

Body-Fitted Level-Set Topology Optimization for Synchronous Reluctance Machines with Free Boundary

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This study proposes a framework for multi-physics topology optimization of a synchronous reluctance machine with the level-set method. Body-fitted meshes are used for accurate finite element analysis (FEA) and criteria evaluation. The novelty lies in including the outer boundary of the rotor in the designable domain with non-penetration constraints. The design can thus attach and detach freely from the outer boundary enabling new designs to emerge.

Index Terms—Design tools, electrical machines, multiphysics, topology optimization, adaptive mesh refinement

I. INTRODUCTION

EFFICIENT design of electrical machines, especially synchronous reluctance machines (SynRMs), is paramount to overcoming the challenges of electrification in the transport industry. These conception problems are inherently multiphysical, with design objectives and constraints taken from magnetic and mechanical analysis. Topology optimization has gained attraction in the past decade in this field thanks to its capability to create innovative and generative designs [1].

In current literature, the density method often associated with SIMP [2] is the most popular being both efficient computationally and relatively easy to implement. However, this approach presents intermediate non-physical materials especially when objectives from competing physics are chosen [2] and are often biased by convergence-enhancing filters. The body-fitted level-set method bypasses these limitations, creating a well-defined design-fitted mesh allowing precise FEA. To the best of the authors' knowledge, this method remains rarely used in electrical machine design with non-linear material [3], preferring non-conforming approaches with fuzzy interfaces like NGnet [4] and fixed-mesh level-set [5].

In this paper, we extend the approach proposed in [3] by making the outer boundary of the rotor optimizable, enabling flux-barrier ends design to derive minimum torque ripple in a closed form. A new non-penetration constraint is introduced to ensure the shape remains within the design domain.

II. BODY-FITTED LEVEL-SET

In the rotor domain Q of the SynRM (Fig4a), the material distribution between the ferromagnetic Ω_f and air Ω_a material domains is represented via a P1-Lagrange level-set field ϕ ,

$$\phi(x) := \begin{cases} \phi(x) < 0, \forall x \in \Omega_f \\ \phi(x) > 0, \forall x \in \Omega_a \\ \phi(x) = 0, \forall x \in \Gamma (= \partial\Omega_i) \end{cases} \quad (1)$$

similar to a signed distance to their interface Γ .

At each optimization step, after updating the level set, the previous mesh is cut at $\{\phi(x) = 0\}$ and re-meshed with the MMG software (Fig.1) providing a conformal mesh.

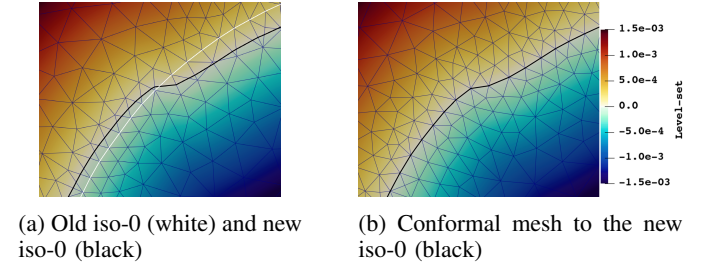


Fig. 1: Re-meshing operation

A custom meshing metric (2) is proposed to better capture the interface with finer mesh elements for $\{|\phi(x)| \ll l_d\}$

$$h(\phi) = h_{min} + \frac{h_{max} - h_{min}}{2} \left(\tanh \left(\frac{|\phi|}{l_d} \right) + 1 \right) \quad (2)$$

where l_d denotes the length over which the element size gradient diffuses from the minimal element size h_{min} to its maximal size h_{max} . Re-meshing is limited to the optimizable domain Q surrounded by a thin band of elements to reduce the computational load while not over-constraining the mesh (see zoom in Fig.4a)

III. OPTIMIZATION PROBLEM

For our analysis, two criteria are chosen: the DQ-saliency [2] and mechanical compliance.

$$\begin{cases} \min_{\Omega_i \subseteq Q} \Delta W_{dq} & \text{(DQ-saliency, magnetostatics)} \\ \text{s.t. } C \leq C_{max} & \text{(Static compliance, mechanics)} \end{cases} \quad (3)$$

Magnetic and mechanical performances of the electrical machine are obtained through FEA. Non-linear magnetic analysis

is performed as in [6]. For mechanics, a linear elasticity model is chosen with a young modulus of 210 GPa, the rotor only subjected to centrifugal forces. The maximum compliance C_{max} is chosen from an initial analysis with a rotation speed of $\omega = 2000$ rpm.

A. Non-linear regularization and non-penetration constraint

The issue with the previous formulation lies in the fact that the bound $\Omega_i \subseteq Q$ is not “visible” to the optimizer even though the sensitivity of magnetic criteria is the highest (by magnitudes) on the rotor’s outer edge. To solve this issue, a custom non-linear regularization scheme is proposed to rescale the sensitivity to magnitudes that are manageable for the optimizer and more homogeneous throughout the design area.

$$\int_{\Omega} h_{max}^2 \nabla w \cdot \nabla v + \int_{\Gamma} (1 + w^{2m}) w \cdot v \, dx = - \int_{\Gamma} \mathcal{D}\mathcal{J}(v), \forall v \quad (4)$$

If $-\mathcal{D}\mathcal{J}(v)$, the shape sensitivity of $\mathcal{J}$ in the direction v , is a descent direction, w the solution of (4) is also a descent direction because the right-hand side of the pde is non-negative for the test function $v = w$. The penalization coefficient m is chosen equal to 3 as a compromise between effective sensitivity magnitude rescaling and speed of the Newton-Raphson solver.

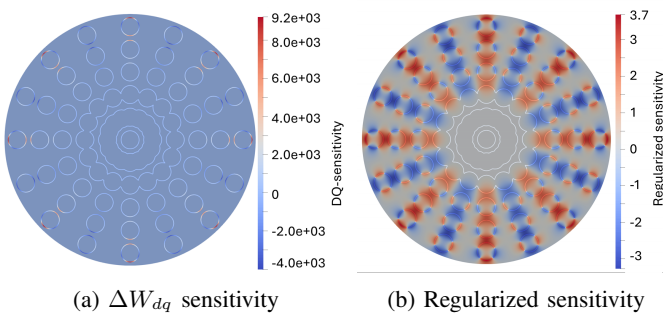


Fig. 2: Regularization procedure

Second, we use a constraint on the L^P norm of the symmetrized distance [7] to the stator inner edge to enforce the size of the air gap and thus the bounds of Q . The sensitivity of this constraint is then given to the Null Space optimizer [8]. We get back a “null space” direction, which allows for the improvement of the objective function while keeping the constraints unchanged (Fig. 3).

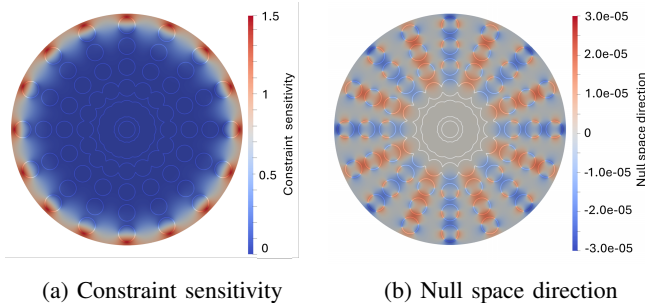


Fig. 3: Assembling a descent direction

IV. CASE STUDY AND RESULTS

A 12-stator tooth, 4-pole pairs SynRM is chosen as a case study to test the proposed method. The winding and geometry are given in Fig.4.

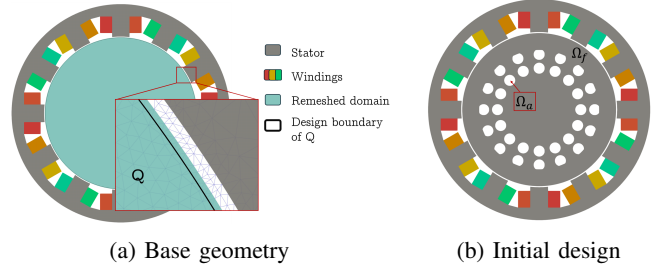


Fig. 4: Geometry and initial design

Optimization is run for 80 iterations taking around 6 hours to complete after reaching a stall. Designs at two stages of the optimization are given in Fig.5 for iterations 40 and 80.

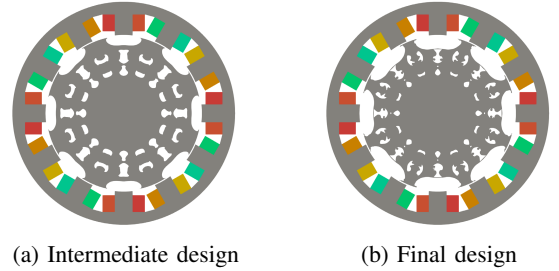


Fig. 5: Evolution of the design

The final design presents the desired non-smooth poles highlighting the effectiveness of the proposed method. In the full paper, criteria more aligned with actual machine design will be used, and further details on the outer bound constraints will be provided along with a comparative analysis with [3].

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