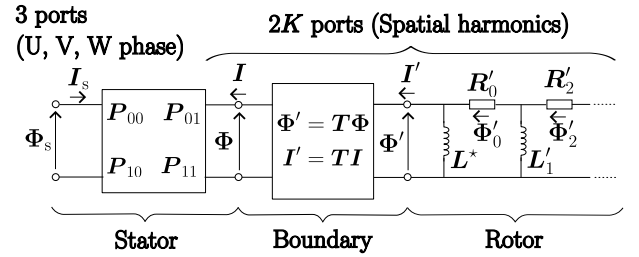


Fig. 1. A-input CLN applied to induction motor (IsHmA). Boundary input of the stator domain and the rotor domain is A.

at the airgap. We found that the corresponding output to A_z is H_θ through the following considerations: the A-input CLN is naturally expressed as a magnetic circuit, where $-A_z \times H_\theta$ signifies magnetic energy flow.

The boundary condition at the gap interface in the case of the H-input CLN was discussed in [3] in detail. First, the boundary condition in the case of sArA is described here. The vector potentials at the boundary surface are expanded into Fourier spatial harmonic components. The Fourier coefficients at the interface of the rotor and stator domains are denoted as $\Phi = (A_{c1}, A_{s1}, \dots, A_{cK}, A_{sK})^\top$ and $\Phi' = (A'_{c1}, A'_{s1}, \dots, A'_{cK}, A'_{sK})^\top$, respectively, where the components over the K -th harmonic are truncated. The harmonic components of magnetic fields are similarly denoted as $I = S(H_{c1}, H_{s1}, \dots, H_{cK}, H_{sK})^\top$ and $I' = S(H'_{c1}, H'_{s1}, \dots, H'_{cK}, H'_{sK})^\top$, where S is the area of the airgap interface. Boundary conditions $H_\theta = H'_\theta$ and $A_z = A'_z$ are represented as

$$I' = TI, \quad \Phi' = T\Phi \quad (1)$$

where $T = \text{blockdiag}(T_1, T_2, \dots, T_K)$ and T_m ($m = 1, \dots, K$) are rotation matrices.

In the case of sHrA, the input of the stator is the magnetic field and the output is the vector potential. Therefore, I and Φ are given as $I = (H_{c1}, H_{s1}, \dots, H_{cK}, H_{sK})$ and $\Phi = S(A_{c1}, A_{s1}, \dots, A_{cK}, A_{sK})$, respectively, and the relations are derived as

$$I' = STI, \quad S\Phi' = T\Phi. \quad (2)$$

B. Rotor domain

This paper extends the single-port A-input CLN described in [4] to a multi-port A-input CLN. The matrix $\mathbf{a}_{2n-1} = (\mathbf{a}_{1,2n-1}, \mathbf{a}_{2,2n-1}, \dots)$ and $\mathbf{e}_{2n} = (\mathbf{e}_{1,2n}, \mathbf{e}_{2,2n}, \dots)$ are matrices of the n -th basis, where the m -th column vectors $\mathbf{a}_{m,2n-1}$ and $\mathbf{e}_{m,2n}$ are field basis vectors when the unit and zero sources are fed into the m -th port and the others, respectively. The first inductance matrix $\mathbf{L}^*$ is given by $\mathbf{L}^{*-1} = \mathbf{a}^{*\top} \mathbf{K} \mathbf{a}^*$ where $\mathbf{a}^*$ is the solution of magnetostatic equation $\mathbf{K} \mathbf{a}^* = \mathbf{0}$ with Dirichlet boundary condition for $\mathbf{A}$, and $\mathbf{K}$ is the stiffness matrix. $\mathbf{e}_0$ is initialized to $-\mathbf{a}^{-1}$ and the first resistance matrix $\mathbf{R}'_0$ is derived from $\mathbf{R}'_0{}^{-1} = \mathbf{e}_0^\top \boldsymbol{\sigma} \mathbf{e}_0$ where $\boldsymbol{\sigma}$ is the conductivity matrix. $\mathbf{a}_{-1}$ is initialized to $\mathbf{0}$. The CLN recurrence relation after $\mathbf{R}'_0$ is the same as the H-input CLN; the distinction lies only in the first step and the initialization.

C. Stator domain

The H-input CLN for the stator domain is described in [3] where the circuit equation is

$$\begin{pmatrix} \Phi_s \\ \Phi \end{pmatrix} = \tilde{\mathbf{L}} \begin{pmatrix} \mathbf{I}_s \\ \mathbf{I} \end{pmatrix} \quad (3)$$

where $\tilde{\mathbf{L}}$ is the inductance matrix and $\mathbf{I}_s = (I_U, I_V, I_W)^\top$ and $\Phi_s = (\Phi_U, \Phi_V, \Phi_W)^\top$ are the three-phase currents and the magnetic fluxes, respectively.

To apply the A-input CLN to the stator domain, whose power source is the three-phase currents $\mathbf{I}_s$, a hybrid matrix instead of an inductance matrix is required, where the circuit equation is given as

$$\begin{pmatrix} \Phi_s \\ \mathbf{I} \end{pmatrix} = \tilde{\mathbf{P}} \begin{pmatrix} \mathbf{I}_s \\ \Phi \end{pmatrix} = \begin{bmatrix} \mathbf{P}_{00} & \mathbf{P}_{01} \\ \mathbf{P}_{10} & \mathbf{P}_{11} \end{bmatrix} \begin{pmatrix} \mathbf{I}_s \\ \Phi \end{pmatrix}. \quad (4)$$

$\mathbf{P}_{00} (\in \mathbb{R}^{3 \times 3})$, $\mathbf{P}_{01} (\in \mathbb{R}^{3 \times 2K})$, $\mathbf{P}_{10} (\in \mathbb{R}^{2K \times 3})$ and $\mathbf{P}_{11} (\in \mathbb{R}^{2K \times 2K})$ are matrix blocks of the hybrid matrix $\tilde{\mathbf{P}} = (p_{ij})$. The corresponding subscripts i to $\mathbf{P}_{00}$ and $\mathbf{P}_{01}$ are 1, 2, 3, and $i = 4, \dots, 2K+3$ for $\mathbf{P}_{10}$ and $\mathbf{P}_{11}$, whereas the corresponding subscripts j to $\mathbf{P}_{00}$ and $\mathbf{P}_{10}$ are 1, 2, 3 and $j = 4, \dots, 2K+3$ for $\mathbf{P}_{01}$ and $\mathbf{P}_{11}$. From the reciprocity, $\mathbf{P}_{01} = -\mathbf{P}_{10}^\top$ is derived. In accordance with the multi-port CLN method, $\mathbf{P}_{00}$ and $\mathbf{P}_{11}$ can be determined by

$$\begin{aligned} p_{ij} &= \int_{\Omega} \nu (\nabla \times \mathbf{A}_i) \cdot (\nabla \times \mathbf{A}_j) d\Omega \\ &= \int_{\Omega} \mathbf{A}_i \cdot \mathbf{J}_j d\Omega + \int_{\partial\Omega} (\mathbf{A}_i \times \mathbf{H}_j) \cdot d\mathbf{S} \end{aligned} \quad (5)$$

where Ω is the analysis domain, $\partial\Omega$ is the boundary of Ω , ν is the reluctivity, and $\mathbf{A}_i$ and $\mathbf{J}_i$ is, respectively, the vector potential and the current density when i -th port's input is unit input and the others are zero. When determining $\mathbf{P}_{01}$, the subscript j ($= 4, \dots, 2K+3$) implies $\mathbf{J}_j = \mathbf{0}$ in Ω , whereas the subscript i ($= 1, 2, 3$) implies $\mathbf{A}_i = \mathbf{0}$ on $\partial\Omega$, in the context of multi-port CLN method. Therefore, from the second line of (5), $\mathbf{P}_{01} = \mathbf{0}$, and consequently $\mathbf{P}_{10} = -\mathbf{P}_{01}^\top = \mathbf{0}$. To avoid this, $\mathbf{P}_{01}$ is given by $\int_{\partial\Omega} (\mathbf{A}_j \times \mathbf{H}_i) \cdot d\mathbf{S}$, and $\mathbf{P}_{10}$ is given by $\int_{\Omega} \mathbf{A}_i \cdot \mathbf{J}_j d\Omega$.

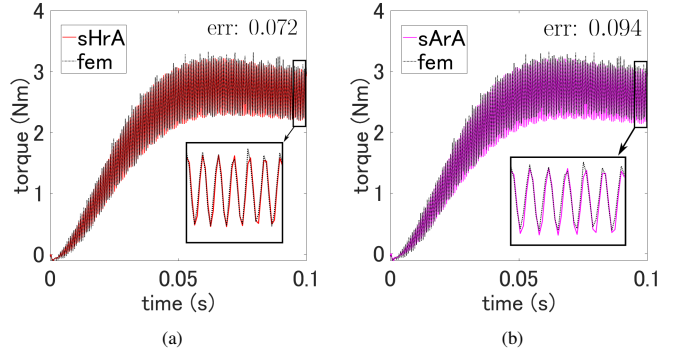


Fig. 2. Transient torque waveforms at slip of 0.15 obtained from (a) sHrA and (b) sArA.

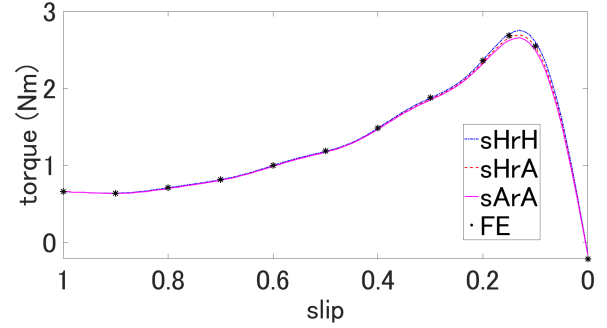


Fig. 3. Speed dependence of average torque obtained by FE analysis and CLNs (sHrH, sHrA, and sArA).

III. COMPUTATIONAL RESULTS

The induction motor was analyzed in finite-element (FE) space. The stator has 24 slots and 2 pole pairs. The rotor consists of an iron core and 34 rotor bars. The three-phase sinusoidal currents are applied at 50 Hz with amplitude 3.56 A. 45 harmonic components are considered ($K = 45$).

Figs. 2(a) and (b) show the transient waveforms of the torque at a slip of 0.15. The results of both the sHrA and sArA methods agree with that of the FE method. Fig. 3 shows the speed dependence of average torque. For comparison, the torque given by the H-input CLN is also included in this figure (denoted as sHrH). The torque is slightly large with the H-input CLN and slightly small with the A-input CLN. The nonlinearization [5] will be discussed in the full paper.

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