

# An Ultra-Fast Numerical Model for AC Losses Evaluation in Superconducting Coils made of HTS tapes

H. Menana

Université de Lorraine, GREEN, F-54000 Nancy, France ; hocine.menana@univ-lorraine.fr

In this work, an integrodifferential model in terms of the electric vector potential is combined with the method of images for the electromagnetic field modeling in coils made of high temperature superconducting tapes. The modeling results are compared to measurements achieved on a Rare-earth Barium Copper Oxide (ReBCO) tape-based coil.

**Index Terms**—AC losses, HTS Coils, Integro-differential Model, Method of images, ReBCO tapes.

## I. INTRODUCTION

MODELING HIGH temperature superconductors (HTS) is mainly based on the evaluation of local quantities due to their complex structures involving heterogeneities, anisotropies and multi-scale dimensions, and to the strong nonlinearities of their electromagnetic properties. It is often a challenging task, where large IT resources and time are needed unless developing ad hoc approaches.

In this work, a model based on an integrodifferential formulation in terms of the electric vector potential combined with the method of images is developed for the electromagnetic field modeling in coils made of HTS tapes, in a 2D axisymmetric approximation. The integro-differential model allows to discretize only the tapes section and the image method, based on the anti-symmetry of the magnetic field on both sides of the coil along the axial direction, allows to reduce this discretization to the half width of the coil (tapes), while the air region is not discretized, reducing the scaling effect problems and avoiding the drawback of defining a certain electrical resistivity in the air region when using magnetic formulations where an equivalent resistivity  $\rho$  is required [1]-[2]. This results in a considerable saving in memory space and computing time. The modeling results are compared to measurements achieved on a Rare-earth Barium Copper Oxide (ReBCO) tape-based coil.

## II. DEVELOPMENTS

The modeled system is a single layer pancake coil made of ReBCO tape which has a multilayer structure of about 0.1mm thickness, where the superconducting layer thickness is about  $1\mu\text{m}$  [3]. All regions of the system, are characterized by the vacuum magnetic permeability  $\mu_0$ . The superconducting region is characterized by a nonlinear electrical resistivity, represented by the power law (Eq.1) where,  $E$  and  $E_c$  are the electric field and its critical value ( $1\mu\text{V/cm}$ ),  $J$  and  $J_c$  are the electric current density and its critical value,  $n$  is the creep exponent characterizing the steepness of the transition from the superconducting to the normal states,  $J_{c0}$  and  $n_0$  are respectively the values of electric current density and the creep exponent at zero magnetic field,  $B_r$  and  $B_z$  are respectively the radial and axial components of the magnetic flux density, and  $B_{0J,n}$ ,  $k_{J,n}$  and  $\beta$  are parameters depending on the material.

$$\begin{cases} \vec{E} = E_c J_c^{-n(B)} (B) \|J\|^{n(B)-1} \vec{J} \\ J_c(B) = J_{c0} \left(1 + B_{0J}^{-1} \sqrt{k_{J,n}^2 B_z^2 + B_r^2}\right)^{-\beta} \\ n(B) = n_0 \left(1 + B_{0n}^{-1} \sqrt{k_n^2 B_z^2 + B_r^2}\right) \end{cases} \quad (1)$$

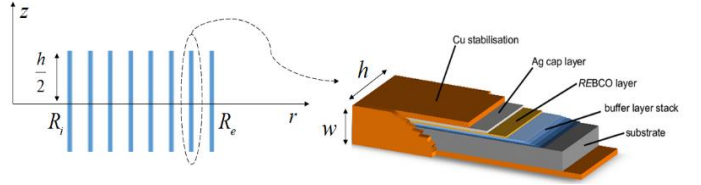


Fig. 1. Modeled system with a schematic layout of REBCO tapes

The electrical current density in the tapes is divided into a source term  $J_s$  and an induced term derived from the electric vector potential  $\vec{T}$ :  $\vec{J} = \vec{J}_s + \text{curl} \vec{T}$ ; this ensures naturally the conservation of the current in the tapes.

In a 2D axisymmetric approach, given the very small thickness of the HTS tape, and the coil being inductive, only the variation of the radial component of the electric vector potential  $T_r$  along the  $z$  axis is considered. The Maxwell-Faraday's law leads thus to the following equation:

$$\begin{cases} \partial_z (\rho \{J_s + \partial_z T_r\}) = \frac{\partial}{\partial t} B_r \\ T_r^t(z=0 \text{ \& } z=h/2) = 0 \end{cases} \quad (2)$$

Each half tape ( $z \geq 0$ ) is discretized into  $N_{et} = N_r \times N_z$  elements of surface  $\Delta S$  in  $r$  and  $z$  directions respectively, resulting in  $N_e = N_t \times N_{et}$  elements for the discretization of the half coil composed of  $N_t$  tapes. The current density is assumed to be uniform in each elementary surface.

In a matrix form, the magnetic flux density in the radial direction is linked to the current density by an integral matrix  $\bar{\bar{M}}$  as follows, where  $\bar{\bar{D}}$  is the derivative matrix with respect to the  $z$  direction:

$$\bar{B}_r = \bar{\bar{M}} \bar{\bar{D}} \bar{T}_r + \bar{\bar{M}} \bar{J}_s \quad (3)$$

Using the backward Euler method for the time discretization, the equation (2) can be written in a matrix form as follows, where  $\bar{\bar{L}}$  represents a second derivative  $\partial_z \partial_z (\cdot)$  matrix:

$$\left( \bar{\bar{M}} \bar{\bar{D}} - \Delta t \left\{ \bar{\bar{\rho}} \bar{\bar{L}} + \bar{\bar{D}} \bar{\bar{\rho}} \bar{\bar{D}} \right\} \right) \bar{T}_r^t = \bar{\bar{M}} \bar{\bar{D}} \bar{T}_r^{t-\Delta t} - \Delta t \bar{\bar{M}} \bar{\partial}_t \bar{J}_s + \Delta t \bar{\bar{D}} \bar{\bar{\rho}} \bar{J}_s \quad (4)$$

Using the image method to consider the magnetic field generated by the half tapes located at ( $z \leq 0$ ) while discretizing only the half tapes located at ( $z \geq 0$ ), the elements  $M(i, j)$  of the matrix  $\bar{\bar{M}}$  are evaluated as follows, where  $E_1$  and  $E_2$  are respectively the complete elliptical integrals of the first and second kind [4]:

$$\begin{cases} M_{i,j} = m_{i,j}^+ + m_{i,j}^- \quad (i \neq j) \\ m_{i,j}^{\pm} = \frac{\mu_0 \Delta S}{4\pi} \frac{k_{i,j}^{\pm} (z_i \pm z_j)}{r_i \sqrt{r_i r_j}} \left( \frac{r_i^2 + r_j^2 + (z_i \pm z_j)^2}{(r_i - r_j)^2 + (z_i \pm z_j)^2} E_2(k_{i,j}^{\pm}) - E_1(k_{i,j}^{\pm}) \right) \\ k_{i,j}^{\pm} = 2\sqrt{r_i r_j} \left( \sqrt{(r_i + r_j)^2 + (z_i \pm z_j)^2} \right)^{-1} \end{cases} \quad (5)$$

### III. RESULTS AND DISCUSSIONS

The system specifications are given in Table 1. The tape is homogenized, considered all superconducting with a critical current density defined for all the tape section [2]. The applied current varies in time as follows:  $I_s(t) = \sqrt{2} I_{rms} \sin(2\pi ft)$ . The period is discretized into a  $N_{\Delta t}$  equal time intervals  $\Delta t$ .

Figure 2 represents the instantaneous losses ( $P_t$ ) evolution with time in the first period. The steady state is reached in the second half period; thus, the average AC losses ( $P$ ) are evaluated on this time interval:  $P = 2T^{-1} \int_{T/2}^T P_t dt$ . Figure 3 represents a comparison between the evolutions of AC losses with the applied current obtained by modeling and measurements. A good agreement is shown at relatively high currents, while a certain discrepancy is observed below. This discrepancy is, in part, due to the uncertainties on the Kim's parameters used in (1), which are determined for a separate sample of HTS tape in applied magnetic field, while, below a certain current, the magnetic field applied to a tape in the coil is close to the self-field. Figure 4 represents the current density repartition in a half tape section at different instants when the steady state is reached. The current density repartition in the other half tape is simply its image. As we can see, at the beginning the current penetrates the tape from the extremity to a certain distance depending on the applied current. When the applied current is cancelled ( $t=T/2$ ), it remains two currents flowing in opposite directions in the tape section keeping a certain magnetization in the tape responsible of the hysteresis losses. The model has been developed on MATLAB®, run on an Intel® Core(TM) i7-8700 CPU @3.2GHz, 64 Gb Processor. The computation time for a point of calculation is  $< 3$  seconds.

TABLE I  
PARAMETERS SPECIFICATION

| Parameter                             | Value                     | Description                     |
|---------------------------------------|---------------------------|---------------------------------|
| $w/h$                                 | 95 $\mu$ m / 4 mm         | Tape thickness / Tape width     |
| $R_i / R_e$                           | 60mm / 62mm               | Inner / outer radii of the coil |
| $N_t$                                 | 6                         | Number of the coil turns        |
| $I_{c0}$                              | 120 A                     | Zero field critical current     |
| $n_0$                                 | 8                         | Zero field creep exponent       |
| $k_J / k_H / B_{0J} / B_{0H} / \beta$ | 0 / 0 / 0.3 T / 0.02T / 1 | Kim's parameters used in (1)    |
| $N_r / N_z / N_{\Delta t}$            | 1 / 15 / 150              | Space and time discretization   |

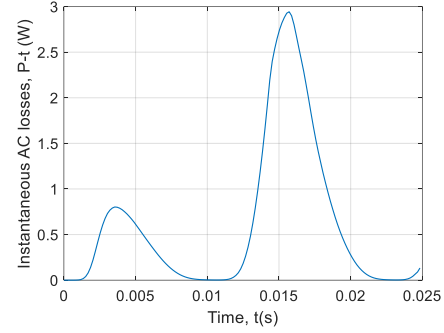


Fig. 2. Instantaneous AC losses in the first period ( $f=40$ Hz,  $I_{rms}=80$ A)

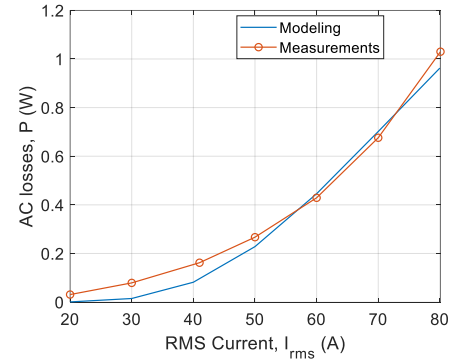


Fig. 3. Average AC losses vs current ( $f=40$ Hz).

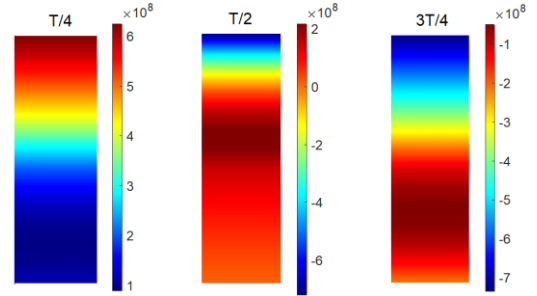


Fig. 4. Current density repartition in a half tape (not to scale) at different instants at the steady state regime ( $f=40$ Hz,  $I_{rms}=80$ A).

### REFERENCES

- [1] B. Shen, F. Grilli and T. Coombs, Review of the AC loss computation for HTS using H formulation», *Supercond. Sci. Technol.*, vol. 33, no. 3, Art. no. 033002, 2020.
- [2] V. M. R. Zermenio, A. B. Abrahamsen, N. Mijatovic, B. B. Jensen and M. P. Sørensen, "Calculation of alternating current losses in stacks and coils made of second generation high temperature superconducting tapes for large scale applications", *J. Appl. Phys.*, vol. 114, 173901, 2013.
- [3] C. Barth et al., "Electro-mechanical properties of REBCO coated conductors from various industrial manufacturers at 77 K, self-field and 4.2 K, 19 T," *Supercond. Sci. Technol.*, vol. 28, no. 4, Feb. 2015, Art. no. 045011.
- [4] E. Durand, "Magnétostatique", Masson and Cie Editeurs, Paris, 1968.