

Machine Learning-based Reduced Order Modelling of Nonlinear Magnetic Devices

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This work presents an approach to efficiently model nonlinear magnetic problems through a machine-learning approach. Proper Orthogonal Decomposition (POD) is employed to reduce the model's dimensionality while machine-learning techniques are employed to find a relationship between the input data and the solution. In this digest, the methodology is demonstrated on a simplified version of the TEAM 36 benchmark problem, involving the induction heating of a steel billet with nonlinear magnetic properties. Results show that the approach accurately captures nonlinear effects while significantly reducing computational demands, with future work focusing on more complex problems.

Index Terms—Dimensionality reduction, Finite element analysis, Machine learning, Nonlinear magnetics.

I. INTRODUCTION

THE Finite Element Method (FEM) is a powerful tool for modeling nonlinear magnetic problems, as it discretizes the governing equations into a large set of partial differential equations. However, in applications such as optimization and model predictive control, there is a strong need to reduce both the computation time required for solving these equations and the memory footprint of the problem. Model Order Reduction (MOR) techniques, such as the Proper Orthogonal Decomposition (POD) [1], address these challenges by projecting the system matrices of the Full Order Model (FOM) onto a lower-dimensional subspace, obtaining a Reduced Order Model (ROM). While MOR techniques are well-suited for linear problems, nonlinear problems introduce additional complexities. Methods like the Discrete Empirical Interpolation Method (DEIM) [2] can deliver excellent results for nonlinear systems but their performance is case-dependent. Furthermore, their intrusiveness can pose challenges, especially when the FOM is generated using commercial software. An alternative strategy involves using machine-learning techniques, to directly establish a relationship between the inputs and the solution. Although these models offer greater flexibility, they also face significant hurdles, such as the high computational cost of generating the training dataset and the challenge of handling the high dimensionality of the solution space. In prior research, the POD algorithm has been employed to reduce the dimensionality of the solution, with machine-learning models (such as Gaussian Process Regression (GPR) [3], Artificial Neural Networks (ANNs) [4] and Radial Basis Functions (RBFs) [5]), subsequently trained to map inputs to the reduced solution vector. Discrepancy modeling strategies, often applied to correct the error between measurements and prediction from a first-principle model [6] could also be employed to correct linear ROMs using data collected from a high-fidelity representation. The novelty of this work lies in the creation of surrogate models for nonlinear magnetic problems through

data-driven techniques, providing a compact representation of the FEM model. The methodology is evaluated on the TEAM benchmark problem 36: “Multi-physics Field Analysis of an Induction Heating Device” [7], [8].

II. CASE STUDY

The induction heating problem described in TEAM 36 consists of a steel billet heated by a solenoidal solid conductor. The billet is made of C45 steel, whose material properties (thermal conductivity, heat capacity, magnetic permeability, electrical conductivity) are dependent on the temperature and on the magnetic field, following [7].

In this digest, the MOR technique was applied to a simplified version of the TEAM 36 problem, where the coil is fed by an uniformly distributed DC current. Moreover, the temperature was fixed at 20°C, rendering the relative permeability a function of the field strength only, following [7]. The 3D problem is approached through the axisymmetric representation:

$$\nabla \cdot \nu'(\Phi) \nabla \Phi = J_{s,\theta}, \quad (1)$$

written in the (r, z, θ) coordinates system. In (1) $\nu' = \frac{1}{r\mu_r\mu_0}$ and $\Phi = rA_\theta$ are the magnetic reluctivity and the magnetic vector potential in the θ direction, scaled by the distance r from the axis of symmetry.

(1) is discretized via an in-house FEM implementation obtaining the nonlinear system of equation:

$$\mathbf{K}(\mathbf{x}) \mathbf{x} = \mathbf{B}\mathbf{u}, \quad (2)$$

where the input $\mathbf{u}$ represents the current flowing in the coil, and the solution vector $\mathbf{x}$ contains the value of Φ at each node of the FEM mesh.

III. METHODOLOGY

POD can approximate the true solution $\mathbf{x}$ by projecting it into a lower dimensional subspace [1]. This is done through a

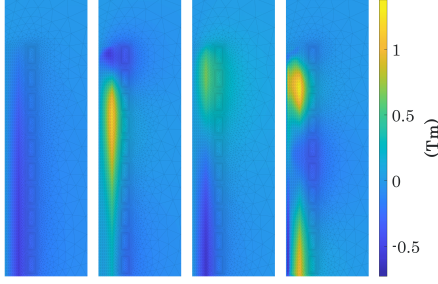


Fig. 1: Spatial distribution of POD modes

projection matrix $\mathbf{V}$, obtained via singular value decomposition of the snapshot matrix, built by concatenating solutions of (2) for different input values. A ROM can be derived by projecting the FOM onto the reduced basis:

$$\mathbf{x} = \mathbf{V}\mathbf{z} \quad (3)$$

$$\mathbf{V}^T \mathbf{K}(\mathbf{x}) \mathbf{V} \mathbf{z} = \mathbf{V}^T \mathbf{B} \mathbf{u}. \quad (4)$$

For a linear FOM, this process would result in a linear ROM with a limited number of states. However, solving the nonlinear ROM requires at each iteration constructing the FOM as a function of $\mathbf{x}$ and then projecting it. While techniques like DEIM allow for obtaining a reduced form of the problem, the effectiveness is case-dependent and the intrusiveness of the approach can be a limit. The POD with interpolation (PODI) technique leverages POD to reduce the dimensionality of $\mathbf{x}$, while interpolation methods are used to find a relationship:

$$\hat{\mathbf{z}} = f(\mathbf{u}). \quad (5)$$

Alternatively, the black-box model is used to find the discrepancy $\delta(\mathbf{u})$ between the results of the linear ROM and the true solution, obtaining:

$$\hat{\mathbf{z}} = \mathbf{z}_{linear}(\mathbf{u}) + \delta(\mathbf{u}). \quad (6)$$

An estimate of the true solution can then be obtained through the reduced basis $\hat{\mathbf{x}} = \mathbf{V}\hat{\mathbf{z}}$. The MOR process introduces two kinds of error. The first one occurs when approximating the true solution as a linear combination of a limited number of modes and can be decreased by augmenting their number, while the second is due to inaccuracies of the chosen model and can be tackled by augmenting the number of data-points and by choosing an adequate model structure.

IV. RESULTS

The FOM was simulated using 101 current values from 0 A to 500 A, then separated into a train and test dataset of 30 and 71 points respectively. POD was used to reduce the dimensionality of $\mathbf{x}$ from 7844 to 4 states presented in Fig. 1, and GPR to find a relationship between the reduced state vector and the input current.

As presented in Fig. 2 the model closely matches the true solution. Moreover, a visual representation of the error between the true solution and the prediction is also reported in Fig. 3.

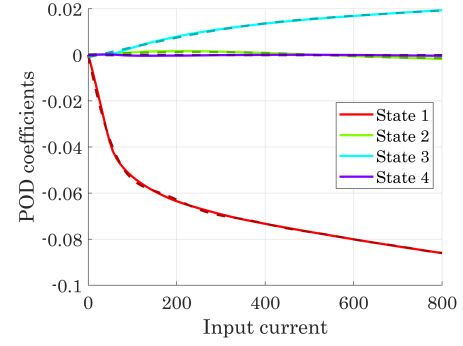


Fig. 2: Reduced state vector as a function of the input current. Continuous lines represent the true solution, while dashed lines are the model predictions.

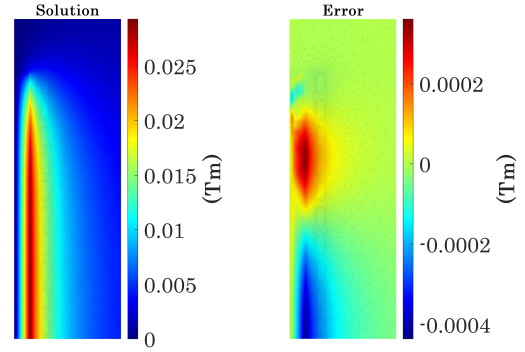


Fig. 3: Solution and prediction error for $I = 240\text{A}$

V. CONCLUSION

While preliminary results show the effectiveness of the approach, the full paper will tailor the method to more complex problems.

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