

$T - A$ formulation for superconducting HTS tapes: the Good, the Bad and the Ugly

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This paper evaluates the $T - A$ formulation, a hybrid approach for modeling the electromagnetic behavior of high-temperature superconducting (HTS) tapes. The method leverages a geometric multi-scale approach by combining a 2D discretization for the superconducting domain, represented by the electric vector potential T , with a 3D discretization for the surrounding domain, described by the magnetic vector potential A . The $T-A$ formulation efficiently handles the high width-to-thickness ratio of HTS tapes, reducing the computational complexity of volumetric modeling while avoiding singularities in the governing equations through resistance-based matrices.

Index Terms—Algebraic formulation, HTS modeling, $T - A$ formulation, Superconducting tapes.

I. INTRODUCTION

HIGH-TEMPERATURE superconducting (HTS) tapes represent a significant breakthrough in high-energy applications. Unlike low-temperature superconductors (LTS), HTS materials operate at elevated temperatures and withstand stronger magnetic fields while carrying high transport currents with minimal energy loss. Among HTS materials, REBCO (rare-earth barium copper oxide) coated conductors stand out as a promising solution to address both technical and economic challenges. These tapes consist of a layered composite structure, with a rare-earth-based superconducting layer enveloped by a copper support, ensuring robust mechanical properties and efficient current transport.

The Good: HTS tapes are distinguished by the exceptionally large width-to-thickness ratio of their superconducting layer, typically ranging from 1000 to 6000 [1]. As a result, typical 3d discretizations required for the numerical electromagnetic solution result in a significantly large number of degrees of freedom. A relatively new approach is to use a 2d discretization for the superconducting domain, where the current field is described by the electric vector potential T , and a 3d discretization of the surrounding domain (usually air) described by the magnetic vector potential A . The $T-A$ formulation [1] exploits the geometric multi-scale peculiarity of the problem using dedicated shell elements, and allows the use of resistance-based matrices, that avoid singularities of the final system when the electrical conductivity is zero in superconductive condition.

The Bad: Despite the growing popularity of the $T - A$ formulation, some authors argue that it cannot be considered a fully three-dimensional electromagnetic model. This is because it does not account for the thickness of the superconducting tape, making it challenging to reliably calculate the local distribution of current density [2].

The Ugly: In addition, in its standard form, the coupling between T and A is imposed as Neumann boundary condition in weak form, with possible violation of the conservation laws [4].

The aim of this paper is to assess the quality of the solutions obtained by the $T - A$ formulation with a quantitative comparison with volumic formulation, showing the benefits and shortcomings.

II. FORMULATION

The non conductive domain is discretized by tetrahedral elements, whereas the superconducting domain is discretized with 2d triangular shell elements, conformal to the faces of the tetrahedra lying on the conductive region. From this simplicial *primal* mesh $\mathcal{G}$, a *dual* mesh $\tilde{\mathcal{G}}$ is derived, based on barycentric subdivision. The $T - A$ method is described in terms algebraic formulation [3].

A. T formulation in the 2d domain

In the 2d domain, $\mathbf{t}$ is the vector of the scalar stream function $t = \vec{T} \cdot \vec{n}$ defined on the nodes, and $\mathbf{i}$ the vector of the electric currents through the edges of the triangular mesh. The electric voltage $\mathbf{e}$ and magnetic flux $\mathbf{b}$ are defined on the dual surface mesh edges and faces, respectively. Defining the classical constitutive resistance matrix $\mathbf{M}_\rho$ and using the edge-to-node $\mathbf{G}_s$ surface incidence matrix, it is possible to write the current field equation

$$\mathbf{G}_s^T \mathbf{M}_\rho \mathbf{G}_s \mathbf{t} = -j\omega \tilde{\mathbf{b}} \quad (1)$$

where $\tilde{\mathbf{b}}$ is the magnetic flux through the dual faces of $\tilde{\mathcal{G}}$.

B. A formulation in the 3d domain

On the 3d mesh, the following integral quantities are defined: the line integral of the magnetic vector potential $\mathbf{a}$ and the magnetic flux $\mathbf{b}$ are defined on the edges and faces, respectively, whereas the magneto-motive force $\tilde{\mathbf{h}}$ and the electric current $\tilde{\mathbf{i}}$ are defined on dual edges and faces, respectively. The face-to-edge incidence matrix $\mathbf{C}$ represents the discrete counterpart of the curl operator. It is possible to show [5] that the magnetic equation is

$$\mathbf{C}^T \mathbf{M}_\nu \mathbf{C} \mathbf{a} = \tilde{\mathbf{i}} \quad (2)$$

where $\mathbf{M}_\nu$ is the discrete constitutive equation such that $\mathbf{h} = \mathbf{M}_\nu \mathbf{b}$ and $\mathbf{C}^T = \bar{\mathbf{C}}$.

C. Coupling terms

Coupling (2) and (1) requires to express the current through dual faces $\tilde{\mathbf{i}}$ in terms of $\mathbf{t}$ and the magnetic flux through the dual faces $\tilde{\mathbf{b}}$ in terms of $\mathbf{a}$. These operation can be done using an interpolation matrix $\mathbf{P}$ and a selection matrix $\mathbf{S}$ whose definition is reported in [3] and omitted here for the sake of conciseness. The final system of equation, in time domain is:

$$\mathbf{M} \frac{d}{dt} \begin{bmatrix} \mathbf{a} \\ \psi \end{bmatrix} + \mathbf{K} \begin{bmatrix} \mathbf{a} \\ \mathbf{t} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix} \quad (3)$$

where:

$$\mathbf{M} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{S} & \mathbf{0} \end{bmatrix}, \mathbf{K} = \begin{bmatrix} \mathbf{C}^T \mathbf{M}_\nu \mathbf{C} & \mathbf{T} \\ \mathbf{0} & \mathbf{G}_s^T \mathbf{M}_\rho(\mathbf{t}) \mathbf{G}_s \end{bmatrix}$$

where the source current can be forced by a suitable Dirichlet boundary conditions on $\mathbf{t}$.

III. SINGLE TAPE MODEL

The $T - A$ formulation is tested versus a standard H -formulation [2] for a single tape with applied alternating current of 140 A at 50 Hz. A tape geometry with a width of 4 mm and a thickness of 1 μm has been studied. For the sake of simplicity, the constant critical current model is considered (i.e. no temperature or field dependence on J_c):

$$\sigma(J) = \frac{J_c}{E_c} \left(\frac{|J|}{J_c} \right)^{n-1} \quad (4)$$

where $J_c = 47.5 \text{ kA/mm}^2$, $E_c = 1 \text{ } \mu\text{V/cm}$ and $n = 35$. The mesh has about 250 k tetrahedral and 8 k triangular elements.

IV. PRELIMINARY RESULTS AND CONCLUSIONS

Fig. 1 shows the current density at 5 ms and 15 ms. The model shows good agreement with the volumic formulation, with the exception of a slight overestimation of the skin effect near the edges of the tape. The loss per cycle is computed as:

$$Q = \int_T \int_V \vec{E} \cdot \vec{J} dV dt \quad (5)$$

Expressed in J/m/cycle. The $T - A$ solution for different input currents is in very good agreement with references, see Fig. 2. The $T - A$ model has an average error of 1.1% with respect to the analytic solution of Norris model: the losses are slightly overestimated at higher current. The $T - A$ formulation seems to be effective calculating local and integral quantities. A thorough analysis with different geometric configuration will be presented at the conference.

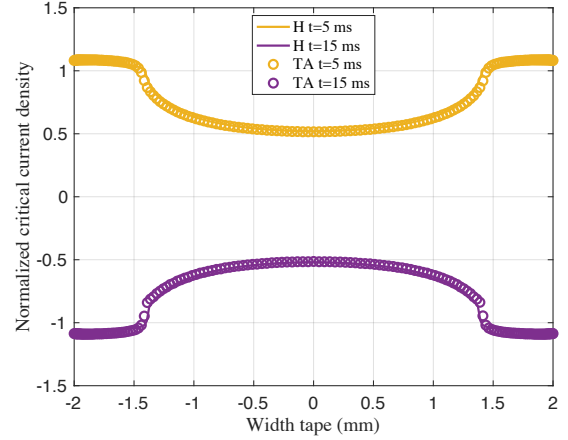


Fig. 1. Current distribution on single tape under a sinusoidal transport current of 140 A.

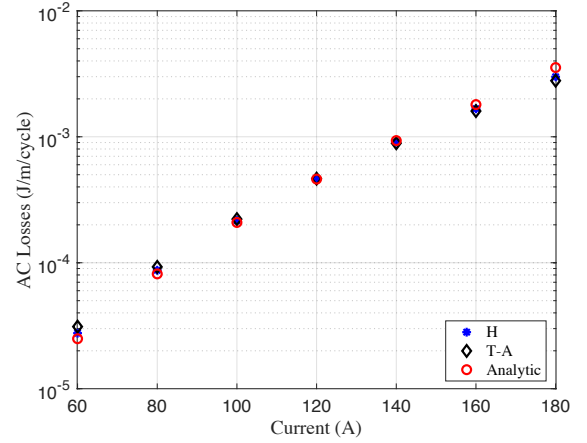


Fig. 2. Comparison of the AC losses of the 2 formulations and the Norris analytical formula.

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