

Air Gap Shape Optimization for Minimizing Proximity Effect Losses in an Inductor

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This paper addresses the constrained gradient-based design optimization of the air gap profile of an inductor with both control points and adaptative mesh deformation based approaches. The objective is to minimize the proximity effect losses inside the conductors while keeping the inductance value constant. This involves a complex permeability model to account for losses, with sensitivities determined via the adjoint method and shape derivatives. It is shown that the optimized air gap profile is curved in order to concentrate the flux lines away from the (fixed) conductors. The implementation uses the open-source finite-element toolbox NGSolve and Mmg remeshing library; all the codes are freely available.

Index Terms—Inductor, AC losses, Parametric optimization, Shape optimization

I. INTRODUCTION

DESIGN optimization is used intensively for the conception of electromagnetic devices. Using gradient-free methods, some work has been done on designing inductors to limit proximity losses due to the fringing effect by optimizing the conductors [1] or the air gap profile [2]. The best configuration prevents the flux lines from crossing the conductors. Even if gradient-based methods generally converge faster to a (local) optimum, they are less common. A possible explanation is the high CPU cost required to compute the derivative of the objective functions using finite differences. Fortunately, mathematical tools like the adjoint method simplify and enhance the efficiency of sensitivity computations. Furthermore, shape derivative computation enables optimization through control points and mesh deformation. To make this mesh deformation adaptive, we leverage the remeshing capabilities of MMG. While shape differentiation is standard in real-valued problems and can even be automated [3], its application to complex-valued problems is not common, although it would be of high interest in electrical engineering. This paper provides a concrete application of the adjoint method and shape derivatives to minimize proximity AC losses in an inductor due to the fringing effect near the air gap. All the Python codes use the NGSolve package and are freely available on <https://doi.org/10.5281/zenodo.14631426>.

II. MATHEMATICAL MODEL

A. Physical equations

We consider the magneto-quasistatic analysis in the frequency domain of a 2D region $\Omega = \Omega_a \cup \Omega_f \cup \Omega_c$, where

- Ω_a is the air region associated with a magnetic permeability $\mu_0 = 4\pi \times 10^{-7}$ H/m;
- Ω_f is the ferromagnetic region associated with a permeability $\mu_f = 1000\mu_0$. The iron losses are neglected;

- Ω_c is the region of electrical conductors. Its complex permeability comes from homogenized models depending on the frequency and configuration of the wires [4]. The adopted value is $\underline{\mu}_c = \mu_0 \exp(-i\delta)$ with $\delta = 0.1$ rad, i being the unit imaginary number such that $i^2 = -1$.

The state equation then reads

$$-\text{div}(\underline{\nu} \nabla \underline{a}) = j, \quad (1)$$

where $\underline{\nu}$ is reluctivity ($\frac{1}{\underline{\mu}_c}$ in Ω_c , $\frac{1}{\mu_f}$ in Ω_f and $\frac{1}{\mu_0}$ in Ω_0), $\underline{a}$ is the out-of-the-plane component of the magnetic vector potential such that $\underline{b} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \nabla \underline{a}$ and j the (real) current density that is the phase reference and assumed to be sinusoidal. The AC losses come from the proximity effect in the conductor that is included in the imaginary part of the permeability and reads

$$P(\underline{a}) = s l_z \pi f \int_{\Omega_c} \text{Im} \left(\frac{1}{\underline{\mu}_c} \right) |\nabla \underline{a}|^2 dx, \quad (2)$$

while the inductance is expressed as

$$L(\underline{a}) = \frac{s l_z}{I^2} \int_{\Omega} \text{Re}(\underline{\nu}) |\nabla \underline{a}|^2 dx, \quad (3)$$

where $s = 4$ is the symmetry coefficient, $l_z = 1$ cm is the z -thickness (fringing is neglected in the z direction), I is the electric current amplitude set to 2 A with 200 turns. The model is discretized and solved using NGSolve.

B. Shape derivatives

The aim is to minimize the AC losses (2) while keeping the inductance (3) to a given value $L_0 = 1$ mH. The initial design is given in Figure 1a.

The sensitivity of objective and constraint functions to a variation of the air gap profile can be evaluated using shape derivatives [3]. In a nutshell, the shape derivative of a given

function g is a linear form defined on a small deformation field $\phi \in (H^1(\Omega))^2$ by the following limit:

$$dg(\Omega)(\phi) = \lim_{\substack{t \rightarrow 0 \\ t > 0}} \frac{g(\Omega + t\phi) - g(\Omega)}{t}. \quad (4)$$

The described methods arise from the flexible definition of ϕ . The shape derivatives of the AC losses and the inductance read

$$dP(\Omega)(\phi) = \text{Re} \left(sl_z \pi f \int_{\Omega_c} \text{Im} \left(\frac{1}{\underline{\mu}_c} \right) A_\phi \nabla \underline{a} \cdot (\nabla \underline{a})^* dx \right. \\ \left. + \int_{\Omega} \underline{\nu} A_\phi \nabla \underline{a} \cdot \nabla \underline{p}_P^* dx - \int_{\Omega_c} j \text{div} \phi \underline{p}_P^* dx \right) \quad (5)$$

$$dL(\Omega)(\phi) = \text{Re} \left(\frac{sl_z}{I^2} \int_{\Omega} \underline{\nu} A_\phi \nabla \underline{a} \cdot (\nabla \underline{a})^* dx \right. \\ \left. + \int_{\Omega} \underline{\nu} A_\phi \nabla \underline{a} \cdot \nabla \underline{p}_L^* dx - \int_{\Omega_c} j \text{div} \phi \underline{p}_L^* dx \right), \quad (6)$$

respectively, with the real 2×2 matrix $A_\phi = \text{div} \phi I_2 - \partial \phi - \partial \phi^\top$. Here, the adjoint state $\underline{p}_P$ solves the equation

$$-\text{div} \left(\underline{\nu}^* \nabla \underline{p}_P \right) = -\frac{\partial P}{\partial \underline{a}}(\underline{a}), \quad (7)$$

and $\underline{p}_L$ is defined analogously by replacing the right-hand side by $-\frac{\partial L}{\partial \underline{a}}(\underline{a})$.

III. OPTIMIZATION RESULTS

In this section, the results of our optimizations are given and compared to a reference design shown in Figure 1a. We use an augmented Lagrangian algorithm, which will be discussed in detail in the extended article. A summary is provided in Table I.

A. Optimization with control points

Defining ϕ as the vertical displacement at the four control points shown in Figure 1b leads to a parametric optimum shown in Figure 2a. The resulting air gap profile is shaped to carry high flux away from the conductors, halving AC losses.

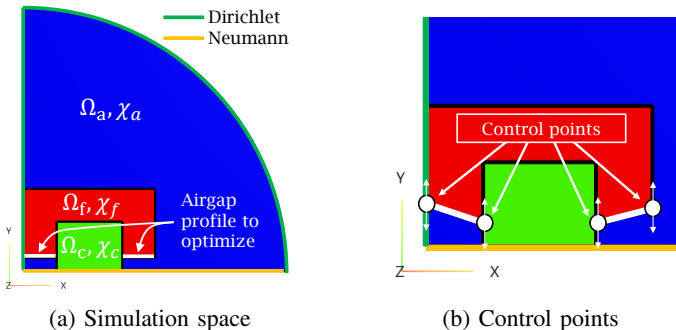


Fig. 1: Simulation and optimization setup.

B. Optimization with adaptive mesh deformation

Defining ϕ as the displacement at the mesh nodes dramatically increases the number of effective control points, leading to greater design diversity. A descent direction is found by solving a variational problem related to the shape derivatives. This enables deformation regularization and partial geometry preservation through the appropriate scalar product and subspace choice. The Mmg library reparametrizes the mesh every 5 iterations, thereby maintaining acceptable quality throughout the optimization. The optimized design in Figure 2b shows a curved air gap profile, achieving lower AC losses but converging more slowly.

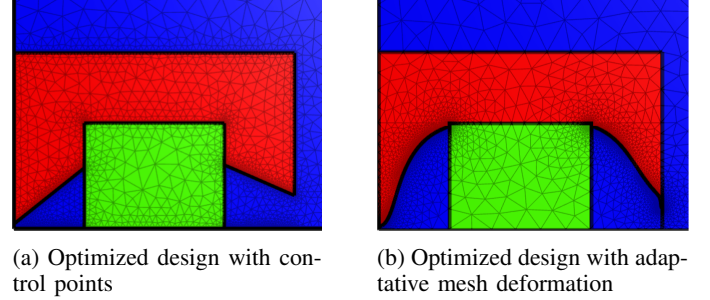


Fig. 2: Final results of the optimizations.

TABLE I: Optimization results.

Optimization Type	Iterations	P (W)	L (mH)
Reference	-	13.16	1.00
Control points	92	6.00	1.00
Adaptive mesh deformation	251	3.82	1.00

IV. CONCLUSION

This paper explains the methodologies of control points and adaptive mesh deformation for shape optimization, effectively reducing proximity-effect losses while maintaining the desired inductance. The latter achieved superior results but requires more iterations, remeshing, and a slightly more complex implementation. The extended article will give more details on the algorithms and computations, especially regarding calculating the shape derivative using complex numbers. In addition, the performance of our methods compared to a standard heuristic and gradient-free optimization algorithm will be discussed.

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