

Reconstruction of Magnetic Material Using Magnetic Dot Sensitivity Analysis

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This paper presents a method for reconstructing magnetic material using magnetic dot sensitivity analysis. The proposed approach introduces the concept of dot sensitivity, which evaluates changes in the objective function when a ferromagnetic material dot is created in the vacant region. This enables simultaneous topology and shape optimization without the need for an initial shape, providing robust identification of target topologies from magnetic flux density data. The method demonstrates feasibility by reconstructing the cross-sectional area of a cast iron cylinder as a numerical example.

Index Terms—Inverse problems, magnetostatics, system identification, and sensitivity analysis.

I. INTRODUCTION

INVERSE PROBLEM in the magnetostatic system have been significant challenges in various applications for decades. The inverse problems include the topology identification of sources or magnetic materials such as non-destructive testing (NDT) and magnetic resonance imaging (MRI). These problems are notoriously difficult to solve due to their inherent characteristics of ill-posedness and underdetermined systems. Although numerous gradient-based methods have been employed for shape optimization, applying these methods to inverse problems often leads to challenges of nonunique solutions or convergence to local optima. Previous studies have optimized the shape of materials using the level-set method and continuum shape sensitivity analysis [1]. These have the advantages of flexibility in defining design variables and independence from discrete models. However, their reliance on an initial shape and the simplicity of optimized geometries limit their applicability to inverse problems. These limitations can be addressed by incorporating continuum topology sensitivity analysis alongside shape sensitivity analysis. Unlike shape sensitivity, topology sensitivity allows analysis over the entire design domain rather than being restricted to specific boundaries. This eliminates the need for an initial shape and reduces the likelihood of converging to local optima, thereby enhancing the robustness of solutions for inverse problems.

This paper proposes a method based on the magnetic dot sensitivity to address inverse problems of reconstructing a magnetic material. The goal of the proposed approach is to identify non-invasively the topology of a target which consists of ferromagnetic material from magnetic flux density data in the air domain. To achieve this, a concept of magnetic dot sensitivity belonging to the continuum topology sensitivity is introduced which is defined as the change in the objective function when a ferromagnetic dot is generated in a vacant region. Simultaneously, shape sensitivity is applied to the boundaries of the generated materials, enabling combined topology and shape optimization to determine the topology of the target.

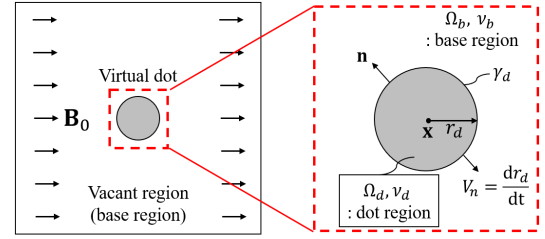


Fig. 1. Virtual dot generation in a base region

II. DERIVATION OF MAGNETIC DOT SENSITIVITY

The magnetic dot sensitivity is the rate of change of the objective function F with respect to the variation in the volume of a virtual dot. To formulate this in 2-D linear magnetostatic systems, a virtual circular dot is supposed to be generated in a vacant region and its surface changes at a rate of V_n , as shown in Fig. 1. The continuum topology sensitivity T_{2D} from the virtual dot is defined as follows.

$$T_{2D}(\mathbf{x}) \equiv \lim_{\gamma_d \rightarrow 0} \frac{\Delta F}{\Delta a} = \lim_{\gamma_d \rightarrow 0} \frac{\oint_{\gamma_d} s(\mathbf{A}, \boldsymbol{\lambda}) V_n d\Gamma}{\oint_{\gamma_d} V_n d\Gamma} \quad (1)$$

where $\mathbf{x}$ is the center of the virtual dot, r_d is its radius, a is its area and γ_d is its interface. The $s(\mathbf{A}, \boldsymbol{\lambda})$ is a continuum shape sensitivity which consists of state variables $\mathbf{A}$ and adjoint variables $\boldsymbol{\lambda}$.

The magnetic shape sensitivity in 2-D magnetostatic systems is expressed by the following equation [1].

$$s(\mathbf{A}, \boldsymbol{\lambda}) = (\nu_b - \nu_d) \mathbf{B}(\mathbf{A}_d) \mathbf{B}(\boldsymbol{\lambda}_b) \quad (2)$$

where $\mathbf{B}$ is the magnetic flux density, ν is the reluctivity which is a reciprocal of the magnetic permeability μ . The notation of b and d represents the base and dot region, respectively.

To compute (2), the magnetic flux density inside the virtual dot which exhibits the properties of ν is determined. The internal magnetic flux density was analytically derived by assuming the creation of an infinitesimal dot within a homogeneous magnetic flux density $\mathbf{B}_0$ [2]:

$$\mathbf{B}_d = \frac{2\nu_b \mathbf{B}_0}{\nu_b + \nu_d} \quad (3)$$

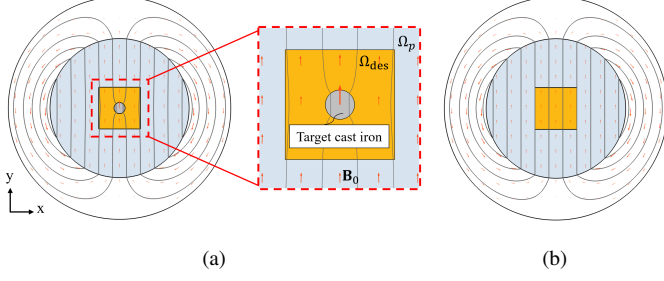


Fig. 2. Numerical modeling (a) reference model (b) initial model

By substituting (2) and (3) into (1), the continuum topology sensitivity in 2-D linear magnetostatic systems, hereafter referred to as dot sensitivity, is derived as follows:

$$T_{2D}(\mathbf{x}) = \frac{2\nu_b(\nu_b - \nu_d)}{\nu_b + \nu_d} \mathbf{B}_b(\mathbf{A})\mathbf{B}_0(\boldsymbol{\lambda}) \quad (4)$$

III. NUMERICAL MODELING FOR MAGNETIC MATERIAL RECONSTRUCTION

To solve the inverse problem using the magnetic dot sensitivity, numerical modeling was implemented. The objective function minimized was formulated as the integration of a residual function between the magnetic flux density $\mathbf{B}_0$ from the reference material and the current magnetic flux density distribution $\mathbf{B}$ in the domain Ω_p :

$$F = \int_{\Omega_p} (\mathbf{B} - \mathbf{B}_0)^2 m_p d\Omega \quad (5)$$

where m_p is a characteristic function of the Ω_p .

The initial state was set as a vacant space and the domain in which the target is identified was assigned as a design domain. The solutions of the state equation and the adjoint equation were computed using the finite element method with COMSOL Multiphysics. A dot was generated at a position corresponding to the minimum value until all values of the dot sensitivity in the design domain become positive. During the generation, the material interface was simultaneously deformed based on the velocity distribution from the shape sensitivity. To express the deformed shape, the level set equation was used:

$$\frac{\partial \phi}{\partial t} + V_n |\nabla \phi| = 0 \quad (6)$$

where ϕ is the level-set function and t is the pseudo-time step.

IV. NUMERICAL EXAMPLES AND RESULTS

To demonstrate the feasibility of the proposed method, a numerical example was conducted to identify the cross-sectional surface area of a basic cylinder cast iron target located in an invisible region. Fig. 2 illustrates the reference model to be identified and the initial model. An external uniform magnetic field was applied in the y direction as a source, and the design domain Ω_{des} was defined as a rectangular region. The region Ω_p was specified as the area enclosing the design domain. During the design process, only the magnetic flux density distribution from the reference material within the

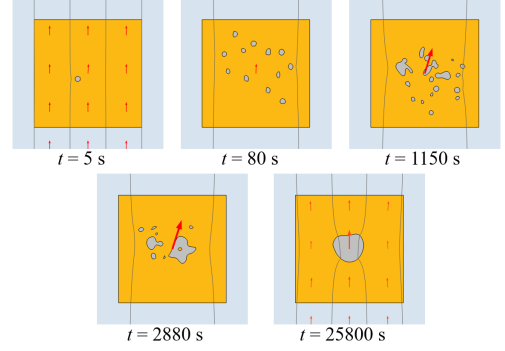


Fig. 3. Results of dot generation with shape optimization

objective function domain is provided as prior information. To numerically demonstrate the effectiveness of the proposed method, prior information was supplied with the simulation results rather than the experimental data.

In this example, determining the position of the dot using the minimum value of dot sensitivity caused dots generated only near the external boundary of Ω_{des} , where the adjoint field magnitude was large. To resolve this issue, the dot sensitivity value was integrated over a circular region with a diameter d centered on each mesh vertex in Ω_{des} . The dot was then generated at the position that had the closest value to (5). The diameter d used in the optimization was determined that had the integral value in the optimum position closest to the objective function in the initial model.

Fig. 3 illustrates the iterative process until the objective function reaches the convergence criterion. In the initial step, a dot was generated near the target center and subsequently, dots were distributed over a wider area around it. However, as the shape optimization progresses simultaneously, the dots begin to cluster closer to the target. In the final model at $t = 25800$ s, the identified material's area was 1281.5 m^2 , showing only a 2.8 % difference from the target area which is 1256.6 m^2 . Additionally, the center position aligns closely with the target, demonstrating significant accuracy.

V. CONCLUSION

This paper proposes a method for reconstructing magnetic materials by using continuum topology sensitivity. To demonstrate its applicability, a numerical example was conducted to identify the shape of a ferromagnetic material. From the results, it was possible to identify the magnetic material shape and position only with the magnetic flux density distribution data. The proposed method can be extended to identify the topology of currents and permanent magnets in the future. Detailed explanations of the dot generation algorithm will be presented in subsequent conference and full papers.

REFERENCES

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