Topology Optimization of Electric Motors using Mesh Projection

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We present a topology optimization framework for electric motors that maximizes the torque under the constraint of the torque ripple and the rotor weight. By projecting the rotor design onto a static background mesh at various orientations, the proposed method eliminates the need for remeshing, simplifying the handling of the relative motion between the stator and rotor. Furthermore, we explore interpolation factors that improve design exploration during optimization. A synchronous reluctance motor (SynRM) is optimized to achieve substantial torque improvement and ripple reduction, demonstrating strong potential for advancing high-efficiency SynRM design.

***Index Terms*—Topology Optimization, Finite Element Analysis, Synchronous Reluctance Motor, Computational Electromagnetics**

I. Introduction

THE design optimization of synchronous reluctance motor (SynRM) focuses on maximizing torque and minimizing ripple [1], [2]. Topology optimization, which explores the design space exhaustively for optimal design features, has been commonly applied to the SynRM motor design [3]. One of the key challenges is to determine suitable interpolation parameters that avoid numerical artifacts. Furthermore, existing methods for handling relative motion between the stator and the rotor involve computationally expensive techniques such as remeshing, periodic boundary conditions [4] or sliding mesh [5].

In this work, we present a novel topology optimization framework that simplifies torque evaluation at various rotor positions. By modifying the density filtering step, we introduce a projection-based approach, mapping the rotor design onto a static background mesh at different orientations, streamlining the computational workflow. Furthermore, we explore the role of interpolation factors on design exploration optimization convergence.

II. Problem Formulation

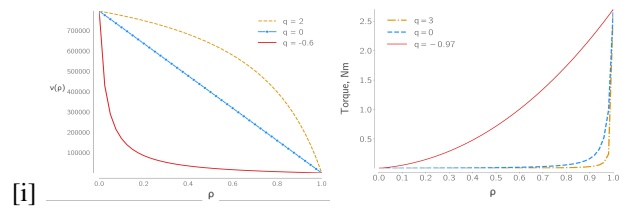
We seek an SynRM rotor topology that maximizes the average torque produced at various rotor angles while constraining the mass of the rotor and the torque ripple. The topology optimization formulation is shown as follows:

$$\begin{aligned} \max_{\rho_e} \quad & T_{avg} \\ \text{s.t.} \quad & T_{ripple} \leq T^*_{ripple} \\ & \sum_{i=1}^{n_e} \frac{\rho_i V}{V_{max}} \leq v_f \\ & \underline{\mathbf{K}}\underline{\phi} = \underline{\mathbf{F}} \quad \forall \quad \theta = 0^\circ, 15^\circ, ... \\ & 0 < \rho_{e_{min}} \leq \rho_e \leq 1 \end{aligned} \tag{1}$$

where. T_{avg} is the average torque produced. The torque at each angle is evaluated using a linear magnetostatic finite element formulation. The torque ripple factor, T_{ripple}, is the ratio of the Root Mean Square Torque (T_{rms}) to the average torque magnitude, with T_{ripple} constrained to be below T^*_{ripple}. A mass fraction constraint of v_f is applied for a lightweight design. Sensitivities are derived using the adjoint method, and the method of moving asymptotes (MMA) [6] is used to solve the optimization problem. Each finite element in the rotor design domain is assigned a design variable, called the material density, $\rho \in [0, 1]$. We use the Rational Approximation of Material Properties (RAMP) scheme to interpolate the material's reluctivity between air(ν_0) and metal(ν_m) as follows:

$$\nu = \nu_m \left[\frac{\rho}{1 + q(1-\rho)} \right] + \nu_0 \left[1 - \frac{\rho}{1 + q(1-\rho)} \right] \tag{2}$$

where q is an interpolation parameter [7]. Figure 1 shows the effect of RAMP interpolation on intermediate densities. The rotor is assigned a uniform intermediate density between 0 and 1, and torque is evaluated for different interpolation parameters q. For $q \geq 0$, most intermediate densities yield similar torque response, causing premature optimization termination. To improve design space exploration, we use $q \in (-1, 0)$, where the effective reluctivity and torque exhibit a concave profile, facilitating smoother convergence in gradient-based optimization.



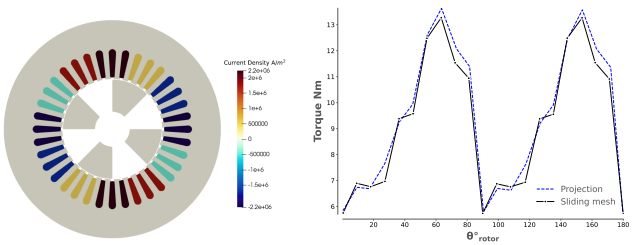
[i]

(a) Reluctivity, $\nu(\rho)$ (b) Torque

Fig. 1: Effect of RAMP interpolation on intermediate design variables. The entire rotor domain is assigned an intermediate value ρ.

III. Proposed Design Projection and Filtering

We propose a simplified projection-based approach to handle the relative motion between the stator and rotor by modifying



(a) SynRM setup with current distribution for validation

(b) Torque evaluation with respect to various rotor orientations

Fig. 2: Comparison between proposed projection-based method and the sliding mesh approach available in COMSOL

the density filtering step in topology optimization. This allows the rotor design to be projected onto a static background mesh at various angles, reducing computational complexity. In topology optimization, filtering prevents mesh-dependency and checkerboard patterns, typically achieved through an element-based spatial filter mapping design variables ρ to filtered physical densities as $\underline{\tilde{\rho}} = \underline{\underline{\mathcal{P}}} \cdot \underline{\rho}$, where $\underline{\underline{\mathcal{P}}}$ is the filter projection matrix computed as follows:

$$\hat{\mathcal{P}}_{i,j} = \left[\max\{1 - \frac{d_{i,j}}{r_0}, 0\}\right]^m \tag{3a}$$

$$\mathcal{P}_{i,j} = \frac{\hat{\mathcal{P}}_{i,j}}{\sum_{j=1}^{n} \hat{\mathcal{P}}_{i,j}} \tag{3b}$$

In (3) r_0 is the characteristic length scale for the zone of influence, d_{ij} the Euclidean distance between element centroid i and j, and $m = 3$ controls the filtering strength. Here, we modify the filtering process to compute d_{ij} between the background mesh centroids and the rotated centroids at rotor angle $\theta = \theta_k$ as described below:

$$\mathcal{R}_{\theta_k} = \begin{bmatrix} \cos\theta_k & -\sin\theta_k \\ \sin\theta_k & \cos\theta_k \end{bmatrix} \tag{4a}$$

$$d_{i,j} = \left\| {c_i}^0 - \mathcal{R}_{\theta_k} c_i^0 \right\|_2 \tag{4b}$$

where ${c_i}^0$ is the centroid of the static background mesh. Consequently, the filtering and the rotor rotation are seamlessly managed together.

The proposed method is validated by evaluating the torque in a SynRM motor with a representative rotor design at various rotor orientations and comparing it with the commercial software COMSOL as shown in Figure 2. The motor is a three-phase, two-pole pair type with axial length of 135 mm, air gap length 2.5 mm, and 18 turns per winding. The peak rated current is $28.284A$ with a phase angle of $108°$. As seen in Figure 2b, our method shows excellent agreement with the commercial software. The slight difference between the two methods can be further minimized by using a finer mesh for projection.

IV. Result

We apply the technique developed to optimize material distribution for the SynRM machine shown in Fig. 2a. Optimization includes two constraints: a maximum volume fraction of 65% and a maximum torque ripple factor of 1.001. The

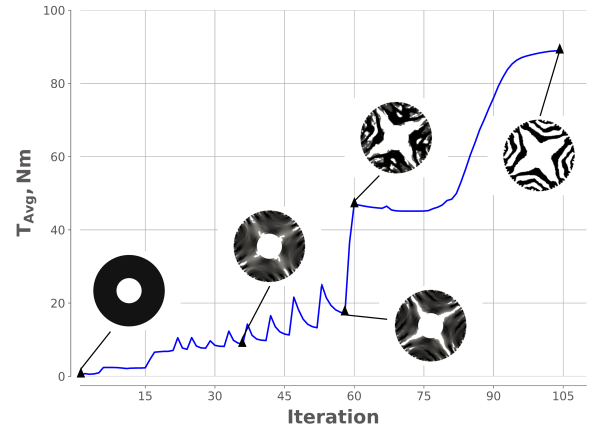


Fig. 3: Rotor design evolution along with maximization of average torque during optimization convergence

torque is evaluated at 12 rotor angles between $0°$ and $180°$ at $15°$ intervals, with a RAMP interpolation factor $q = -0.7$, and rotor design variables initialized uniformly at 0.9. Figure 3 shows rotor design evolution through iterations until convergence. The optimal design features a distinct periodic pattern with air barriers to guide the magnetic fields, achieving an average torque of $88.91Nm$ and a ripple factor of 1.0009. At convergence, the rotor has 47% mass fraction, achieving a lightweight design with high torque and low ripple.

V. Concluding Remarks

We present a projection-based technique for rotor design in topology optimization, which simplifies rotor rotation handling through density filtering on a static background mesh, eliminating the need for remeshing or sliding mesh methods. We apply the method to optimize a SynRM rotor to achieve a lightweight design with a maximum average torque with minimal torque ripple. The RAMP interpolation with suitable profiles improves design space exploration. Future work will extend this approach to include multiphysics constraints and material non-linearity.

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