

Meshing Strategies for Shape Optimization in Electromechanics based on the Virtual Element Method

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The Virtual Element Method (VEM) is a recent and successful method for the numerical solution of partial differential equations. The methodological background of the VEM is presented and its application in magnetic field analysis is shown; accordingly, the numerical solution of non-linear magnetostatic problems is worked out in two-dimensional domains. In this paper, we are interested in the analysis and synthesis of an internal permanent magnet motor. In this framework, the VEM is particularly suitable thanks to its flexibility in handling generally shaped polygons, specifically the possibility of treating in a smart way both hanging nodes and mesh gluing.

Index Terms—Finite element method, magnetostatics, polygonal meshes, virtual element method.

I. INTRODUCTION

THE Virtual Element Method (VEM) is a recent and successful method for the numerical solution of partial differential equations [1]. The VEM can be considered as an extension of the Finite Element Method (FEM) that can make use of very general decompositions of the computational domain into polygonal or polyhedral elements. The Virtual Element Method allows for a generic degree of accuracy, that corresponds to the polynomial degree in standard FEM, furthermore it exhibits an excellent robustness with respect to mesh distortions. The VEM can be particularly useful for grid adaptation, moving meshes and discretizing complex geometries. Moreover, the VEM and the FEM are perfectly compatible. Hence it is possible (and advisable) to use Virtual Elements *only* in those parts of the domain where they are needed.

In the framework of magnetostatic problems, the VEM was developed both for both the potential and the Kikuchi [2] formulations. The subject of this article is to further develop the method for the magnetic analysis of a class of permanent magnet motors for electric tractions considering the non-linear B-H curve of ferromagnetic materials and the nodal VEM space for the potential formulation described in [2]. To obtain such results, an original technique for handling a sequence of rotor positions is presented by generating two different meshes (one for the rotor and one for the air-gap/stator regions) and gluing them according to the angle.

II. POTENTIAL FORMULATION

We consider the classical potential formulation in a two dimensional domain Ω , which consists of searching for the scalar A , i.e., the only non zero component of the three dimensional vector potential associated with the magnetic induction field $\mathbf{B} = \text{curl } A$, such that

$$\text{curl}(\mu^{-1} \text{curl } A) = j + j_{\text{pm}} \quad \text{in } \Omega, \quad (1)$$

plus boundary conditions on $\partial\Omega$.

The two-dimensional curl and the scalar curl are defined as

$$\text{curl } A := (\partial_2 A, -\partial_1 A) \quad \text{and} \quad \text{curl } \mathbf{A} := \partial_1 A_2 - \partial_2 A_1, \quad (2)$$

respectively. In Equation (1), j is the assigned current density and j_{pm} is a fictitious current density defining permanent magnetization (current sheet model), while μ represents the material permeability. The quantity μ implicitly depends on the unknown potential providing the source of the nonlinearity of the problem at hand.

It turns out to be

$$\begin{aligned} \text{curl}(\mu^{-1} \text{curl } A) &= \partial_1(-\mu^{-1} \partial_1 A) - \partial_2(\mu^{-1} \partial_2 A) \\ &= -\text{div}(\mu^{-1} \nabla A), \end{aligned} \quad (3)$$

so Equation (1) becomes

$$-\text{div}(\mu^{-1} \nabla A) = j + j_{\text{pm}} \quad \text{in } \Omega, \quad (4)$$

subject to appropriate boundary conditions.

III. INTERIOR PERMANENT-MAGNET MOTOR

An Interior Permanent-Magnet motor characterized by four poles and 12 stator slots is considered as a case study, see Fig. 1. The external and the rotor diameters are 68 mm and 30 mm, respectively, while the air-gap width is 0.5 mm. The permanent magnet exhibits a radial magnetization with remanent field equal to $\mathbf{B}_0 = 1$ T and coercive field equal to $\mathbf{H}_c = 7.957 \cdot 10^5$ Am⁻¹. Furthermore, we assume that the electrical steel has no remanence due to the inherent hysteresis properties.

In this example, according to the experiments already shown in [2], we use the projection operators to compute the cogging torque, i.e., the torque acting on the rotor when the three-phase current in the stator slots is zero (no-load operation).

Such quantity is crucial to design a permanent magnet motor since it takes into account the tendency of the permanent magnet axis to align with the direction that corresponds to

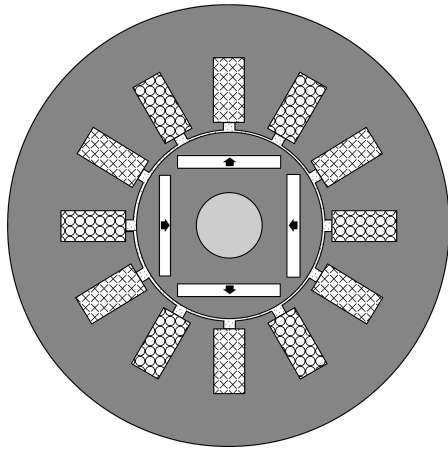


Fig. 1. The geometry of the 4-pole motor taken into account.

the minimum energy stored in the motor. Although its value is substantially lower than the running torque due to the on-load current, cogging torque could be responsible of annoying vibrations occurring during the normal on-load operation of a small motor like the one here considered.

The cogging torque has to be computed considering different angular positions of the rotor. To generate *all* these meshes, we employ the flexibility of VEM in gluing meshes. Indeed, we propose the following strategy: we generate stator and rotor mesh, separately, and then glue them together with the desired angle, see Fig. 2.

We exploit the standard current sheet model for simulating the permanent magnet effect and we use a fixed point iterative procedure to handle the non-linear permeability. In Figure 3, we show the magnitude of the magnetic induction at an angle of 21° .

To compute the torque, we use the Maxwell stress tensor approach considering a cylindrical surface co-axially located with respect to the rotation axis as the integration surface accordingly. The resulting torque-angle curve exhibits a period of 30° as expected, it has zero mean value and, finally, its maximum value is approximately 0.15Nm at 5° , while the minimum is -0.15Nm at 25° .

IV. CONCLUSION

This is a preliminary study on the possibility of using the VEM to compute the torque curve that exploits the flexibility of the VEM in meshing subdomains and gluing them together. A further development of this technique is to consider different types of rotor meshes that may incorporate flux barriers. Such a study will be straightforward since the part of the domain that is changing is *only* the rotor region. Moreover, this meshing procedure allows us also to set and solve an optimisation problem on the shapes of both permanent magnet and flux barriers, acting just in a small and limited region of the computational domain. All these aspects will be covered in the full length paper.

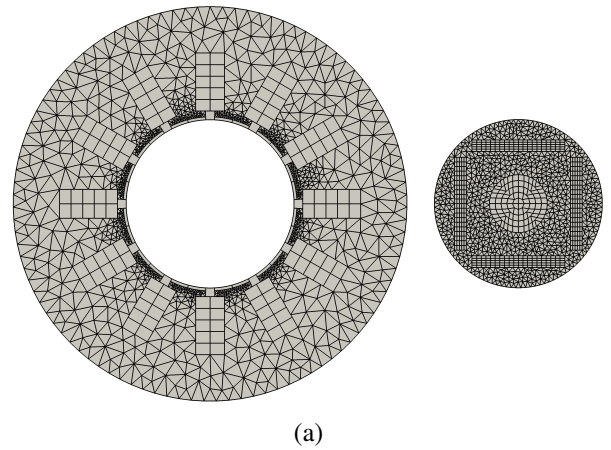


Fig. 2. (a) stator and rotor mesh, (b) mesh glued with a detail of the hanging-nodes generated by the gluing procedure.

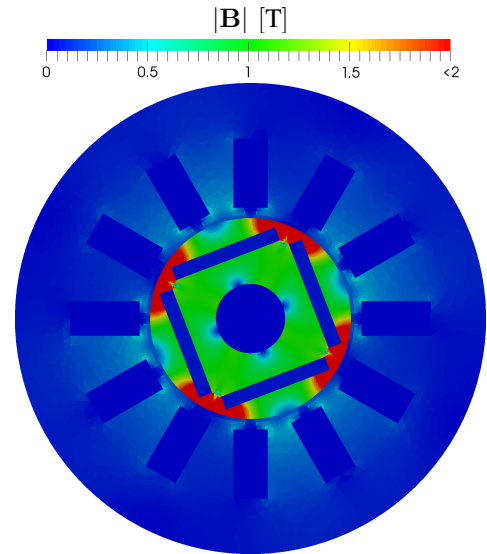


Fig. 3. Magnetic induction map at an angle of 21° .

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