

Consistency and Convergence Analysis of Co-Simulation Techniques for Field-Circuit Coupled Problems

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This work compares different techniques for co-simulation when dealing with field-circuit coupled problems. The convergence behavior of the coupled system is investigated using different time-stepping techniques and coupling methods, such as source and parameter coupling. The speed-up of the methods with regard to accuracy is explored, and challenges which can occur in the context of differential algebraic systems are elaborated.

Index Terms—Co-Simulation, Field-Circuit Coupling, Multirate Physics.

I. INTRODUCTION

APPLYING Co-Simulation techniques for field-circuit coupled problems are common to speed up coupled simulations. Instead of a large monolithic system of equations, the individual subproblems, i.e., field and circuit, are solved independently on time windows [1], [2]. For field-circuit coupled problems, this allows different tools to be used to solve the circuit equations and the magnetic field problem, taking into account the different time dynamics and multirate behavior.

There are several ways in which the exchange of information can be organized in a co-simulation setting [3]. With *source coupling*, currents i and voltages u are exchanged, either as a value or as a time-dependent function. In [3] it is shown that in most cases there is a benefit if the field problem is voltage driven and represented as a current source in the circuit. With *parameter coupling* the field problem is represented as an equivalent model in the circuit, e.g. a possibly time-dependent inductance and source. A schematic representation of the two common coupling approaches is illustrated in Fig. 1.

The simulation sequence for can be chosen: Depending on the dynamics, this can be classified as *fast first* or *slow first*. Another option is to iterate over the time windows multiple times to further increase the simulation accuracy. This concept is known as waveform relaxation [4]. This work revisits co-simulation and in particular linearization approaches to study their (higher-order) convergence behavior; see also [5].

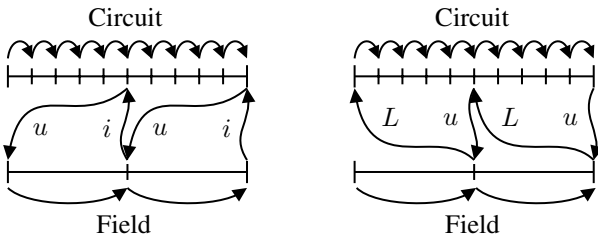


Fig. 1. Co-simulation approaches. Left: Source coupling with circuit (fast) first. Right: Parameter coupling with field (slow) first.

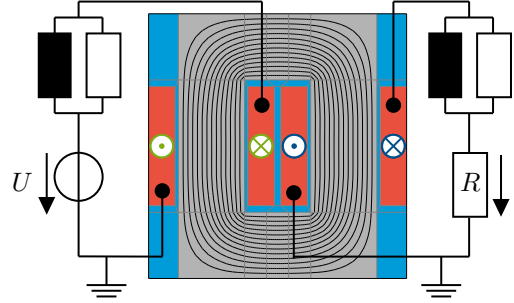


Fig. 2. Field-Circuit coupled example circuit consisting of a two dimensional magnetoquasi-static field simulation and a voltage-driven RL-circuit.

II. PROBLEM DESCRIPTION

The co-simulation techniques are analyzed based on a simple field-circuit test problem. For this, the two-dimensional transformer model given in Fig. 2 is coupled to a circuit consisting of a power source connected to the left and a load connected to the right coil. Additional resistances and inductances are added to ensure dynamical circuit behavior. The simulation is carried out in transient magneto-quasistatic and static settings with nonlinear material behavior. The voltage source enforces a sinusoidal excitation such that the iron core reaches saturation. Then, the first period is simulated, and the time-stepping error of the coil currents is compared to a reference solution, which is obtained by a highly resolved monolithic computation. The reference currents of the coils are illustrated in Fig. 3.

It can be observed that the currents are not sinusoidal as a result of initial transients and the nonlinear material model. After 0.05s the iron core saturates, resulting in a current increase for the primary side.

III. MODELING

The field system is discretized with the A -formulation using finite elements. This gives rise to differential-algebraic equations (DAEs) [6] with (singular) mass matrix $\mathbf{M}_\sigma$, (nonlinear) stiffness matrix $\mathbf{K}_\nu$ and unknowns $\mathbf{a}$. The circuit equations are

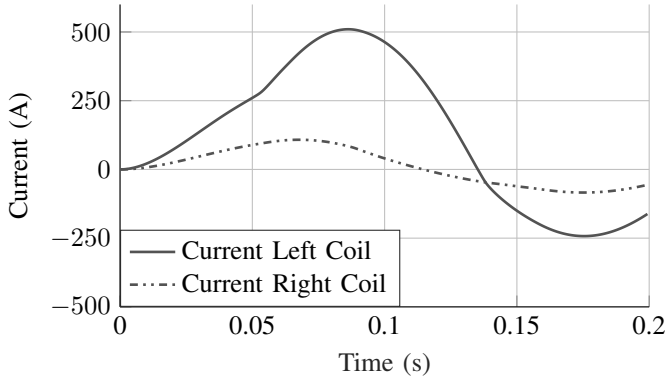


Fig. 3. Reference solution for the currents. The turn ratio of the coils is reflected by the scaling of the currents.

modeled using Nodal Analysis (MNA) in flux/charge-oriented form [7]. Note that this leads to even more complicated DAEs and may require extra care to ensure consistency, especially when choosing the time integration scheme. The system is given by the (singular) mass matrix $\mathbf{M}_c$, the stiffness matrix $\mathbf{K}_c$, sources $\mathbf{S}_c$ and unknowns $\mathbf{u}$, e.g., including the nodal potentials. The field and circuit are coupled here via the stranded conductor model, which uses $\mathbf{X}_s$ to distribute the currents $\mathbf{i}_s$ in the field domain and $\mathbf{Y}_s$ to define the connections to the circuit [8]. The fully coupled system is then given by

$$\begin{bmatrix} \mathbf{M}_\sigma & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_c & \mathbf{0} \\ \mathbf{X}_s^\top & \mathbf{0} & \mathbf{0} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} \mathbf{a} \\ \mathbf{u} \\ \mathbf{i}_s \end{bmatrix} = \begin{bmatrix} -\mathbf{K}_\nu(\cdot) & \mathbf{0} & \mathbf{X}_s \\ \mathbf{0} & \mathbf{K}_c & \mathbf{Y}_s \\ \mathbf{0} & \mathbf{Y}_s^\top & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{a} \\ \mathbf{u} \\ \mathbf{i}_s \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{S}_c \\ \mathbf{0} \end{bmatrix}$$

The equivalent lumped (differential) inductance can be calculated using the relationship [8]

$$\mathbf{L} = \mathbf{X}_s^\top \left(\frac{d \mathbf{K}_\nu(\mathbf{a}) \mathbf{a}}{d \mathbf{a}} \right)^{-1} \mathbf{X}_s. \quad (1)$$

IV. PRELIMINARY RESULTS

Fig. 4 presents results for parameter coupling in the case of magnetostatics. The exchange of values is as shown in Fig. 1 on the right with the lumped inductance calculation according to (1). The currents for both coils are plotted for the circuit-first and field-first configuration. It is clearly visible that the simulation order of field and circuit makes a difference for the results. If the circuit is simulated first, there is no updated information for the inductance values, which can result in inconsistent initial conditions, that is, a violation of the circuit equation. This becomes a problem for the trapezoidal rule which leads to nonphysical oscillatory behavior. This is also reflected in the convergence study shown in Fig. 5. Integration with the trapezoidal rule (TRA) leads to higher errors even compared to the implicit Euler (IE). Simulating the field (slow) first restores the convergence order.

V. SUMMARY AND OUTLOOK

We have recalled the concept of co-simulation for field-circuit coupled problems and presented numerical issues due to a lack of consistency. The full paper will present the mathematical understanding and implications due to the DAE character of the system and extend the findings to quasistatics.

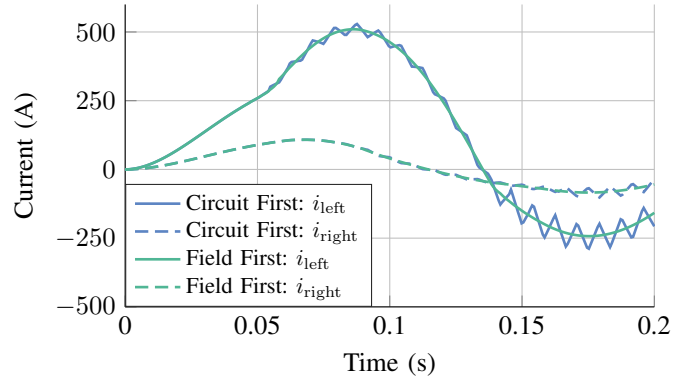


Fig. 4. Comparison of co-simulation results for parameter coupling: Circuit first versus field first with trapezoidal rule.

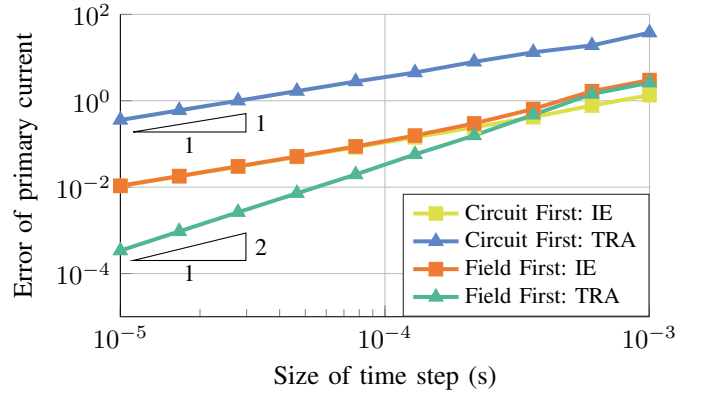


Fig. 5. Convergence study comparing for parameter coupling: Implicit Euler (IE) versus Trapezoidal Rule (TRA).

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REFERENCES

- [1] P. Zhou, D. Lin, W. N. Fu, B. Ionescu, and Z. J. Cendes, "A general co-simulation approach for coupled field-circuit problems," *IEEE Trans. Magn.*, vol. 42, no. 4, pp. 1051–1054, 04 2006.
- [2] E. Lange, F. Henrotte, and K. Hameyer, "An efficient field-circuit coupling based on a temporary linearization of FE electrical machine models," *IEEE Trans. Magn.*, vol. 45, no. 3, pp. 1258–1261, 2009.
- [3] S. Schöps, H. De Gersem, and A. Bartel, "A cosimulation framework for multirate time-integration of field/circuit coupled problems," *IEEE Trans. Magn.*, vol. 46, no. 8, pp. 3233–3236, 07 2010.
- [4] J. K. White, F. Odeh, A. L. Sangiovanni-Vincentelli, and A. E. Ruehli, "Waveform relaxation: Theory and practice," *Transactions of the Society for Computer Simulation*, vol. 2, no. 1, pp. 95–133, 1985.
- [5] S. Schöps, H. De Gersem, and A. Bartel, "Higher-order cosimulation of field/circuit coupled problems," *IEEE Trans. Magn.*, vol. 48, no. 2, pp. 535–538, 02 2012.
- [6] A. Nicolet and F. Delincé, "Implicit Runge-Kutta methods for transient magnetic field computation," *IEEE Trans. Magn.*, vol. 32, no. 3, pp. 1405–1408, 05 1996.
- [7] U. Feldmann and M. Günther, "CAD-based electric-circuit modeling in industry I: mathematical structure and index of network equations," *Surv. Math. Ind.*, vol. 8, no. 2, pp. 97–129, 1999.
- [8] S. Schöps, H. De Gersem, and T. Weiland, "Winding functions in transient magnetoquasistatic field-circuit coupled simulations," *COMPEL*, vol. 32, no. 6, pp. 2063–2083, 09 2013.