

A Multi-objective Topology Optimization Methodology and its Application to an Electromagnetic Actuator Prototype

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In this paper, a novel hybrid multi-objective optimization (MOO) algorithm is proposed by combining a beluga whale optimization (BWO) with an improved NSGA-II. The original BWO is transformed to a MOO version and NSGA-II is enhanced by using opposite-learning initialization and revised crowding-distance strategy. Besides, material field series expansion method is adopted to form a multi-objective topology optimization methodology. To validate the proposed methodology, topology of an electromagnetic actuator prototype is optimized.

Index Terms—Topology optimization, multi-objective optimization, evolutionary algorithm, material field series expansion.

I. A NOVEL HYBRID MULTI-OBJECTIVE OPTIMIZATION ALGORITHM

A. Beluga whale optimization

Due to its advantages such as, good search efficiency, simple and easy to be implemented, the original BWO [1] and its variants have been widely applied to various theoretical and practical optimization problems. Inspired from the behaviors of beluga whales, including swimming, predation and whale fall in the sea, exploration, exploitation and whale fall phase are formulated using mathematical models.

The three different phases, namely, exploration, exploitation and whale fall are decided according to two parameters called balance factor and whale fall probability, denoted by $B_f = B_0(1 - t/2t_{max})$ and $W_f = 0.1 - 0.05t/t_{max}$, respectively. t is the current iteration number and t_{max} is the maximum iteration number, B_0 randomly changes between (0, 1) at each iteration. When $B_f > 0.5$, exploration phase happens; when $W_f \leq B_f \leq 0.5$, exploitation happens; while if $B_f < W_f$, whale fall phase executes.

For exploration phase, the positions of search agents are determined by the pair swim of beluga whales, and the positions for beluga whales are updated as follows

$$X_{i,j}^{t+1} = \begin{cases} X_{i,p_j}^t + (X_{r,p_1}^t - X_{i,p_j}^t)(1 + r_1) \sin(2\pi r_2), & j = \text{even} \\ X_{i,p_j}^t + (X_{r,p_1}^t - X_{i,p_j}^t)(1 + r_1) \cos(2\pi r_2), & j = \text{odd} \end{cases} \quad (1)$$

where $X_{i,j}^{t+1}$ is the new position for the i th beluga whale on the j th dimension, p_j is a random number selected from d -dimension, X_{i,p_j}^t denotes the position of the i th beluga whale on p_j dimension, X_{r,p_1}^t is the current positions for r th beluga whale (r is a randomly selected beluga whale). r_1 and r_2 are two random numbers between (0, 1), which are used to enhance the random operators in exploration phase.

For exploitation, the strategy of Levy flight [2] is adopted to enhance the convergence. The position of i th beluga whale is updated by

$$X_i^{t+1} = r_3 X_{best}^t - r_4 X_i^t + C_1 \cdot L_F \cdot (X_i^t - X_i^t) \quad (2)$$

where X_i^t and X_r^t are current position for the i th beluga whale and a random beluga whale, X_i^{t+1} is the new position of the i th beluga whale, X_{best}^t is the best position among beluga whales, r_3 and r_4 are random numbers between (0, 1) and $C_1 = 2r_4(1 - T/T_{max})$ is the random jump strength that measuring the intensity of Levy flight.

L_F is the Levy flight function, which is calculated by

$$L_F = 0.05 \times \frac{u \times \sigma}{|v|^{1/\beta}} \quad (3)$$

$$\sigma = \left(\frac{\Gamma(1+\beta) \times \sin(\pi\beta/2)}{\Gamma((1+\beta)/2) \times \beta \times 2^{(\beta-1)/2}} \right)^{1/\beta} \quad (4)$$

where Γ represents for gamma function, u and v are normally distributed random numbers, β is constant equals to 1.5.

For whale fall phase, the agent position is updated by

$$X_i^{t+1} = r_5 X_i^t - r_6 X_r^t + r_7 X_{step} \quad (5)$$

where r_5 , r_6 and r_7 are random numbers between (0, 1), X_{step} is the step size of whale fall that follows

$$X_{step} = (\mathbf{u}_b - \mathbf{l}_b) e^{(-C_2 t/t_{max})} \quad (6)$$

where $C_2 = 2W_f n$ (n is the population size), $\mathbf{u}_b$ and $\mathbf{l}_b$ is the upper boundary vector and lower boundary vector of variables, respectively.

B. Flowchart of the hybrid algorithm

Flowchart of the proposed hybrid MOO algorithm is shown in Fig. 1. The original NSGA-II is improved by adopting opposition-based learning scheme and exclusive crowding-distance strategy [3]. After the initialization, NSGA-II and multi-objective BWO (MOBWO) are carried out in parallel; for NSGA-II, simulated binary crossover and polynomial mutation are used to generate offspring population; for MOBWO, exploration, exploitation and whale fall phase are executed according to balance factor B_f and whale fall probability W_f . Similarly, new positions of search agents will be generated. A greedy strategy is used to select the best population size individuals among the current population, namely, offspring generated from NSGA-II and new positions generated from MOBWO as population for next iteration.

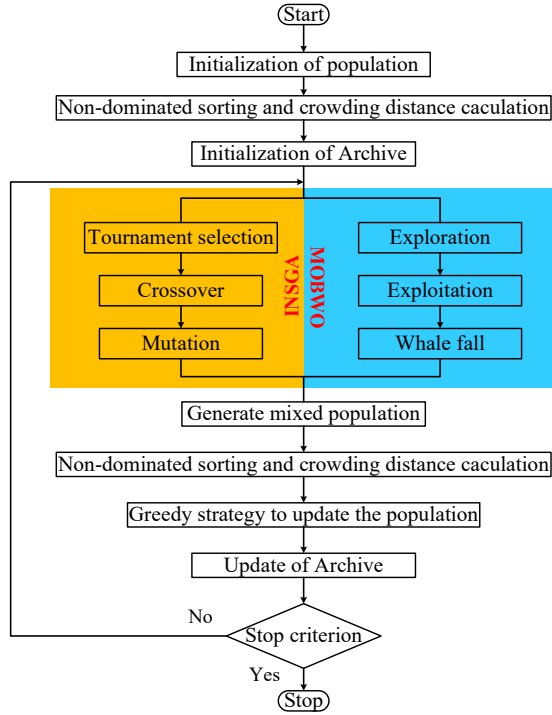


Fig. 1. Flowchart of the proposed hybrid algorithm.

C. Performance test

Standard test functions [4] are solved to verify the performance of the hybrid algorithm. 30 independent runs for each algorithm are performed and the performance is compared using convergence metric γ and diversity metric Δ . Population size for each algorithm is set to be 100 and maximum iteration number for NSGA-II and MOBWO is 250, and for hybrid algorithm is 125.

TABLE I
STATISTICAL RESULTS OF CONVERGENCE METRIC

$f(\cdot)$	Hybrid		NSGA-II		MOBWO	
	E(γ)	$\sigma(\gamma)$	E(γ)	$\sigma(\gamma)$	E(γ)	$\sigma(\gamma)$
KUR	1.33e-2	8.35e-4	1.43e-2	1.15e-2	1.84e-2	1.72e-3
ZDT1	1.21e-3	1.19e-4	3.37e-1	9.40e-2	2.26e-3	1.39e-3
ZDT2	7.75e-4	5.05e-4	6.13e-1	8.79e-2	1.03e-3	2.08e-3
ZDT3	4.80e-3	6.52e-4	5.57e-2	1.30e-2	3.72e-3	5.53e-4
ZDT4	2.61e-3	1.58e-3	2.18e0	3.76e-1	4.83e-3	4.88e-3
ZDT6	2.57e-2	3.85e-2	9.33e-1	9.13e-1	2.71e-2	2.98e-2

TABLE II
STATISTICAL RESULTS OF DIVERSITY METRIC

$f(\cdot)$	Hybrid		NSGA-II		MOBWO	
	E(Δ)	$\sigma(\Delta)$	E(Δ)	$\sigma(\Delta)$	E(Δ)	$\sigma(\Delta)$
KUR	3.55e-1	1.86e-2	5.00e-1	3.26e-2	4.11e-1	2.39e-2
ZDT1	2.81e-1	2.49e-2	6.50e-1	5.57e-2	5.48e-1	1.43e-1
ZDT2	5.25e-1	3.28e-1	8.50e-1	1.63e-1	9.49e-1	1.43e-1
ZDT3	6.26e-1	1.42e-2	7.65e-1	4.07e-2	1.11e0	1.50e-1
ZDT4	4.34e-1	1.71e-1	7.91e-1	4.16e-2	5.90e-1	2.12e-1
ZDT6	5.83e-1	4.50e-1	1.33e0	1.08e-1	9.99e-1	1.27e-1

II. TOPOLOGY OPTIMIZATION OF AN ELECTROMAGNETIC ACTUATOR PROTOTYPE

To lower computational cost of topology optimization, a material-field series expansion (MFSE) method [5] is introduced to reduce the number of design variables. A material-field function $\varphi(\mathbf{x}) \in [-1, 1]$ is defined in the entire design domain to represent structural topology. The distribution of material with position is determined by the value of material-field function

$$\mathbf{x} \in \begin{cases} \Omega_{void}, & \text{if } -1 \leq \varphi(\mathbf{x}) < 0 \\ \Omega_{solid}, & \text{if } 0 < \varphi(\mathbf{x}) \leq 1 \end{cases} \quad (7)$$

Given N_p observation points distributed equally in the design domain, the material field is expressed as

$$\varphi(\mathbf{x}) = \sum_{k=1}^{N_p} \left(\eta_k \frac{1}{\sqrt{\lambda_k}} \boldsymbol{\psi}_k^T \mathbf{C}_d(\mathbf{x}) \right) \quad (8)$$

where vector $\mathbf{C}_d(\mathbf{x}) = \{C(\mathbf{x}, \mathbf{x}_1) C(\mathbf{x}, \mathbf{x}_2) \dots C(\mathbf{x}, \mathbf{x}_{N_{pt}})\}^T$, $\mathbf{C}$ is the correlation matrix of material-field function related to distance having exponential form; λ_k and $\boldsymbol{\psi}_k$ are the k th eigenvalue and normalized eigenvector of matrix $\mathbf{C}$, respectively; η_k is the coefficient to be determined.

For computational efficiency, (8) is truncated by accounting for only the M largest eigenvalues and corresponding eigenvectors. The reduced series expansion of the material field is rewritten as

$$\varphi(\mathbf{x}) \approx \sum_{k=1}^M \left(\eta_k \frac{1}{\sqrt{\lambda_k}} \boldsymbol{\psi}_k^T \mathbf{C}_d(\mathbf{x}) \right) = \boldsymbol{\eta}^T \boldsymbol{\Lambda}^{-1/2} \boldsymbol{\Psi}^T \mathbf{C}_d(\mathbf{x}) \quad (9)$$

where $\boldsymbol{\eta} = \{\eta_1 \eta_2 \dots \eta_M\}^T$ is the vector of undetermined coefficients, $\boldsymbol{\Lambda} = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_M)$ is a diagonal matrix and $\boldsymbol{\Psi} = \{\boldsymbol{\psi}_1 \boldsymbol{\psi}_2 \dots \boldsymbol{\psi}_M\}$ is matrix with dimension $N_p \times M$.

Topology of a magnetic actuator prototype [6] is optimized to maximize the electromagnetic force and minimize the material consumption simultaneously. By adopting MFSE method, the original number of design variables is reduced from 1080 to 172, and a typical optimized topology is shown in Fig. 2, of which the electromagnetic force is increased by 28.74% and the material cost is reduced by 30.35% compared with the original design. Details of the proposed multi-objective topology optimization methodology will be included in full paper.

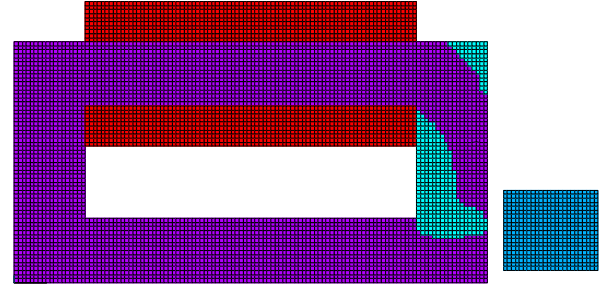


Fig. 2. Typical optimized topology.

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