

A Semi-Analytical Method for the Analysis of Electrical Machines with Complex 3D Salient Geometries

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This research presents a Semi-Analytical method to analyze air gap fields of electrical machines with complex salient geometries and that, a priori, require a 3-dimensional (3D) approach. When comparing the performance of a Radial Variable Flux Reluctance Machine (RVFRM) to that of an Axial Variable Flux Reluctance Machine (AVFRM), the computational cost of the process is increased due to the 3D nature of the latter machine. Approximations of radial flux machines with a 2-dimensional (2D) cross section are a standard procedure, and have further been combined with analytical methods based on conformal mapping. Consequently, this research establishes an equivalent procedure for axial flux machines based on applying a semi-analytical method on already researched 2D slicing of axial flux Permanent Magnet (PM) based machines.

Index Terms—Axial and radial flux, electric machines, semi-analytical model, variable flux reluctance machine.

I. INTRODUCTION

ELECTRICAL MACHINES utilizing Permanent Magnet (PM) materials are recognized to be the most power dense and efficient nowadays. However, the market for Rare-Earth PM (REPM) materials is volatile, making them costly. In addition, PMs can become a fault source as they can become demagnetized at high speeds and temperatures. As a result, interest in PM-free topologies has grown in favor of robustness and reduced costs.

Despite their relatively lower efficiency and power density, reluctance-based motors are promising candidates for being used as alternatives. PM-based axial flux topologies are reported to have higher power and torque densities than their radial counterparts [1]. Thus, it is intended to investigate whether the same applies to reluctance-based motors to counteract the decrease in power and torque densities experienced in the radial flux reluctance-based machines when compared to PM-based machines. The benchmark for this paper are Variable Flux Reluctance Machines (VFRM) as the Radial VFRM (RVFRM) is provided by [2].

Comparing RVFRM and Axial VFRM (AVFRM) means dealing with the 3-dimensional (3D) nature of AVFRM, which can become challenging as the 3D Finite Element Method (FEM) is computationally expensive. Therefore, reducing the complexity of the problem is of interest, even for tasks of machine design and optimization. The reduction of a 3D problem to a 2D problem by modeling the cross-section of a radial flux machine is common practice. A similar approach is presented by [3] for axial flux PM machines by slicing the machine at different radii, resulting in 2D linear machine problems. Furthermore, aiming for analytical solutions can further reduce the computational cost as is done in [4][5] by applying Schwarz-Christoffel (SC) conformal mapping. The contribution of this paper is combining the principles presented in [3] and [4] to establish a process which allows for computationally inexpensive analysis of the AVFRM in early design stages.

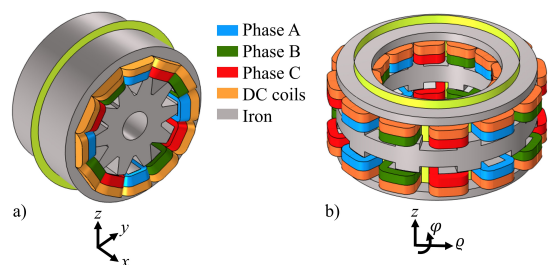


Fig. 1. Full geometry of a (a) 12/10 pole radial variable flux reluctance machine and (b) an equivalent axial variable flux reluctance machine. The yellow disk and cylinder boundaries represent the slices for 2D approximation.

Section II presents the benchmarks for both the RVFRM and AVFRM. Section III explains the reduction from a 3D to a 2D problem of the geometry and the whole process based on conformal mapping is given in Section IV. The conclusion and future work are given in Section V.

II. RADIAL AND AXIAL VFRM BENCHMARKS

Axial flux machines can adopt different structures such as single-rotor-single-stator, double-rotor-single-stator, double-stator-single-rotor or multi-stage. The topology investigated in this paper has a rotor between two stators as shown in Fig. 1(b). This allows for the compensation of the strong axial magnetic pull present in single-rotor-single-stator designs. To compare the AVFRM and RVFRM topologies, an equivalent AVFRM of the RVFRM in [2] shown in Fig. 1(a) is developed. The volume and the current densities fed to the coils are kept the same for both machines. The AVFRM has a larger diameter and shorter axial length following the trend of axial flux machines adopting a more pancake shape.

III. REDUCTION FROM A 3D TO A 2D PROBLEM

Machine design and optimization are time consuming and computationally expensive. To avoid it, this work approximates

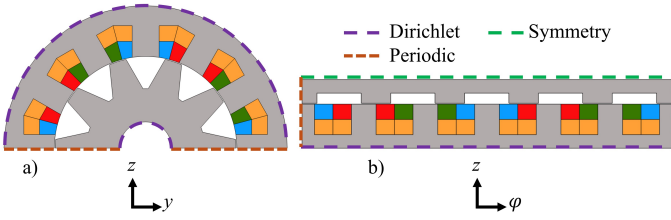


Fig. 2. 2D sliced geometries for 2D analysis for (a) the radial variable flux reluctance machine exploiting half symmetry and (b) the axial variable flux reluctance machine exploiting quarter symmetry.

the 3D geometry of the AVFRM with a 2D slice shown by the yellow cylinder in Fig. 1(b) following the method presented in [3]. The model exploits the symmetry of the AVFRM and only half the machine is considered. Furthermore, only one stator is modeled, thus exploiting a second case of symmetry. The slice is taken at the middle radius R_{slice} , between the inner R_{in} and outer R_{out} radii. This results in the simulation of a linear VFRM (LVFRM) in 2D with a length of πR_{slice} , a height of $l_{\text{ax,a}}$ and a depth of $R_{\text{out}} - R_{\text{in}}$. The resulting geometry is shown in Fig. 2(b). The model of the RVFRM also covers half the machine and is based on simulating the cross section of the machine with a slice as shown in Fig. 2(a) and indicated by the yellow disk in Fig. 1(a). Given the symmetry, periodic boundary conditions are imposed at the ends to ensure continuity of the results and the 2D LVFRM also requires a symmetry boundary condition where the rotor is cut in half. The back iron cores follow Dirichlet boundary conditions.

IV. COMPUTATION OF ELECTROMAGNETIC PROBLEM

With the 2D geometry reductions given in the previous section, it is possible to perform analyses with 2D FEM which are considerably faster than 3D FEM. For example, the transient simulation over one electrical period for the 3D AVFRM took 32 min with a coarse mesh, whereas the 2D AVFRM only took 4 min. A preliminary comparison with 2D FEM to compare both machines in Fig. 2 showed that both produced a similar average torque with the AVFRM outperforming the RVFRM by 1.5 %.

However, 2D FEM still depends on the quality of the mesh, which may not only influence the simulation time but also the accuracy of the results. Therefore, as an alternative to FEM, a further step is taken to solve the problem analytically as shown in [4]. The method is based on SC conformal mapping. It simplifies the air gap geometry and utilizes a known analytical solution in the simplified domain. Finally, the solution is remapped back into the complex domain. The resulting process is presented in Fig 3. However, this step comes with its own drawbacks. First, due to the complex geometry of the air gap, finding the mapping function f becomes a numerical integration problem. Secondly, the chosen analytical approach may lead to further simplifications, e.g. coils having to be described by line currents at discrete points and the stator and rotor materials having to be modeled by linear magnetic materials.

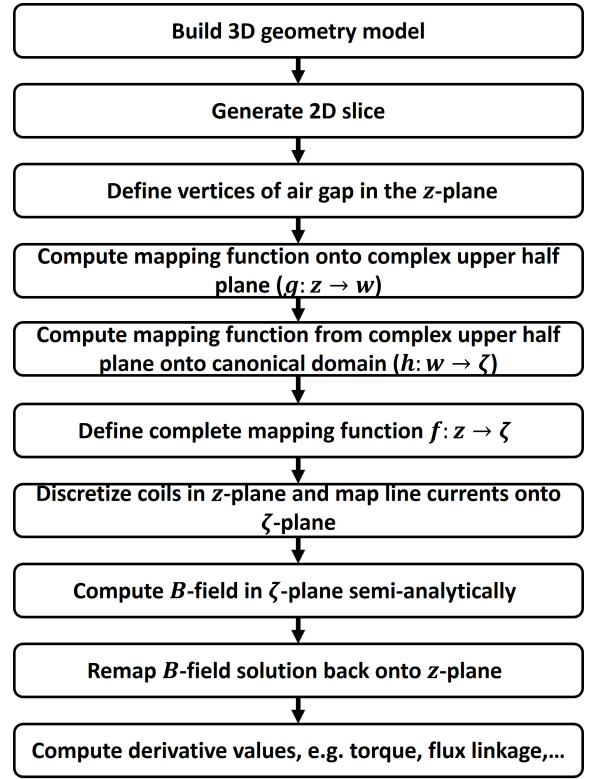


Fig. 3. Process workflow to semi-analytically analyze axial variable flux reluctance machines in a 2D environment implementing Schwarz-Christoffel conformal mapping.

V. CONCLUSION AND FUTURE WORK

This paper shows the process of simplifying the analysis of an AVFRM by slicing the 3D geometry into a 2D problem in order to apply an analytical solution based on Schwarz-Christoffel conformal mapping. This process is capable of reducing the computation time by at least a factor of 8, allowing for faster design and optimization in early machine development stages.

In the full paper, an extended description of the solution process based on Schwarz-Christoffel conformal mapping will be presented. The results of the process will be showcased on an optimized AVFRM. Further, validation through direct comparison to both 2D and 3D FEM will be presented, to show to what degree the workflow can accurately describe the performance of the machines.

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