

# Dynamic hysteresis model based on a Fourier neural operator

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In this digest, a Fourier neural operator (FNO) is used to model the dynamic hysteresis behavior of ferromagnetic materials. First, a 1D finite element (FE) model is built, in which, both hysteresis and eddy current are considered. By applying a sinusoidal source at the boundary (magnetic field  $H$ ) with different frequencies and peak values, a bunch of corresponding dynamic magnetic flux density ( $B$ ) are generated. Then, all dynamic hysteresis loops (DHLs) are split into 2 groups: training data and test data. Attention is paid to feasibility and accuracy of proposed method in DHLs modeling.

*Index Terms*—Dynamic hysteresis model, Fourier neural operator, eddy current losses, finite element model

## I. INTRODUCTION

**S**TUDYING the magnetic characteristics and modeling the ensuing losses are essential for analyzing and optimizing electromagnetic devices. However, the intrinsic saturation and hysteresis characteristics of ferromagnetic materials make them harder to model the devices given the high nonlinearity [1]. Most models nowadays disregard hysteresis and eddy current, assuming the core is made of a reversible and nonconducting material. While such simplifications streamline the modeling process, they often lead to significant inaccuracies, especially when predicting complex hysteresis behaviors. This limitation becomes particularly problematic in applications where precision is critical [2]. A well-designed model not only enhances predictive accuracy but also facilitates the practical implementation of advanced magnetic systems, enabling more reliable and optimized designs for real-world applications. With the development of data science, machine learning (ML) and especially neural networks (NNs) are widely used in ferromagnetic material modeling, including multilayer perceptron (MLP) [3], recurrent neural network (RNN) [4], long short term memory (LSTM), convolutional neural network (CNN) [5]. In most cases, only static hysteresis is considered, neglecting the frequency effects and eddy currents. Besides, a substantial amount of data is often required to train these models.

A new type of neural network: the neural operator is proposed based on the universal operator approximation theory: a NN with a single hidden layer can approximate accurately any nonlinear continuous functional and nonlinear operator [6]. Different from physics-informed neural network (PINN), which is trained to solve one specific partial differential equation (PDE), the neural operator can be trained to model the differential operator itself [7]. There are mainly two types of neural operators: the deep operator network (DeepONet) [7] and the Fourier neural operator (FNO) [8], which have been successfully applied to complex problems in fluid dynamics, heat transfer but rarely in electromagnetism. DeepONet consists of two networks: the branch net for input functions and the trunk for the output domain coordinates. In FNO, fast Fourier transform (FFT) is used to transform complex integral operations into simple multiplications in the Fourier domain. Given the capability of FNO to directly incorporate

time sequences into training data, which effectively represents frequency information, this work leverages FNO to develop a dynamic hysteresis model of ferromagnetic material.

## II. METHODS AND IMPLEMENTATION

### A. Fourier neural operator (FNO)

FNO proposed in [8] is formulated as an iterative architecture. As shown in Fig 1, the input  $H(t)$  is first lifted to a higher dimensional representation  $f_0(t) = \mathcal{P}(H(t))$  by the local transformation  $\mathcal{P}$ , which is a simple fully-connected NN. In this way, more channels are created to represent more details of the input. Then, several FNO units are applied for updates  $f_i \rightarrow f_{i+1}$ , where  $i = 0, 1, \dots, I$ . The final updated  $f_I$  is projected by a local transformation  $\mathcal{Q}$ . In each iteration, the update is by:

$$f_{i+1}(t) := \alpha(W f_i(t) + \mathcal{K}(a; \phi) f_i)(t), \quad \forall t \in D \quad (1)$$

where  $D$  is a bounded open set,  $W$  is a linear transformation, and  $\alpha$  is a non-linear activation function whose action is defined component-wise,  $\mathcal{K}(a; \phi)$  is a bounded open set, with  $a$  as the input function space and  $\phi$  as the learnable parameters of the kernel function. In FNO,  $\mathcal{K}(a; \phi)$  is chosen as a kernel integral transformation parameterized by a neural network as:

$$(\mathcal{K}(a; \phi) f_i)(t) := \int_D \kappa(t, y, a(t), a(\tau); \phi) f_i(\tau) d\tau, \quad \forall t \in D \quad (2)$$

where  $\kappa$  is a neural network parameterized by  $\phi \in \Theta_{\mathcal{K}}$ ,  $\Theta_{\mathcal{K}}$  is the finite-dimensional parameter space for kernel  $\mathcal{K}$ . Then, replacing the kernel integral operator by a convolution operator defined in the Fourier space and letting  $\kappa(t, \tau, a(t), a(\tau)) = \kappa(t - \tau)$  in (2), it could be computed via a FFT:

$$(\mathcal{K}(\phi) f_i)(t) = \mathcal{F}^{-1}(\mathcal{R}_{\phi} \cdot (\mathcal{F} f_i))(t), \quad \forall t \in D \quad (3)$$

where  $\mathcal{F}$  denotes the Fourier transform of a function and  $\mathcal{F}^{-1}$  its the inverse.  $\mathcal{R}_{\phi}$  is the built model parameterized by  $\phi \in \Theta_{\mathcal{K}}$ .

### B. Finite element model for data generation

A one-dimensional (1D) finite element (FE) model is built to generate a bunch of dynamic hysteresis loops.

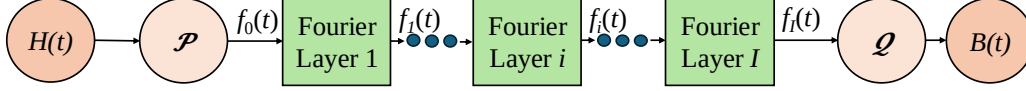


Fig. 1: FNO procedure. With  $\mathcal{P}$  for lifting,  $\mathcal{Q}$  for transforming and several Fourier layers in between.

Jiles-Atherton (JA) equations with parameters as  $M_{\text{sat}} = 1.47 \times 10^6$  A/m,  $a = 3.6$  A/m,  $k = 10.4$  A/m,  $c = 0.03$  and  $\alpha = 3.2 \times 10^{-6}$  are inserted in. In addition, the model incorporates eddy current losses to account for the frequency effects. By applying different magnetic field sources ( $H$ ) with varying peak values and frequencies,  $B - H$  hysteresis curves are computed for various scenarios. These curves are derived by averaging the  $B$  values over the entire domain and correlating them with the corresponding applied  $H$ . A total of 27 different frequencies of  $H$ , ranging from 5 Hz to 1000 Hz, are applied to the FE model. For each frequency, the peak value of  $H$  is varied among 100, 500 and 1000 A/m, resulting in  $27 \times 3 = 81$  distinct groups of dynamic hysteresis loops (DHLs). Each DHL contains 101 data points. To account for frequency differences, time  $t$  is also included in the training data as an additional feature. The dataset is divided into training, validation and test sets, with 80% used for training and 10% each reserved for validation and testing.

### III. RESULTS

After 2000 iterations, taking 12 mins on CPU of an Apple M1 Max system with its ARM-based architecture, the training loss decreased to approximately  $1e^{-4}$ , and the test loss reached a value lower than  $1e^{-3}$ . The predictions generated by the trained FNO model and their corresponding reference values, shown in Fig. 2, are nearly identical. Additionally, the hysteresis loss calculated using (4) as the area of the DHLs (after denormalization), is provided in Table I. The errors are mostly under 1%, confirming both the feasibility and the high accuracy of proposed FNO-based DHL model.

$$P_V = \frac{1}{T} \int_{B(0)}^{B(T)} H(t) dB(t). \quad (4)$$

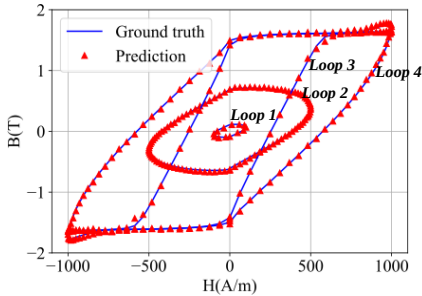


Fig. 2: Predicted DHLs from FNO and reference

TABLE I: Losses by FNO and reference for DHLs in Fig. 2

| Loops | Reference<br>(J/m <sup>3</sup> ) | Predictions<br>(J/m <sup>3</sup> ) | Relative<br>Error (%) |
|-------|----------------------------------|------------------------------------|-----------------------|
| 1     | 26.93                            | 26.93                              | 0.03%                 |
| 2     | 873.30                           | 894.11                             | 2.38%                 |
| 3     | 1641.19                          | 1656.06                            | 0.91%                 |
| 4     | 3496.13                          | 3502.29                            | 0.18%                 |

### IV. CONCLUSION

In this digest, a Fourier neural network is built to model the dynamic hysteresis of ferromagnetic materials with different peak values under different frequencies. With only less than 70 groups of data, the established operator model can accurately predict the corresponding DHLs outside of the training data. Besides, hysteresis losses are calculated using generated DHLs, with the relative error to reference values mostly lower than 1%, which shows the great ability of proposed model for capturing dynamic magnetic hysteresis. In the full paper, cases with more frequencies and peak values would be studied. In addition, more strategies would be applied to FNO to increase the accuracy and the generalization ability of model further.

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