

A magneto-elastic energy-based hysteresis model with magnetization as internal dissipation variable

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The energy-based hysteresis model with dry friction type dissipation using magnetization as internal variable is extended considering a magnetoelastic Gibbs free energy. Magnetic field and mechanical stress loadings are considered and a multiscale model is used to represent the anhysteretic behavior. An explicit evaluation scheme inspired by the vector play approach is proposed. By comparison with uniaxial measurements, the model is shown to generate reasonable piezomagnetic loops and gives very satisfying results in a case with stress and field varying simultaneously.

Index Terms—Magnetomechanical effects, ferromagnetic materials, magnetic hysteresis.

I. INTRODUCTION

The energy-based hysteresis model is an attractive alternative to the popular Preisach and Jiles-Atherton models for the representation of magnetic hysteresis. It presents intermediate computational cost and strong thermodynamical basis [1][2]. The energy-based approach, and particularly its approximation as a vector play model, was applied to the representation of magnetoelastic behavior under constant stress [3] [4]. To extend this approach to magnetoelastic behavior under varying stress, the use of magnetostriction as an additional internal variable was proposed in [5]. Here we propose to keep only magnetization as internal variable, and we derive a magneto-elastic model directly from the expression of the Gibbs free energy and the dry-friction dissipation. An approximation, similar to the one used in the vector play approach, is also proposed to avoid minimization. Results are compared to uniaxial measurements for piezomagnetic loops and for a loading with simultaneous variations of stress and field.

II. MODELING

Considering only magnetic field ($\vec{H}$) loading and using magnetization ($\vec{M}$) as internal variable to account for dissipation processes, the material behavior is described through the definition of a Gibbs free energy $g(\vec{H}, \vec{M})$ such that [6]:

$$\vec{B} = -\frac{\partial g}{\partial \vec{H}}, \quad (1a)$$

$$D = -\frac{\partial g}{\partial \vec{M}} \cdot \dot{\vec{M}} \geq 0. \quad (1b)$$

where $\vec{B} = \mu_0 (\vec{H} + \vec{M})$ is the magnetic flux density, D is the dissipation power density and $\dot{\vec{M}}$ is the time derivative

of the magnetization. Assuming dry-friction like processes, a dissipation function, linear in $\dot{\vec{M}}$, is defined as

$$\phi_d(\dot{\vec{M}}) = \mu_0 \kappa \|\dot{\vec{M}}\| = \frac{\partial \phi_d}{\partial \dot{\vec{M}}} \cdot \dot{\vec{M}} = \mu_0 \kappa \frac{\dot{\vec{M}}}{\|\dot{\vec{M}}\|} \cdot \dot{\vec{M}} \quad (2)$$

where κ is a constant (independent on $\vec{M}$) model parameter. For small enough magnetization steps, a finite difference approximation is made and condition (1b) is then verified if [2]:

$$\vec{0} \in \frac{\partial}{\partial \vec{M}} \left[g + \mu_0 \kappa \|\vec{M} - \vec{M}_{(p)}\| \right] \quad (3)$$

where $\partial/\partial \vec{M}$ is the subgradient with respect to $\vec{M}$. To account for magneto-mechanical loadings, we consider now that the Gibbs free energy $g(\vec{H}, \boldsymbol{\sigma}, \vec{M})$ depends also on the second order stress tensor $\boldsymbol{\sigma}$. To conditions (1) we must add:

$$\boldsymbol{\epsilon} = -\frac{\partial g}{\partial \boldsymbol{\sigma}} \quad (4)$$

where $\boldsymbol{\epsilon} = \mathcal{C}^{-1} \boldsymbol{\sigma} + \boldsymbol{\epsilon}^\mu$ is the total strain, $\mathcal{C}$ is the stiffness tensor and $\boldsymbol{\epsilon}^\mu$ the magnetostriction strain. To satisfy this condition and (1a), the Gibbs free energy is written as:

$$g(\vec{H}, \boldsymbol{\sigma}, \vec{M}) = \mu_0 \left(-\frac{1}{2} \|\vec{H}\|^2 - \vec{H} \cdot \vec{M} \right) - \frac{1}{2} \mathcal{C}^{-1} \boldsymbol{\sigma} : \boldsymbol{\sigma} + \mu_0 f(\vec{M}, \boldsymbol{\sigma}) \quad (5)$$

with $\partial f/\partial \vec{M} = \vec{H}_{rev}$ and $\partial f/\partial \boldsymbol{\sigma} = -\frac{1}{\mu_0} \boldsymbol{\epsilon}^\mu$ defining the reversible behavior. We then have:

$$\frac{\partial g}{\partial \vec{M}} = -\mu_0 (\vec{H} - \vec{H}_{rev}) = -\mu_0 \vec{H}_{irr} \quad (6)$$

where $\vec{H}_{rev}$ is the reversible field and $\vec{H}_{irr} = \vec{H} - \vec{H}_{rev}$ is the irreversible field. The magnetoelastic model is then obtained by calculating $\vec{M}$ that verifies (3) at each step, and the corresponding magnetostriction $\boldsymbol{\epsilon}^\mu(\vec{M}, \boldsymbol{\sigma})$. While

$$\left\| \frac{\partial g}{\partial \vec{M}} \right\| = \mu_0 \|\vec{H}_{irr}\| < \mu_0 \kappa \quad (7)$$

magnetization remains unchanged $\vec{M} = \vec{M}_{(p)}$. If $\|\vec{H}_{irr}\| = \kappa$, (3) gives

$$\frac{\partial g}{\partial \vec{M}} + \mu_0 \kappa \frac{\vec{M} - \vec{M}_{(p)}}{\|\vec{M} - \vec{M}_{(p)}\|} = \vec{0} \quad (8)$$

so

$$\vec{H}_{irr} = \vec{H} - \vec{H}_{rev} = \kappa \frac{\vec{M} - \vec{M}_{(p)}}{\|\vec{M} - \vec{M}_{(p)}\|} \quad (9)$$

The relation between $\vec{H}_{rev}$ and $\vec{M}$ being known (defined by the anhysteretic behavior $\vec{M} = \vec{M}_{an}(\vec{H}_{rev}, \sigma)$), this implicit equation can be solved for $\vec{H}_{rev}$ under imposed loading $(\vec{H}, \sigma)$. Under constant stress, an explicit solution can be obtained considering the vector play approximation:

$$\vec{H}_{irr}^{vp} = \kappa \frac{\vec{H} - \vec{H}_{rev(p)}}{\|\vec{H} - \vec{H}_{rev(p)}\|} \quad (10)$$

where $\vec{H}_{rev(p)}$ is the magnetic field at previous step. We propose to extend this approximation to the magnetoelastic context considering:

$$\vec{H}_{irr} = \kappa \frac{\vec{M} - \vec{M}_{(p)}}{\|\vec{M} - \vec{M}_{(p)}\|} \quad (11)$$

where $\vec{M}$ is an approximation of the new magnetization:

$$\vec{M} = \vec{M}_{an}(\vec{H} - \vec{H}_{irr}^{vp}, \sigma) \quad (12)$$

From this, the reversible field, the magnetization $\vec{M} = \vec{M}_{an}(\vec{H}_{rev}, \sigma)$ and the magnetostriction $\epsilon^\mu = \epsilon_{an}^\mu(\vec{H}_{rev}, \sigma)$ can be updated. If $\|\vec{H}_{irr}\| < \kappa$, we solve:

$$d\vec{M} = \frac{\partial \vec{M}_{an}}{\partial \vec{H}_{rev}} d\vec{H}_{rev} + \frac{\partial \vec{M}_{an}}{\partial \sigma} d\sigma = \vec{0} \quad (13)$$

for the reversible field which changes with stress in such a way it keeps the magnetization constant. The variation of irreversible field considering field and stress variations is $d\vec{H}_{irr} = d\vec{H} - d\vec{H}_{rev}$.

III. APPLICATION

The model described above is extended to consider a distribution of κ^k as is usually done in the multi-cell energy-based model [1][2][3]. A stress dependence of the κ^k parameters is also considered [3][5]. The anhysteretic behavior is modeled by the simplified (uniform magnetic field and mechanical stress, no localization process) multi-scale approach with a simple 4 grains $\langle 111 \rangle$ fiber texture [5]. The κ^k parameters are identified using only uniaxial symmetric magnetic hysteresis loops $\vec{B}(\vec{H})$ under constant stress. More details can be found in [3]. Measurements and modeling results are shown in Fig 1: piezomagnetic loops under different values of constant magnetic field and, magnetic and piezomagnetic behavior for a configuration with simultaneous variations of magnetic field and mechanical stress.

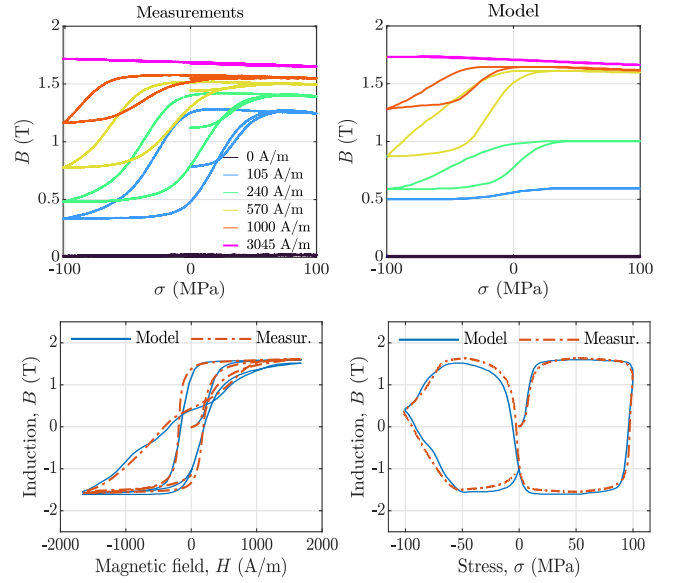


Fig. 1. Measurements and model results: piezomagnetic loops (top), field and uniaxial stress variations (bottom).

IV. DISCUSSION

The proposed magnetoelastic hysteresis is thermodynamically consistent (it ensures positive dissipation at all time) and its parameters can be identified from hysteresis loops $\vec{B}(\vec{H})$ under constant stress. Under constant stress, the model is equivalent to the magnetic energy-based model. Under stress variations, reversible and irreversible field may change, and piezomagnetic hysteresis is accounted for. The obtained piezomagnetic loops reasonably match with measurements for magnetic fields above 240 A/m. Below this value, the hysteresis loops amplitude decreases quickly. A similar decrease should appear in measurements, but below 100 A/m. In the test case with variations of field and stress, model matches very well with experiment. Further tests will be made, similar to the ones in [7], to check the properties of the proposed vector play type approximation.

REFERENCES

- [1] F. Henrotte, A. Nicolet, and K. Hameyer, "An energy-based vector hysteresis model for ferromagnetic materials," *COMPEL*, vol. 25, pp. 71-80, 2006.
- [2] K. Jacques, "Energy-based magnetic hysteresis models-theoretical development and finite element formulations," *IEEE Access*, PdD thesis, ULiège-Université de Liège, 2018.
- [3] L. G. da Silva, A. Abderahmane, M. Domenjoud, L. Bernard, and L. Daniel, "An Extension of the Vector-play Model to the Case of Magneto-elastic Loadings," *IEEE Access*, pp. 1-13, DOI:10.1109/ACCESS.2022.3222833, 2022.
- [4] H. Zhao, Y. An, J. Liu, "An enhanced Energy-Based hysteresis model for electrical steel sheets considering mechanical stress," *JMMM*, vol. 606, pp. 172347, Elsevier, 2024.
- [5] L. G. da Silva, L. Bernard, M. Domenjoud, and L. Daniel, "A magneto-elastic vector-play model including piezomagnetic behavior," *JMMM*, vol. 609, pp. 172439, Elsevier, 2024.
- [6] C. Miehe, B. Kiefer, D. Rosato, "An incremental variational formulation of dissipative magnetostriction at the macroscopic continuum level," *International Journal of Solids and Structures*, vol. 48, n°13, pp. 1846-1866, Elsevier, 2011.
- [7] L. Prigozhin, V. Sokolovsky, J.W. Barrett, and S.E. Zirka, "On the energy-based variational model for vector magnetic hysteresis," *IEEE TMag*, vol. 52, n°12, pp. 1-11, IEEE, 2016.