

Pareto Meta-Front for the Optimal Design of Reconfigurable Devices: an Application in Electromechanics

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Reconfigurable devices are of paramount importance in many applications and a method inspired to Pareto-like optimality can generate a sequence of configurations, each of which is the best trade-off between design criteria. Accordingly, an original concept of Pareto meta-front is presented: given a family of fronts depending on a variable parameter, the meta-front is defined as the curve which joins all the closest-to-utopia points. It describes the variation of the best combinations of two conflicting design criteria against a cost parameter like *e.g.* a material property. The case study considered in order to prove the validity of the proposed method is the optimal shape design of an interior permanent-magnet motor (IPM) which is assumed to be magnetically reconfigurable, meaning that the variable parameter indexes a set of several permanent-magnet materials, each of which is featured by its own demagnetization curve.

Index Terms—Finite element method, permanent-magnet motors, reconfigurable devices, multiobjective Pareto optimization.

I. INTRODUCTION

Reconfigurable devices are of paramount importance in many applications and their optimal design is an open issue. Usually, in fact, the result of a multiobjective optimization process is a front of indifferent solution: the device corresponding to a specific optimal solution is supposed to be non reconfigurable. This means that if shape or supply or material change, the updated configuration in general is no longer optimal. In turn, the challenge is to develop a method inspired to Pareto-like optimality able to generate a sequence of configurations, each of which is the best trade-off between design criteria against a variable parameter indexing variations of shape or supply or material. In this respect, three broad classes of reconfigurable devices could be categorized: -mechanically reconfigurable, when the device shape changes during its operation (controllable shape morphing, *e.g.* joint actuators in robotic systems); -electrically reconfigurable (*e.g.* metamaterials for millimetric waves, capable of modifying their radiation properties in a controlled and reversible manner, [1]); -magnetically reconfigurable (*e.g.* material properties of permanent magnets machines). The paper focuses on the last class and presents an original method for helping the design optimization of reconfigurable motors.

II. PARETO META-FRONT THEORY

Let a bi-objective minimization problem, characterized by convex Pareto fronts, be considered. The following can be stated:

given a parameter space $\Lambda \subset \mathbb{R}$
 assigned design criteria

$$(f_1(x, \lambda), f_2(x, \lambda)), x \in \Omega, \lambda \in \Lambda \quad (1)$$

with $\Omega \subseteq \mathbb{R}^{n_v}$ feasible design region and λ a scalar parameter, a family of fronts is originated by varying parameter λ and solving the relevant minimization problem (1). Accordingly, each front identifies a unique utopia point U in the

objective space. A meta-front can then be defined as follows: find the solution point

$$\tilde{f}_\lambda = (\tilde{f}_1, \tilde{f}_2), \lambda \in \Omega \quad (2)$$

along the Pareto front relevant to parameter λ such that the distance

$$d(\tilde{f}_\lambda, U_\lambda), \lambda \in \Omega \quad (3)$$

is minimum. By varying parameter λ , a set of points (2) is originated: the resultant locus in the objective space is a curve joining all the closest-to-utopia points, called Pareto meta-front, which in general is not a front. In the case of reconfigurable devices, the meta-front provides the best compromises trading off the (f_1, f_2) pair of criteria against variable parameter λ .

The following remarks can be put forward:

- i) problem constraints are assumed to be fulfilled;
- ii) prescribed a value of λ , solution point (2) exists because it belongs to a front of feasible solutions;
- iii) given a distance metric, such a point is unique because utopia is unique by definition;
- iv) it follows that the meta-front exists and is unique;
- v) should the fronts be non-convex, existence would still hold, but nothing could be *a priori* said as per uniqueness;
- vi) distance (3) is here computed according to the Euclidean norm, but it can be alternatively computed using *e.g.* the infinite norm.

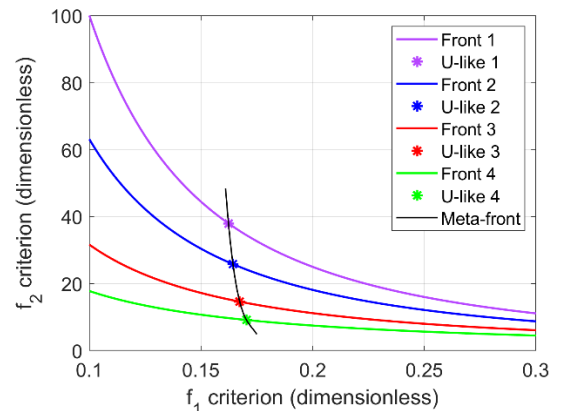


Fig. 1. Family of four Pareto fronts and the relevant meta-front.

A conceptual scenario is represented in Fig.1 where a family of four fronts is shown along with the associated meta-front. The meta-front describes the variation of the best combinations of two conflicting design criteria against a cost parameter like e.g. a material property.

The case study here considered to prove the validity of the proposed method is the optimal shape design of a magnetically reconfigurable IPM, where the variable parameter indexes a set of several permanent-magnet materials, each of which featured by its own demagnetization curve; in particular, it could be set:

$$\lambda = |BH|_{max} \quad (4)$$

referring to the highest specific energy.

III. PERMANENT MAGNET MOTOR OPTIMAL DESIGN

The IPM model [2] is shown in Fig.2 along with four design variables defining rotor geometry $g=(d, l, b, h, \alpha)$. The variation ranges for the design variables are shown in Table I.

TABLE I
VARIATION RANGE FOR THE DESIGN VARIABLES

	d (mm)	l (mm)	b (mm)	h (mm)	α (deg)
Min	1	10	0.5	8	1
Max	3	22	2	12	5

The objective functions are:

$f_1(g)$, the average running torque (T_r), to be maximised;

$f_2(g)$, the highest cogging torque (T_c), to be minimised.

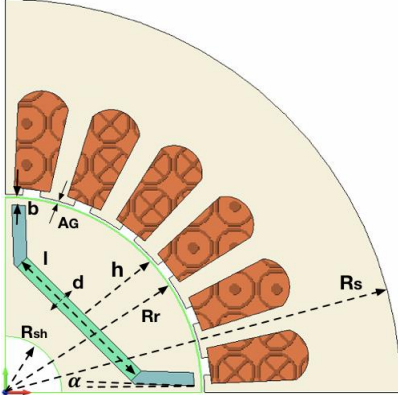


Fig. 2. IPM model: 5D design vector $g = (l, d, h, b, \alpha)$.

RESULTS

In Fig. 3 a family of six Pareto optimal fronts, relevant to several permanent-magnet materials, is shown: each front has been identified by means of the NSGA-II algorithm (20 individuals, 100 generations). The closest-to-utopia points are the knee points of the fronts and they belong to the Pareto meta-front: details are reported in Table II, where the relevant values of X-space coordinates (design vector) and F-space coordinates (objective vector) are accordingly shown. In turn, each line in Table II corresponds to a unique value of the λ parameter; specifically, the λ vector reads as follows: NdFeB, $\lambda = 236.7 \cdot 10^3 \text{ Jm}^{-3}$, PM10, $\lambda = 198.9 \cdot 10^3 \text{ Jm}^{-3}$, SmCo, $\lambda = 173.4 \cdot 10^3 \text{ Jm}^{-3}$, PM08, $\lambda = 127.3 \cdot 10^3 \text{ Jm}^{-3}$, AlNiCo5, $\lambda = 43.9 \cdot 10^3 \text{ Jm}^{-3}$, Ferrite

$\lambda = 30.7 \cdot 10^3 \text{ Jm}^{-3}$. It can be noted that λ values and related knee-point positions scale monotonically.

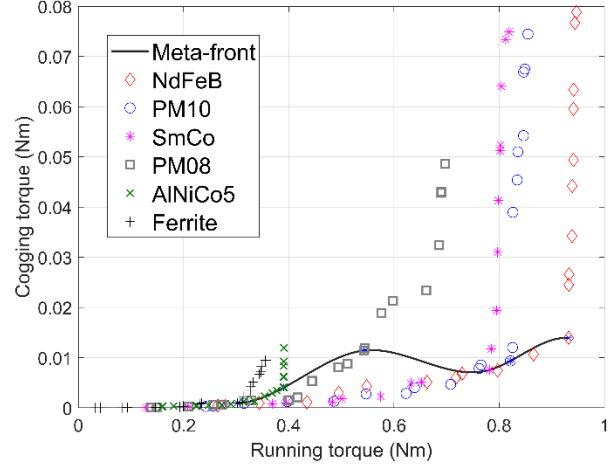


Fig. 3. Family of six fronts in the objective space with evidence of Pareto meta-front.

TABLE II
PARETO META FRONT SOLUTIONS

Material	d (mm)	l (mm)	b (mm)	h (mm)	α (deg)	f_1 T_r (Nm)	f_2 T_c (mNm)
NdFeB	3	22	0.5	8.2	1.68	-0.93	13.9
PM10	2.8	22	0.5	8	1.95	-0.82	9.3
SmCo	2.9	21.9	0.5	8	1.82	-0.78	7.5
PM08	2.4	21.4	1.1	10.6	3.42	-0.54	11.4
AlNiCo5	3	22	0.7	8.4	1.65	-0.39	4
Ferrite	2.4	22	0.5	8.3	3.25	-0.33	3.5

After Fig.3 and Table II, at a first glance it appears that for the considered class of problems the meta-front identifies the same optimal geometry across several PM materials. In other words, the knee points of the fronts in the objective space might correspond to a unique geometry in the design space.

In the full-length paper a thorough discussion of results in both objective and design space will be presented.

IV. ACKNOWLEDGMENTS

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