

A Novel Energy-Density Based Approach: From Confinement to Radiation in Cylindrical EM Structures

A. Pirisi¹, R. E. Zich¹, *Member, IEEE*

¹Politecnico di Milano, Milano, 20133, Italy, andrea.pirisi@polimi.it

This work analyzes the transition between quasi-stationary and propagative regimes in electromagnetic wave propagation, considering cylindrical capacitor and inductor configurations as a case study. Starting from the conditions defined by Max Abraham and the subsequent contributions of Schelkunoff—who emphasized energy balance as a crucial variable—the spatio-temporal energy density as a fundamental state variable is here presented. By applying Maxwell's equations through Hertz's method, a rigorous framework to describe energy confinement phenomena in quasi-stationary regimes and their transition to propagative behaviors has been outlined with respect to any cylindrical geometry. The results aim to provide guidelines for designing and optimizing wireless power systems, reconfigurable metamaterials, and advanced electromagnetic devices operating in stratified or anisotropic media.

Index Terms— Electromagnetic propagation, Electromagnetic modeling, Electromagnetic analysis, Inductive power transmission.

I. INTRODUCTION

THE accurate modeling of higher-order effects is increasingly crucial for the design of 6G wireless networks, advanced resonant systems, and modern electromagnetic (EM) devices. In [1], it has been introduced an analytical model that defines the transition from quasi-static confinement to radiative behavior for any EM structure with cylindrical symmetry.

Building on the classical foundations of Abraham [2] and Schelkunoff [3], [4], who respectively addressed spatial constraints and energy balance, the proposed approach provides a predictive criterion that incorporates both spatial and temporal dynamics, enhanced by Hertz's method [5], and generalizes these foundational concepts through a unified space–time energy-density formulation, complementary to simulation-based visualizations of field transitions [6], [7].

Since any complex EM geometry can be locally approximated by infinitesimal cylindrical elements, our formulation can be naturally integrated into finite element method (FEM) solvers, supporting the accurate analysis of higher-order effects and improving the design of massive MIMO systems, and next-generation wireless applications.

II. FOUNDATIONAL APPROACHES

A. Max Abraham's Condition

As recalled in [1], under sinusoidal excitation, the Maclaurin expansion of the scalar potential shows that simultaneous influence across all points requires vanishing the second-order term, leading to the classical non-propagative condition formulated by Abraham [2]:

$$\left| \frac{-j\omega R}{v} \right| \ll 1 \Rightarrow \omega R \ll v \quad (1)$$

where ω is the excitation frequency and v the wave speed.

B. Sergei Schelkunoff's Condition

Schelkunoff introduces an alternative criterion by examining energy accumulation in physical systems [3], [4]. He distinguishes between electric ξ_e and magnetic ξ_m energy

storage. In systems dominated by electric phenomena, such as capacitors, the energy balance is characterized by:

$$\frac{\max(\xi_m)}{\max(\xi_e)} \ll 1 \quad (2)$$

Similarly, in magnetic-dominated systems, such as inductors, energy balance is characterized by:

$$\frac{\max(\xi_m)}{\max(\xi_e)} \gg 1 \quad (3)$$

Abraham's condition considers only the Cartesian distance between the source and the perturbed point, neglecting the system's geometry and internal energy distribution.

Schelkunoff demonstrated that this limitation can lead to failures in accurately describing EM phenomena. From his analysis, a form factor coefficient K emerges as a tool to address these limitations effectively:

$$\frac{\max(\xi_m)}{\max(\xi_e)} = \frac{1}{K} \left(\frac{\omega R}{v} \right) \ll 1 \Rightarrow \omega R \ll K v \quad (4)$$

III. THEORETICAL MODEL

The proposed approach combines Abraham's spatial conditions and Schelkunoff's energy balance with the concept of distributed energy density. To illustrate this framework, two representative systems have been analyzed: a finite cylindrical capacitor and an inductor sharing the same geometric symmetries, as shown in Fig. 1.

A. Field Analysis

Integrating ME using Hertz's method reveals higher-order contributions, refining the representation of dynamic EM phenomena. Field equations for the capacitor, detailed in (5) and (6), are expressed in the phasor domain, where ρ and φ refer to cylindrical coordinates, and n denotes the iteration order according to Hertz's method [5].

$$\bar{E}_{z,2n}(\rho) = \frac{(j\omega a/v)^{2n}}{(2^n n!)^2} \bar{E}_z(0) \quad (5)$$

$$\bar{H}_{\varphi,2n+1}(\rho) = \frac{j(\omega a/v)}{\mu v(2n+2)} \bar{E}_{z,2n}(\rho) \quad (6)$$

B. Energy Analysis

Our extended analysis, aligned with the spirit of the Chu-Adler-Fano model, includes impedance calculations. From these results emerges a function K , linked to system geometry, excitation frequency, and iteration order:

$$K_n(\Lambda) = \frac{(j\Lambda)^{2n}}{(2^n n!)^2}, \quad \Lambda = \frac{a\omega}{v} \quad (7)$$

where a represents the largest Lagrangian dimension.

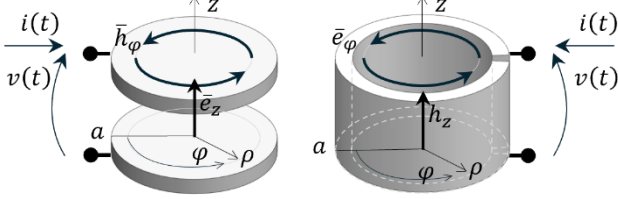


Fig. 1. Systems geometries and integration domains.

We also derived Schelkunoff's condition for the same capacitive system, with the result shown in (10) generalizing Abraham's condition. Although derived from the series expansion of the electric field, it depends only on spatial energy distribution, while the function K also depends on frequency. By neglecting temporal energy distribution, it can therefore be regarded as a necessary but not sufficient condition.

$$\omega a = (2\sqrt{2}) v \quad (8)$$

To overcome this limitation and derive a more general and comprehensive condition, we performed an extensive energy analysis comparing electric and magnetic field contributions. Surprisingly, even though this analysis was carried out on two fundamentally different systems—a finite cylindrical capacitor and a finite cylindrical inductor sharing only geometric symmetry—both yielded the same result:

$$\omega a \ll M(\Lambda, N) v \Rightarrow \Lambda \ll M(\Lambda, N) \quad (9)$$

where a function M emerges, accounting for both Schelkunoff's form factor, excitation frequency and the iteration order N at which the analysis is considered sufficiently accurate:

$$M(\Lambda, N) = \frac{\sum_{n=0}^N \frac{K_n(\Lambda)}{\sqrt{4n+2}}}{\sum_{n=0}^N \frac{K_n(\Lambda)}{(2n+2)\sqrt{4n+4}}} \quad (10)$$

Using (7) and truncating the series in (10) at the first term reduces condition (9) to the Schelkunoff's condition (8), emphasizing the broader applicability of this formulation, as shown in (11). This suggests that the derived condition could represent a broader, more universal model, extending beyond the foundational works.

$$\omega a \ll M(\Lambda, 0) v = (2\sqrt{2}) v \quad (11)$$

IV. IMPLICATIONS AND APPLICATIONS

The proposed framework is based on a single fundamental state variable: the space-time energy density, emerging from the combination of Schelkunoff's energy-balance approach and Hertz's potential method. This unified formulation enables the prediction of the transition from confined quasi-static behavior

to radiative propagation, without introducing new physical models or modifying standard numerical formulations.

Since the criterion is dimensionless and local, it can be evaluated directly on each finite element. The Energy-Density Transition Surface $M(\Lambda, N)$, exemplified in Fig. 2, can be computed from FEM solutions without modifying any solver. Knowing in advance where the transition from confinement to radiation occurs enables targeted mesh refinement to better resolve radiative effects. Moreover, the transition criterion provides a natural a posteriori error indicator, highlighting regions where the discretization must be improved.

Practical applications include the design and performance optimization of massive MIMO antenna arrays, high-frequency structures with strong mutual coupling, and next-generation wireless and sensing systems, where precise control of field confinement and radiation governs overall system efficiency.

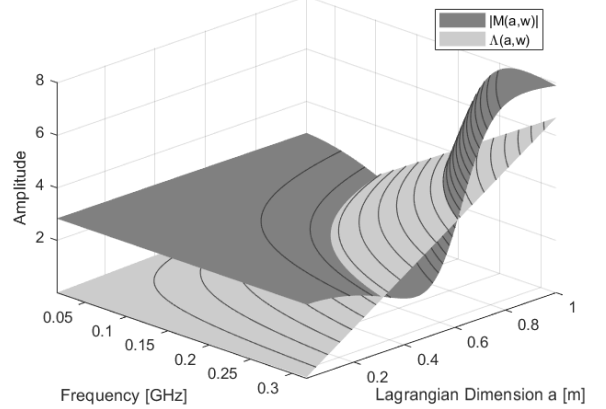


Fig. 2. $M(a, \omega)$ and $\Lambda(a, \omega)$ valued with $N = 2$ and $v = 3 \cdot 10^8 \text{ m/s}$.

V. CONCLUSION

A theoretical extension of Abraham's and Schelkunoff's contributions is presented, introducing spatio-temporal energy density as a key variable for predicting and modeling the transition between quasi-stationary and radiative regimes.

The framework applies to any EM structure with cylindrical symmetry and can be locally integrated into FEM solvers, opening opportunities for efficient simulation and design of advanced materials and next-generation wireless devices.

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