

Reduced NGnet Method for Topology Optimization of Electromagnetic Devices

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This paper introduces a reduced normalized Gaussian network (NGnet) method for topology optimization, which effectively reduces the optimization complexity while preserving the advantages of NGnet such as coordinate system independence and no gray scale generation. In this method, the basis functions are derived by truncated eigendecomposition of the kernel matrix composed of the NG functions. Comparative optimization results demonstrate that this method effectively balances the complexity and diversity of topological structures. It provides an effective approach for achieving more manufacturing-friendly structures.

Index Terms—Topology optimization, Gaussian basis functions, eigendecomposition.

I. Introduction

TOPOLOGY OPTIMIZATION (TO) [1] offers the ability to generate novel structures beyond the scope of parameter optimization, playing a crucial role in advancing the performance of electromagnetic devices. The normalized Gaussian network (NGnet) method [1-2], based on normalized Gaussian (NG) basis functions, has been extensively applied in the design of rotating machines and other devices. Leveraging the locality and smoothness of Gaussian basis functions, the NGnet method enables diverse and smoothly varying topological structures. Using more basis functions can enhance topological diversity and performance but also increases computational cost and the possibility of generating complicated structures that are not suitable for manufacturing. Luo [3] proposed the material-field series-expansion which performs truncated singular value decomposition of the kernel matrix composed of the Gaussian bases to obtain the reduced basis functions for topology optimization. However, the lack of normalization may result in improper basis functions, as will be shown in this paper.

To address these issues, we propose here the reduced NGnet method which performs truncated eigendecomposition of the kernel matrix constructed from NG basis functions. By retaining only the dominant eigenvalues and their corresponding eigenvectors, we construct a new reduced set of basis functions. The reduced NGnet method significantly reduces dimensionality while improving computational efficiency, allowing for optimal solutions with simplified structures and high manufacturability.

II. Reduced NGnet Method

We start from the original NGnet method in a multidimensional space. Let $x, x_i \in R^d$. The shape function $y(x)$ is represented as a sum of kernel functions $k(x, x_i)$ with weight w_i :

$$y(x) = \sum_{i=1}^n k(x, x_i) w_i, \quad (1)$$

where $k(x, x_i)$ is based on normalized Gaussian functions:

$$k(x, x_i) = \frac{G_i(x, x_i)}{\sum_{j=1}^n G_j(x, x_j)}, \quad (2)$$

$$G_i(x, x_i) = \frac{1}{(2\pi)^{d/2} \sigma^d} \exp\left(-\frac{1}{2\sigma^2} \|x - x_i\|^2\right). \quad (3)$$

In matrix form, equation (1) becomes:

$$y(x) = k^T w, \quad (4)$$

where $k = [k(x, x_1), \dots, k(x, x_n)]^T$ is the kernel vector, and $w = [w_1, \dots, w_n]^T$ is the weight vector. The kernel matrix K is defined as:

$$K_{ij} = k(x_i, x_j), \quad i, j = 1, \dots, n, \quad (5)$$

and its eigenvalue decomposition is given by:

$$K = U \Lambda U^T, \quad (6)$$

where $U = [u_1, \dots, u_n]$ is the matrix of eigenvectors, and $\Lambda = \text{diag}(\lambda_1, \dots, \lambda_n)$ is the diagonal matrix of eigenvalues. By projecting the kernel vector k and weight vector w into the eigenvector space, the shape function can be expressed as:

$$y(x) = \sum_{j=1}^n \frac{1}{\lambda_j} (u_j^T w) (k^T u_j), \quad (7)$$

where we treat $k^T u_j$ as the new basis functions, and $\frac{1}{\lambda_j} (u_j^T w)$ represents the new weights. For simplicity, let:

$$v_j(x) = k^T u_j, \quad c_j = \frac{1}{\lambda_j} u_j^T w, \quad (8)$$

To reduce dimensionality, only the top m eigenvalues and corresponding eigenvectors are retained. The value of m is determined by the following criterion:

$$\frac{\sum_{j=1}^n \lambda_j - \sum_{j=1}^m \lambda_j}{\sum_{j=1}^n \lambda_j} \leq \epsilon, \quad (9)$$

Then the approximated shape function is given by:

$$y(x) \approx \sum_{j=1}^m c_j v_j(x). \quad (10)$$

We have plotted the topology structure images for different values of m in Fig. 1. It can be observed that as m decreases, the details in the images gradually decrease, but the main features of the original image are preserved.



Fig. 1: Topologies under a different basis numbers.

Fig. 2 and Fig. 3 illustrate the first five new basis functions, extracted from normalized and unnormalized Gaussian basis function networks, respectively. The new basis functions in Fig. 2 have proper characteristics; lower-order functions are relatively flat, while higher-order functions capture more complex details of NGnet. For the following optimization, the basis functions in Fig. 2 will be used.

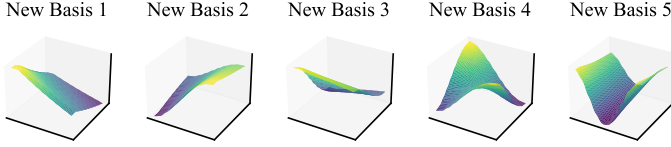


Fig. 2: New basis functions from normalized Gaussian.

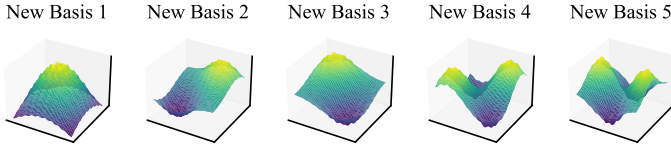


Fig. 3: New basis functions from unnormalized Gaussian.

III. Optimization Results

We take the design of a Permanent Magnet Synchronous Motor (PMSM) rotor as an example to compare our method with the NGnet method. The design model is shown in Fig. 4. We here solve a multi-objective problem including magnetization direction[4]. For the NGnet method, 17 NG bases with $\sigma = 6$ mm were uniformly placed in the rotor's design region (Fig. 4a). For the proposed method, 60 NG bases with $\sigma = 3$ mm were used (Fig. 4b). The design objective is to maximize torque (T_q) while minimizing torque ripple (T_{rip}), formulated as a multi-objective optimization problem:

$$\begin{aligned} \min_{\mathbf{x}} \quad & \mathbf{F}(\mathbf{x}) = [-T_q(\mathbf{x}), (T_{rip}(\mathbf{x}))], \\ \text{subject to} \quad & V(\mathbf{x}) \leq V_0(\mathbf{x}), \end{aligned} \quad (11)$$

where V denotes the total volume of the magnet. We conducted optimizations with NSGA-II, an evolutionary algorithm, based on NGnet method with 17 NG bases and 60 NG bases, and proposed method with 17 and 15 new bases, respectively. Notably, the proposed method starts with 60 NG bases and reduces them to 17 and 15 new bases through eigendecomposition. The optimal pareto fronts

and iteration histories are shown in Fig. 5. Compared to the NGnet method, the proposed method achieves a better Pareto fronts, especially when using 15 basis functions. This is because the proposed method extracts 15 new important bases from 60 NG bases, preserving diversity while reducing computational complexity. The corresponding optimal structures are shown in Fig. 6.

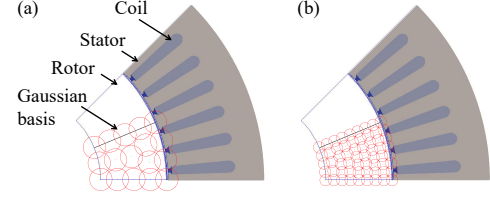


Fig. 4: Gaussian bases in the design region for (a) 17 bases, and (b) 60 bases.

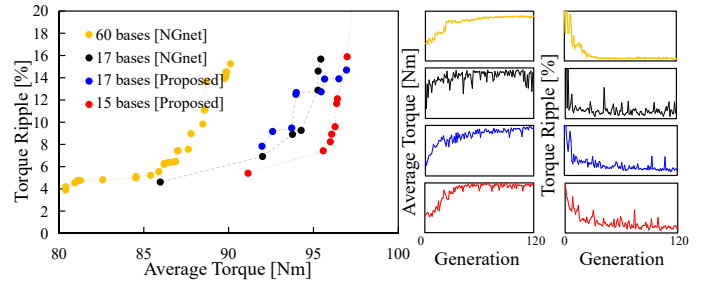


Fig. 5: Pareto fronts from NGnet and proposed method.

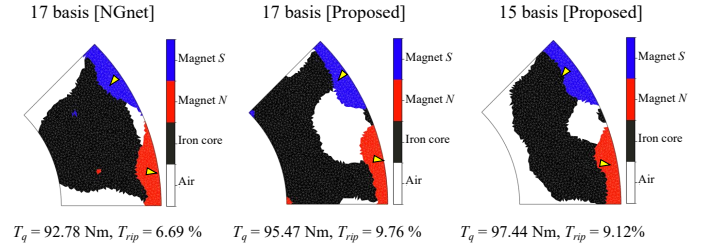


Fig. 6: Best topologies from NGnet and proposed method.

IV. Conclusion

This paper proposes a reduced NGnet method that requires fewer variables, promotes diverse topological structures, accelerates convergence, and aids in discovering manufacturable designs. This method would be particularly useful for the multi-material and three-dimensional topology optimization. A more detailed comparison with other methods will be given in the extended version.

References

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