

Semi-Analytical Modeling of an Equivalent Circuit for Multi-Turn Coils Considering Parasitic Capacitance and Eddy Current Loss

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This paper presents a semi-analytical method for modeling equivalent circuits of multi-turn coils considering parasitic capacitance and eddy current loss. The proposed method incorporates complex resistances and inductances into an RLC circuit based on the winding structure of a multi-turn coil. This enables the analysis of the eddy current loss resulting from skin and proximity effects within circuit analysis. Circuit parameters are determined using electrostatic and homogenization-based finite element analyses. The results demonstrate a significant difference in inductive-capacitive behavior between the proposed and conventional RLC circuits, primarily due to the inclusion of eddy current losses.

Index Terms—Coils, eddy currents, equivalent circuits, parasitic capacitance.

I. INTRODUCTION

DEVELOPING accurate modeling methods for multi-turn coils that account for inductive-capacitive phenomenon is essential, particularly at the high frequencies encountered in power electronic devices using SiC and GaN. Among the methods for inductive-capacitive field computation, the Darwin model, which employs the $\mathbf{A} - \varphi$ formulation, has been recently proposed [1]–[2]. This approach enables the simultaneous consideration of eddy and displacement currents. However, the Darwin model lacks robustness across a wide frequency range, leading to increased computation time and complicating the analysis of inductive-capacitive phenomenon in multi-turn coils under various driving conditions.

The equivalent circuit method has been proposed as a simplified numerical analysis method for multi-turn coils [3]. By utilizing an RLC circuit based on the winding structure of a multi-turn coil, this method allows for a detailed evaluation of high-frequency behavior between windings due to parasitic capacitance. Additionally, an extended RLC circuit method was proposed, which incorporates eddy and displacement currents [4]. In this method, the resistances of the RLC circuit based on the winding structure are updated at each time step using finite element eddy current analysis. However, the computation time increases with driving frequency because the coil region must be subdivided into elements smaller than the skin depth. To address these limitations, a hierarchical RLC circuit coupled with a homogenization-based finite element analysis (FEA) was recently proposed [5]. This approach can represent multiple resonant peaks while accounting for eddy current loss without increasing computation time. Nevertheless, it requires pre-measured data over a wide frequency range to determine the circuit parameters. Moreover, understanding the physical meaning of each circuit element in this model remains challenging.

This study proposes an equivalent circuit modeling method for multi-turn coils that accounts for parasitic capacitance and

eddy current loss. The proposed method introduces complex resistances and inductances into an RLC circuit based on the winding structure. These circuit elements represent the skin and proximity effects in multi-turn coils, enabling the evaluation of inductive-capacitive phenomenon, including eddy current loss, without coupling with FEA, as in previous studies [4]–[5]. The complex circuit parameters are semi-analytically determined using the homogenization method [6], whereas the parasitic capacitances are determined using electrostatic FEA. To validate the proposed method, the frequency characteristics of a 100-turn air-core coil are compared with those of conventional RLC circuits, highlighting the influence of eddy current loss.

II. SEMI-ANALYTICAL MODELING METHOD FOR RLC CIRCUIT BASED ON WINDING STRUCTURE OF MULTI-TURN COIL

An example of the proposed RLC circuit, which includes complex resistances and inductances, is shown in Fig. 1. The parasitic capacitances C_{ij} are determined using electrostatic FEA, as in previous studies [3]–[4]. The complex inductances $\hat{L}_{ij}$ are determined using homogenization-based FEA [6] as follows:

$$\text{rot } \mathbf{v}(\text{rot } \mathbf{A}) = \mathbf{J}, \quad (1)$$

where $\mathbf{v}$, $\mathbf{A}$, and $\mathbf{J}$ denote the reactance tensor, magnetic vector potential, and current density, respectively. In the homogenization method [6], the magnetic property is analyzed to set

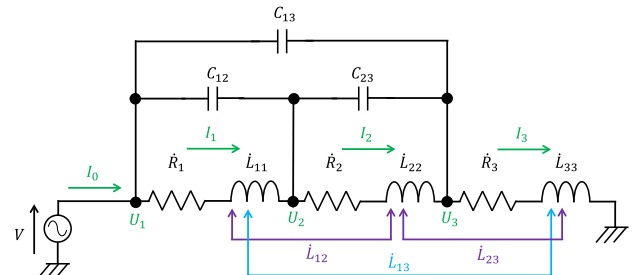


Fig. 1. Proposed RLC circuit for a three-turn coil.

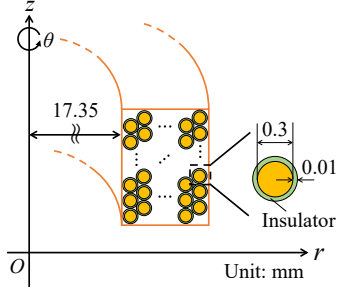
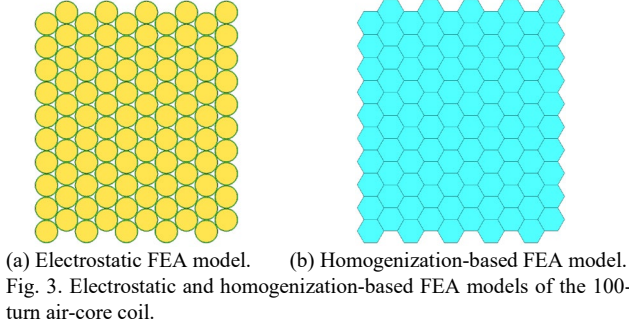


Fig. 2. 100-turn air-core coil model (10 × 10 turns).



(a) Electrostatic FEA model. (b) Homogenization-based FEA model.
Fig. 3. Electrostatic and homogenization-based FEA models of the 100-turn air-core coil.

the macroscopic complex permeability (μ) in the coil region. This approach accounts for the proximity effect without requiring subdivision into elements smaller than the skin depth, significantly decreasing the computation time required to determine the complex inductance values. In contrast, the complex resistances $\hat{R}_i$ representing the skin effect are computed analytically as follows [6]:

$$\hat{R}_i = R_{DC}^{(i)} \frac{z J_0(z)}{2 J_1(z)}, \quad (2a)$$

$$z = a(1 - j)/\delta, \quad (2b)$$

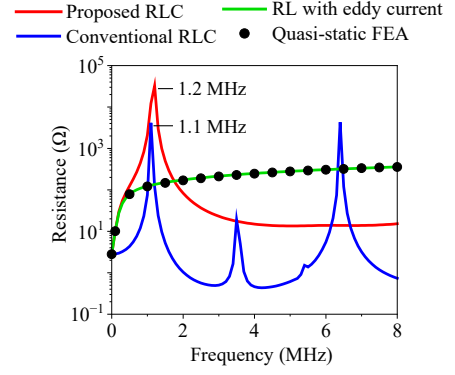
where $R_{DC}^{(i)}$, $J_i(z)$ ($i = 0, 1$), a , j , and δ denote the DC resistance of the i -th turn, i -th Bessel function, strand radius, imaginary unit, and skin depth, respectively. The inductive-capacitive phenomenon of the multi-turn coil, considering both the skin and proximity effects, is observed by solving the following circuit equation:

$$\left\{ j\omega \begin{bmatrix} [L] & 0 \\ 0 & [C] \end{bmatrix} + \begin{bmatrix} [R] & [K] \\ [K]^t & 0 \end{bmatrix} \right\} \begin{bmatrix} \mathbf{I} \\ \mathbf{U} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\beta} \end{bmatrix}, \quad (3)$$

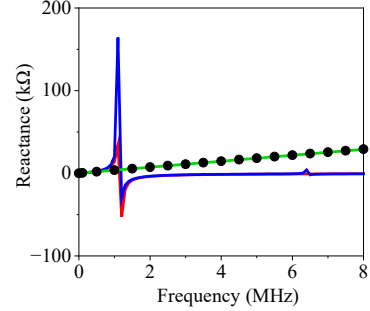
where ω , $[L]$, $[C]$, and $[R]$ denote the angular frequency, inductance, capacitance, and resistance matrices, respectively. $[K]$ is a special matrix for the current and voltage vectors ($\mathbf{I}$ and $\mathbf{U}$, respectively). $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are the source vectors for the input voltage V .

III. NUMERICAL RESULTS

This study compared the frequency characteristics of a 100-turn air-core coil, shown in Fig. 2, using the proposed and conventional RLC circuits. The electrostatic and homogenization-based FEA models for determining the proposed circuit parameters are illustrated in Fig. 3. To evaluate the complex inductance for each turn, the homogenized coil region was subdivided



(a) Resistance.



(b) Reactance.

Fig. 4. Frequency characteristics of the 100-turn air-core coil.

into hexagonal blocks, as shown in Fig. 3(b). The frequency characteristics of the 100-turn air-core coil obtained using the proposed and conventional RLC circuits are shown in Fig. 4. In addition, the frequency characteristics obtained from conventional quasi-static FEA and RL circuit, which ignores the parasitic capacitances in the proposed RLC circuit, are presented for validation of the complex resistances and inductances. The frequency responses from the quasi-static FEA and RL circuits align closely, indicating that the proposed RLC circuit can accurately evaluate the inductive-capacitive phenomenon, including the eddy current loss across a wide frequency range. Moreover, unlike conventional circuits, the proposed RLC circuit exhibits only a single resonant peak, with the complex resistances and inductances significantly dominating the high-frequency region due to the eddy current loss. In the presentation, we will compare the numerical results obtained from the proposed RLC circuit with experimental results.

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