

A Novel EC Model for Carbon Fiber Reinforced Polymers Defect Detection and Classification

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A novel eddy current model for Carbon Fiber Reinforced Polymers is proposed, using a multiplicative perturbation of the conductivity tensor to represent defects while preserving anisotropy. Implemented within a variational finite element framework, it ensures physical consistency and stability. Applied to a Boeing 777-300ER rudder, the model achieves improved defect localization, anisotropic characterization, and distinct electromagnetic response mapping, enhancing non-destructive testing accuracy.

Index Terms—Eddy Currents, anisotropic conductivity, Carbon Fiber Reinforced Polymers, Defect Detection, FEM.

I. INTRODUCTION

CARBON Fiber Reinforced Polymer (CFRP) are anisotropic and heterogeneous materials susceptible to conductivity-altering defects. We propose a novel Eddy Current (EC) model based on a multiplicative perturbation of the conductivity tensor, preserving anisotropy and ensuring physical consistency. Implemented within a FEM framework and validated on the 3D geometry of a Boeing 777-300ER rudder, the model achieves superior defect detection and characterization compared to traditional methods.

II. DEFECTS AND CONDUCTIVITY IN CFRP

$\Omega \subset \mathbb{R}^3$ models the rudder geometry by associating each point with a position vector $\mathbf{x}$, allowing an accurate definition of fiber orientation, conductivity tensor, and defect distribution. In the local frame aligned with the fibers, the conductivity is $\boldsymbol{\sigma}_0(\mathbf{x}) = \text{diag}(\sigma_{\parallel}, \sigma_{\perp}, \sigma_{\perp})$, with $\sigma_{\parallel}$ and $\sigma_{\perp}$ representing conductivities along and across the fibers [1]. The local fiber orientation is described by the rotation matrix $\mathbf{R}(\theta(\mathbf{x}))$, leading to the anisotropic conductivity tensor, $\boldsymbol{\sigma}(\mathbf{x}) = \mathbf{R}(\theta(\mathbf{x})) \cdot \boldsymbol{\sigma}_0 \cdot \mathbf{R}^T(\theta(\mathbf{x}))$ that with defects is extended as $\boldsymbol{\sigma}'(\mathbf{x}) = \mathbf{R}(\theta(\mathbf{x})) \cdot \boldsymbol{\sigma}_0 \cdot \mathbf{T}_d(\mathbf{x}) \cdot \mathbf{R}^T(\theta(\mathbf{x}))$. The tensor $\mathbf{T}_d(\mathbf{x})$ introduces a local perturbation to the conductivity, independent of $\boldsymbol{\sigma}_0$, modeling directional or scalar reductions while preserving symmetry and positive energy dissipation. Outside defects, $\mathbf{T}_d(\mathbf{x})$ becomes the identity matrix, so $\boldsymbol{\sigma}'(\mathbf{x}) = \boldsymbol{\sigma}(\mathbf{x})$ maintaining physical consistency. The new multiplicative formulation here used offers a more accurate, anisotropic, and physically consistent defect modeling compared to additive approaches [1], improving the separation between material properties and defect effects for high-fidelity simulations.

III. DESCRIPTION OF THE DOMAIN

$\Omega \subset \mathbb{R}^3$ models the rudder of a Boeing 777-300ER, a CFRP structure with a height of 7.3m, a base width of 3.2m, a

This work was supported by the project FAIR (CUP H23C22000860006), funded by the European Union under "NextGenerationEU", by the MUR, and by "Italia Domani - PNRR".

TABLE I: $\mathbf{T}_d(\mathbf{x})$ for various defect.

Type of Defect/Innovative Feature	$\mathbf{T}_d(\mathbf{x})$ (Innovative Feature)
Subsurface Delamination (Gaussian out-of-plane directional profile)	$\mathbf{I} - (1 - \delta)e^{-\frac{(z-z_d)^2}{2\sigma_z^2}} \mathbf{e}_z \otimes \mathbf{e}_z$, $\delta \in (0, 1)$, controls the conductivity reduction, z_d is the mean depth of the defect, σ_z is the effective thickness of the damaged region
Oriented Fiber Breakage (Arbitrarily oriented damage modeling)	$\mathbf{I} - (1 - \delta)\mathbf{d} \otimes \mathbf{d}$
Conductive Inclusion (Localized directional conductivity increase)	$\mathbf{I} + \gamma\chi_D(\mathbf{x})(\mathbf{n} \otimes \mathbf{n})$, $\gamma > 0$ is the conductivity amplification factor, $\chi_D(\mathbf{x})$ is the characteristic function of the defect region
Distributed Porosity (Isotropic attenuation field)	$(1 - \epsilon(\mathbf{x}))\mathbf{I}$, $\epsilon(\mathbf{x}) \sim \mathcal{GP}(\bar{\epsilon}, k(\mathbf{x}, \mathbf{x}'))$, $\bar{\epsilon} \in (0, 1)$ is the mean attenuation factor, $\mathcal{GP}$ denotes a Gaussian process with covariance kernel k

top width of 1.0m, and thickness varying from 0.2mm to 0.1mm. It is represented as a 3D curvilinear layered body made of N plies, each subdomain Ω_k having fiber orientation θ_k according to the lay-up sequence $[0^\circ/+45^\circ/-45^\circ/90^\circ]^n$, with $n \in \mathbb{N}$. Each point $\mathbf{x}$ is associated with a rotation matrix $\mathbf{R}(\theta(\mathbf{x})) \in \text{SO}(3)$ to transform vectors from the local to the global frame. Defects are modeled as subsets $\mathcal{D} = \bigcup_{\ell=1}^M D_\ell$, where each D_ℓ represents a localized defect, leading to a partition of Ω into undamaged regions and defective regions where the conductivity $\boldsymbol{\sigma}'(\mathbf{x})$ is locally altered.

IV. EC MODEL AND WEAK ENERGETIC FORMULATION

The behavior of conductive materials follows the EC model, neglecting displacement currents. Using the magnetic vector potential $\mathbf{A}(\mathbf{x})$ and the electric scalar potential $\varphi(\mathbf{x})$, the governing equation is [2] $\nabla \times (\mu^{-1} \nabla \times \mathbf{A}(\mathbf{x})) + \boldsymbol{\sigma}(\mathbf{x})(-j\omega \mathbf{A}(\mathbf{x}) - \nabla \varphi(\mathbf{x})) = \mathbf{J}_e(\mathbf{x})$, where μ is the magnetic permeability, ω the angular frequency, and $\mathbf{J}_e(\mathbf{x})$ the impressed current density. The time-harmonic electric field is

$\mathbf{E}(\mathbf{x}) = -j\omega\mathbf{A}(\mathbf{x}) - \nabla\varphi(\mathbf{x})$. Given that $\sigma'(\mathbf{x})$ is real and symmetric, and using $(\mathbf{E}(\mathbf{x}))^*$ as the complex conjugate of $\mathbf{E}(\mathbf{x})$, the energy functional becomes

$$\mathcal{E}[\mathbf{A}(\mathbf{x}), \varphi(\mathbf{x})] = \int_{\Omega} \left[\frac{1}{\mu} |\nabla \times \mathbf{A}(\mathbf{x})|^2 + \sigma(\mathbf{x}) |\nabla\varphi(\mathbf{x}) + j\omega\mathbf{A}(\mathbf{x})|^2 \right] d\mathbf{x}$$

in which the real part of the functional is non-negative, consistent with Joule energy dissipation. Using the associative property of matrix products, after some calculation, the dissipated energy can be rewritten as $\mathcal{E}_{\text{diss}} = \int_{\Omega} \mathbf{e}(\mathbf{x})^* \cdot (\sigma_0 \mathbf{T}_d(\mathbf{x})) \cdot \mathbf{e}(\mathbf{x}) d\mathbf{x}$ where $\mathbf{e}(\mathbf{x}) = \mathbf{R}^T(\theta(\mathbf{x}))\mathbf{E}(\mathbf{x})$. The variation of the dissipative term regarding $\mathbf{T}_d(\mathbf{x})$ is $\delta\mathcal{E}_{\text{diss}} = \text{Tr}(\mathbf{S}(\mathbf{x}) \cdot \delta\mathbf{T}_d(\mathbf{x}))$, where $\mathbf{S}(\mathbf{x}) = \sigma_0 \cdot \mathbf{e}(\mathbf{x}) \otimes \mathbf{e}^*(\mathbf{x}) = \sigma_0 \cdot \mathbf{R}^T(\theta(\mathbf{x}))\mathbf{E}(\mathbf{x}) \otimes \mathbf{E}^*(\mathbf{x})\mathbf{R}(\theta(\mathbf{x}))$ ¹. The variational problem consists of finding $(\mathbf{A}(\mathbf{x}), \phi(\mathbf{x})) \in H(\text{curl}, \Omega) \times H^1(\Omega)$ such that, for all virtual variations $(\delta\mathbf{A}(\mathbf{x}), \delta\phi(\mathbf{x}))$, $a((\mathbf{A}, \phi), (\delta\mathbf{A}, \delta\phi)) = L(\delta\mathbf{A})$ where $a((\mathbf{A}, \phi), (\delta\mathbf{A}, \delta\phi)) = \int_{\Omega} \mu^{-1} (\nabla \times \mathbf{A}) \cdot (\nabla \times \delta\mathbf{A})^* d\mathbf{x} + \int_{\Omega} (-j\omega\delta\mathbf{A} - \nabla\delta\phi)^* \cdot \sigma'(\mathbf{x}) \cdot (-j\omega\mathbf{A} - \nabla\phi) d\mathbf{x}$ and $L(\delta\mathbf{A}) = \int_{\Omega} \mathbf{J}_e(\mathbf{x}) \cdot \delta\mathbf{A}^*(\mathbf{x}) d\mathbf{x}$.

V. FEM DISCRETIZATION OF THE EC PROBLEM

The rudder geometry was discretized into about 120,000 tetrahedral elements using first-order Nedge elements, with mesh refinement near edges and at the base to improve accuracy where stresses and defects are more likely, ensuring a balance between geometric precision, numerical stability, and computational efficiency for accurate defect modeling [3]. Let $V_h \subset H(\text{curl}, \Omega)$ be the space of Nedge elements of the first kind used to approximate $\mathbf{A}_h(\mathbf{x})$, and let $W_h \subset H^1(\Omega)$ be the space of Lagrangian elements (e.g., P1 elements) used to approximate $\phi_h(\mathbf{x})$. The approximated solutions are given by $\mathbf{A}_h(\mathbf{x}) = \sum_{i=1}^{N_A} \alpha_i \phi_i(\mathbf{x})$, $\phi_h(\mathbf{x}) = \sum_{j=1}^{N_{\phi}} \varphi_j \eta_j(\mathbf{x})$, where $\phi_i(\mathbf{x}) \in V_h$ are the vector-valued basis functions and $\eta_j(\mathbf{x}) \in W_h$ are the scalar-valued basis functions. The unknown coefficient vectors are $\alpha \in \mathbb{C}^{N_A}$ and $\varphi \in \mathbb{C}^{N_{\phi}}$. The discretized $\mathbf{E}(\mathbf{x})$ reads $\mathbf{E}_h(\mathbf{x}) = -j\omega\mathbf{A}_h(\mathbf{x}) - \nabla\phi_h(\mathbf{x})$. The magnetic stiffness matrix $\mathbf{K}$ was obtained by discretizing the magnetic energy via FEM, integrating the curls of the basis functions weighted by μ^{-1} . The anisotropic conductivity matrix $\mathbf{C}$ was derived by inserting $\mathbf{E}(\mathbf{x})$, expressed through the potentials, into the energy formulation and integrating with $\sigma'(\mathbf{x})$ over Ω , thus capturing anisotropy and defects. The coupling matrix between $\phi(\mathbf{x})$ and $\mathbf{A}(\mathbf{x})$ was assembled by integrating the scalar product of $\nabla\phi(\mathbf{x})$ with $\mathbf{E}(\mathbf{x})$, while the right-hand side vector $\mathbf{f}$ was computed by integrating $\mathbf{J}_e(\mathbf{x})$ with the vector basis functions. These steps lead to the construction of the final complex algebraic system. The FEM coefficients for $\mathbf{A}_h(\mathbf{x})$, ϕ_h , and $\mathbf{f}(\mathbf{x})$ were used to assemble matrices accounting for anisotropy and defects, computed with first-order Nedge and Lagrangian elements and accurate quadrature at Gauss points; the final system was efficiently solved with a preconditioned GMRES method.

VI. SIMULATIONS AND RESULTS

For the simulations, we focused on a restricted area of the rudder near the fuselage (highly prone to defect development).

¹ $\otimes$ represents the tensor product (or outer product) between vectors.

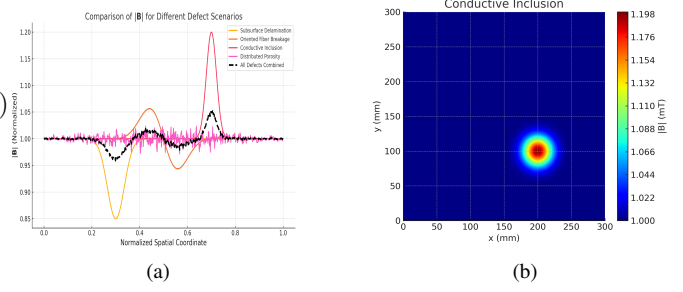


Fig. 1: Normalized $|\mathbf{B}|$ for various defect scenarios (a) and detailed map for a conductive inclusion (b), showing a localized increase along the preferential direction due to its critical impact.

In Fig. 1(a), the delamination is characterized by a conductivity reduction factor $\delta = 0.5$, a mean depth $z_d = 2$ mm, and an effective thickness $\sigma_z = 0.5$ mm; the oriented fiber breakage is modeled with a reduction factor $\delta = 0.7$ and a damage direction $\mathbf{d}$ aligned at 45° to the fiber axis; the conductive inclusion is defined by a conductivity amplification factor $\gamma = 2$ and localized in a small defect region $D(\mathbf{x})$; the distributed porosity is represented by an isotropic attenuation field $(1 - \epsilon(\mathbf{x}))I$, where $\epsilon(\mathbf{x})$ follows a Gaussian process with mean $\bar{\epsilon} = 0.3$ and covariance kernel $k(\mathbf{x}, \mathbf{x}')$ describing spatial correlation. Compared to [1], the proposed energy-based FEM model with anisotropy and fiber rotation captures local effects more accurately and in greater detail. Conductive inclusions notably increase $|\mathbf{B}|$ and can mask weaker defects, while nonlinear interactions are correctly modeled, avoiding the overestimation of additive approaches. For space reasons, only the $|\mathbf{B}|$ map for conductive inclusions, identified as the most critical defects, is shown in Fig. 1(b).

High-resolution 2D maps showed localized perturbations: conductive inclusions caused a 20% increase in $|\mathbf{B}|$, subsurface delaminations led to a 15% reduction, and oriented fiber breakages generated strong anisotropic distortions, all accurately captured through local modifications of the anisotropic conductivity tensor. Combined defects resulted in nonlinear smoothing without overestimating, preserving each defect's signature. The multiplicative approach with local fiber orientation effectively models conductivity variations, preserves anisotropy, and enhances defect detection and reconstruction. Future work will focus on inverse problem strategies, multiphysics monitoring, and experimental validation on CFRP samples.

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