

Lightning Preliminary Breakdown Pulses Model Optimization

Daniele Mestriner¹, *Member, IEEE*, Martino Nicora¹, *Member, IEEE*, Renato Procopio¹, *Senior Member, IEEE*, Massimo Brignone¹, *Member, IEEE*, Alessandro Formisano², *Senior Member, IEEE*, Ehsan Akbari Sekehravani², Sami Barmada³, *Senior Member, IEEE*, and Shayan Dodge³

¹DITEN, University of Genoa, Italy

²Engineering Department, Università degli Studi della Campania, Italy

³DESTEC, University of Pisa, Italy

Preliminary Breakdown Pulses (PBP) are a series of electromagnetic signals observed tens of milliseconds prior to the first Return Stroke (RS) in a lightning flash. Due to the complexity of the phenomenon and since the risk associated with PBPs is typically lower than that produced by RSs, few studies in the literature exist about PBP modeling. However, observation of PBPs and accurate models may be useful to develop RS prediction tools, allowing to include preventive actions in the actual lightning protection strategies. This contribution presents a preliminary study for the optimization of the parameters of a PBP discharge current numerical model for Cloud-to-Ground (CG) flashes simulations. This is to be intended as the first step to obtain statistical distributions of the model parameters from a large dataset of experimental measurements.

Index Terms—Lightning, Preliminary Breakdown Pulses, Optimization

I. INTRODUCTION

PBPs represent one of the most complicated processes in the lightning discharge mechanism. As noted in [1], PBPs in negative CG lightning result from discharges between differently charged regions of the thundercloud, predominantly propagating in the vertical direction.

An accurate PBP model should be: i) able to obtain reliable EM fields prediction and ii) integrated with the one describing the RS, looking for correlation between the PBP electromagnetic characteristics and the main parameters of the relevant RS. Among the existing PBP models available in the literature, that proposed in [2] (labelled from now on as “MTLK model”) has been selected for the purposes of this work. The main advantage of the MTLK model [2] is the analogy with typical transmission line engineering models for the RS current (e.g., the Modified Transmission Line model with Exponential decay, MTLE [3]).

The goal of our research is to solve an inverse problem, i.e. determine the optimal parameters of the MTLK model current $i(z, t)$ [2] by fitting experimental data (i.e., minimizing the error between the computed E-fields and the measured PBP signals), in order to develop a reliable and relatively fast simulation tool. In particular, the statistical distributions of model parameters will be the final aim of this research activity. This in principle would allow to simulate random PBP events that may be useful for research and risk assessment purposes.

The optimization procedure is performed with the Multi-variate Ant Colony Algorithm for Continuous Optimization (MACACO) toolbox [4] which allows, for each event, to find the set of parameters best fitting the data (the output of the model is compared with field measurements). In this contribution, a preliminary study on the performance of the optimization procedure is assessed.

II. METHODOLOGY

The MTLK model for PBPs considers the PBP discharge channel as a vertical thin-wire radiator (transmission line) extending between the altitudes H_1 and H_2 and assumes the traveling current as a continuous function of time t and height z . Such a current is expressed as

$$i(z, t) = f(z)I \left(t - \frac{z - H_1}{v} \right) \quad t \geq 0 \quad H_1 < z < H_2 \quad (1)$$

where I is the current injected at the bottom of the discharge channel located at H_1 and traveling upward with a constant speed v (lower than the speed of light in vacuum c), and f models the variations of the current depending on z . The MTLK model assumes the current pulse starting at zero when $z = H_1$, increasing until a certain height H_m is reached, and decaying back to zero when $z = H_2$. Thus, assuming the Kumaraswamy distribution [5] for the shape of the current traveling along the transmission line, the authors of [2] proposed

$$f(z) = \left(\frac{z - H_1}{H_2 - H_1} \right)^{a-1} \left[1 - \left(\frac{z - H_1}{H_2 - H_1} \right)^a \right]^{b-1} \quad (2)$$

with $H_1 < z < H_2$, and a and b being shape parameters. As far as the waveform of the current source I at the bottom of the transmission line (and present in eq. (1)), in accordance with [6], an asymmetric Gaussian function is assumed in [2]. This function is expressed as

$$I(t) = \begin{cases} Ae^{-[\alpha(t-t_1)]^2} & 0 \leq t \leq t_1 \\ Ae^{-[\alpha \frac{(t-t_1)t_1}{(t_2-t_1)}]^2} & t > t_1 \end{cases} \quad (3)$$

where t_2 is the ending time of the considered time window, t_1 is the rise time of the current pulse, and A is the amplitude parameter. The parameter α controls the asymmetric shape of the current pulse; for computation purposes $\alpha_{ind} = \alpha t_2$ is

used in place of α . In participle, by applying the Position By Fast Antenna (PBFA) technique [7], integrating data from more sensors, it is possible to obtain the (x, y, z) location of the maximum peak PBP current. In such a case, for the MTLK model, H_m would be assumed equal to the PBFA z -coordinate prediction. However, for the sake of generality, here the optimization procedure is implemented considering H_m as a free parameter, together with H_1 , H_2 , a , b , t_1 , t_2 , α_{ind} , A , and v . The cost function to minimize in the optimization problem is

$$\rho_{norm} = \frac{1}{N} \sum_{i=1}^N \sum_{n=1}^{L_i} \sqrt{\frac{1}{L_n} \left[\frac{E_{mi}(t_n) - E_{ci}(t_n)}{\Delta E_{mi,pp}} \right]^2} \quad (4)$$

where N is the total number of E-field sensors and L_i is the number of samples in waveforms recorded by the i -sensor. $E_{mi}(t_n)$ is the electric field measured by the i -sensor at $t = t_n$, and $E_{ci}(t_n)$ is the corresponding value given by computations. The normalizing factor $\Delta E_{mi,pp}$ is the peak-to-peak electric field measured at the i -sensor. The vertical electric field produced by a current pulse traveling along a vertical radiator observed at (x_0, y_0, z_0) , is given by the Maxwell's equations formulation provided in [8]. Then, the maximum condition for f are set: introducing the linear transformation $x = \frac{z-H_1}{H_2-H_1}$ ($0 < x < 1$), imposing $f'(x_m) = 0$ and considering $0 < x_m = \frac{H_m-H_1}{H_2-H_1} < 1$, it is possible to obtain a relationship between the shape parameters a and b , i.e.

$$b = \frac{x_m^a + a - 1}{ax_m^a} \quad (5)$$

So, in this way we get one less unknown. Finally, the optimization procedure is implemented: i) the waveform samples $E_{mi}(t_n)$ of two events (PBP-4 and PBP-2) are extrapolated from Figures 4 and 6 of [2], and ii) the MACACO tool [4] is run to find the best set of parameters which fit the data (i.e., the vertical E-field measured at four different locations, i.e., distances D from the source).

III. RESULTS AND DISCUSSION

Results are reported in Figure 1 for the PBP-4 at one of the measuring location (results on the other locations as well as those of PBP-2 are similar, but are omitted for the sake of brevity). The corresponding model parameters are reported in Table I, where also the parameters obtained by the authors of [2] are proposed. It can be observed that the optimum points are completely different between the two methods, suggesting that the same field shape can be reproduced with different configurations of the model parameters. Finally, to keep a reduced CPU time, a large time step is assumed ($1\mu s$). In these conditions, the cost function obtained with MACACO is higher than that of [2]. This choice is driven by the necessity of applying this procedure to a large number of events in order to obtain reliable statistical distributions of the parameters of the MTLK model current. This work is part of a project aiming at designing a system able to detect PBPs, perform a prediction of the risk associated with the incoming RS and eventually disconnect the electric infrastructure under its protection.

TABLE I
OUTPUT (INPUT) PARAMETERS FOR PBP-4 AND PBP-2 MTLK MODELS
AS REPORTED IN [2] AND AS CALCULATED WITH MACACO.

Parameter	PBP-4 Ref. [2]	PBP-4 MACACO	PBP-2 Ref. [2]	PBP-2 MACACO
x_0 (m)	(-24653)	(-24653)	(-952)	(-952)
y_0 (m)	(-20106)	(-20106)	(-1902)	(-1902)
z_0 (m)	(0)	(0)	(0)	(0)
H_m (m)	(5530)	6120	(5824)	6221
H_1 (m)	5244	5974	5706	6017
H_2 (m)	6260	7266	6625	7360
t_1 (μs)	19.9	16.3	25	20
t_2 (μs)	46.7	52.04	46.7	49.2
v ($\times 10^8$ m/s)	1.1	1.4	1.2	1.3
α_{ind}	9.2	13.67	8.7	13.22
a	1.5	2.69	1.2	2.93
b	2.9	3.38	2.9	3.35
A (kA)	119	150	101	163
ρ_{norm}	4.70×10^{-2}	0.14	4.60×10^{-2}	0.43

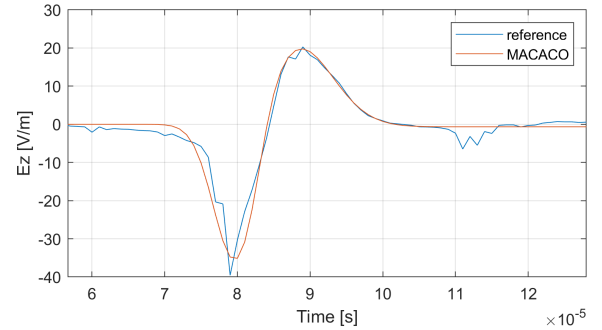


Fig. 1. Results with MACACO (time step $1\mu s$). Waveform comparison of MTLK model (red) to measured data (blue) for the PBP-4 at $D = 17$ km.

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