

Finite Element Motor Analysis for Sensorless Neutral-Point Rotor Angle Estimation in the Frequency Domain

R. Meisinger¹, W. Gruber¹, *Senior Member, IEEE* and S. Silber²

¹ Institute of Electric Drives and Power Electronics, Johannes Kepler University Linz, 4040 Linz, Austria

² LCM Linz Center of Mechatronics, 4040 Linz, Austria

Sensorless rotor angle estimation methods are becoming increasingly important due to the growing demands on electric motors. However, their application is often constrained by insufficient rotor angle precision and additional energy loss. This paper addresses these challenges under dynamic operating conditions by presenting a method to predict the optimal operating range of a permanent magnet synchronous motor with respect to injection frequency and loads when using zero-sequence voltage with a high-frequency injection method for rotor angle detection. The approach is based on a finite element analysis of inductance variations across different loads over one electrical rotation in frequency domain.

Index Terms—Finite element analysis, Frequency domain analysis, Permanent magnet machines, Sensorless control

I. INTRODUCTION

PERMANENT magnet synchronous motors (PMSMs) are widely used due to their high efficiency. Accurate rotor angle estimation, typically achieved with sensors, is crucial but introduces drawbacks like additional cost and wiring, higher failure rates, and increased space requirements. To overcome these limitations, sensorless methods have been developed, particularly for low-speed operation, utilizing phase current or zero-sequence voltage measurements [1]. This paper focuses on a high-frequency (HF) injection method which uses zero-sequence voltage [2], [3] and aims to predict the optimal injection frequency, injection voltage and loss.

The paper first derives the mathematical model for a PMSM for HF injection. Then, a finite element analysis (FEA) in frequency domain is conducted for a low-power PMSM. Finally, these results are used to evaluate the optimal injection frequency and the operating range for the motor under investigation, whose cross-section is shown in Fig. 2.

II. MATHEMATICAL MODEL

To extract the rotor angle using HF injection, the saliency of the machine is exploited. In a HF model, changes in voltage due to back electromagnetic force and voltage drops across resistance can be neglected at standstill and low speeds [2]. In Fig. 1 the measurement setup is illustrated, which includes a balanced resistor network used as reference to mitigate the effects of neutral point voltage (U_{N0}) caused by modulation and inverter non-linearity for detecting zero-sequence voltage (U_{NR}). Therefore the motor must be star-connected.

The zero-sequence voltage is calculated for applying a stator phasor with an alternating amplitude of $U_{\text{peak, inj}}$, so that a square wave with an injection frequency f_{inj} is injected into the direction θ_{inj} in the stator frame expressed as

$$U_{A0} = U_{\text{inj}} \cos(\theta_{\text{inj}}), \quad (1)$$

$$U_{B0} = U_{\text{inj}} \cos\left(\theta_{\text{inj}} - \frac{2}{3}\pi\right), \quad (2)$$

$$U_{C0} = U_{\text{inj}} \cos\left(\theta_{\text{inj}} + \frac{2}{3}\pi\right) \quad (3)$$

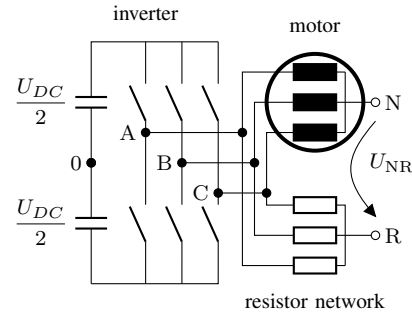


Fig. 1. Zero-sequence voltage measurement scheme.

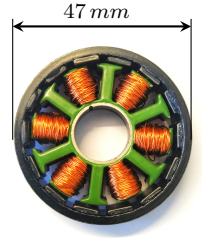


Fig. 2. Cross section of the investigated motor

with the injected voltage

$$U_{\text{inj}}(t) = U_{\text{peak, inj}} (2 \text{rect}(f_{\text{inj}} t \pmod{1}) - 1). \quad (4)$$

The HF phase voltages of the motor can be expressed as [2]

$$\begin{bmatrix} U_{AN} \\ U_{BN} \\ U_{CN} \end{bmatrix} = \underbrace{\begin{bmatrix} L_{aa} & M_{ab} & M_{ac} \\ M_{ba} & L_{bb} & M_{bc} \\ M_{ca} & M_{cb} & L_{cc} \end{bmatrix}}_{L(i_d, i_q, \alpha_e, f)} \begin{bmatrix} di_a/dt \\ di_b/dt \\ di_c/dt \end{bmatrix} \quad (5)$$

for low speed. L and M represent the self and mutual phase inductance, while i_a , i_b , and i_c denote the phase currents. For the analysis presented in this paper, FEA is performed to calculate the inductance matrix in equation (5) for various current levels (direct-axis and quadrature-axis currents i_d , i_q), at multiple rotor angles, and across a range of frequencies over one electrical period using *FEMM* in a magnetostatic simulation. As a result, the two dimensional inductance matrix in equation (5) extends to six dimensions, incorporating dependencies on current, rotor angle, and frequency. The dependency on current and frequency is not explicitly indicated in the formula symbols due to simplicity. Considering the sum of current derivatives to be zero

$$\frac{di_a}{dt} + \frac{di_b}{dt} + \frac{di_c}{dt} = 0 \quad (6)$$

and [3]

$$U_{A0} = U_{AN} + U_{NR}, \quad (7)$$

$$U_{B0} = U_{BN} + U_{NR}, \quad (8)$$

$$U_{C0} = U_{CN} + U_{NR} \quad (9)$$

the zero-sequence voltage U_{NR} can be calculated in dependency of the injection angle and the phase self- and mutual inductance. Further, the phase self and mutual inductance is modeled as

$$L_{aa} = L_0 - L_2 \cos(2\alpha_e + \alpha_{Ls,2}) - L_4 \cos(4\alpha_e + \alpha_{Ls,4}), \quad (10a)$$

$$M_{bc} = M_0 - M_2 \cos(2\alpha_e + \alpha_{Ms,2}) - M_4 \cos(4\alpha_e + \alpha_{Ms,4}). \quad (10b)$$

The inductance L_{bb} , L_{cc} and all the mutual inductances are computed likewise adapting the subscripts and applying a phase shift of $2\pi/3$ or $-2\pi/3$ respectively. α_e is the electric rotor angle. For considering saturation effects, $\alpha_{Ls,2}$, $\alpha_{Ls,4}$, $\alpha_{Ms,2}$ and $\alpha_{Ms,4}$ are introduced [2]. The coefficients L_0 , L_2 , L_4 , M_0 , M_2 and M_4 stand for the DC, second and fourth harmonic order of the self and mutual inductance. These values are determined by a fast Fourier transformation (FFT) of L_{aa} and M_{bc} from the FEA result. With equation (10) the zero-sequence voltage in dependency of the injected voltage, the injection angle and the rotor angle is given by

$$U_{NR} = U_{inj}(t) (A_c \cos(\theta_i + 2\alpha_e + \theta_{sat,Ac}) + B_c \cos(\theta_i - 4\alpha_e + \alpha_{sat,Bc}) + C_c \cos(\theta_i + 8\alpha_e + \alpha_{sat,Cc})) / (1 + D_c \cos(\alpha_e + \alpha_{sat,Dc})). \quad (11)$$

Here, A_c , B_c , C_c , and D_c and the angles due to saturation $\theta_{sat,Ac}$, $\theta_{sat,Bc}$, $\theta_{sat,Cc}$ and $\theta_{sat,Dc}$ are coefficients derived from the phase inductance coefficients in equation (10). The analytic solution for the coefficients is omitted in this short paper for brevity.

III. SIMULATION AND RESULTS

To identify the optimal injection frequency and operating range, the coefficients in equation (11) are analyzed. High sensitivity in rotor angle detection is crucial to minimize the injection voltage amplitude and additional loss. This requires a large coefficient A_c , often referred to as the main anisotropy. In contrast, the coefficients B_c and C_c , which cause disturbances, need to be kept as small as possible. If B_c or C_c exceed A_c , rotor angle estimation using the described sensorless method becomes even impossible, thereby restricting the operating range. The coefficients can be determined in two ways: by deriving the DC, second and fourth order inductance and saturation angles from an FFT and substituting these into the analytical equation, or by directly calculating U_{NR} numerically from the inductance matrix and fitting the equation (11). Since numerical approach also accounts for higher order inductance effects, it is preferred. The analysis results are shown in Fig. 3. They reveal that the main anisotropy A_c is high at low frequencies, suggesting a reduced injection frequency. For the

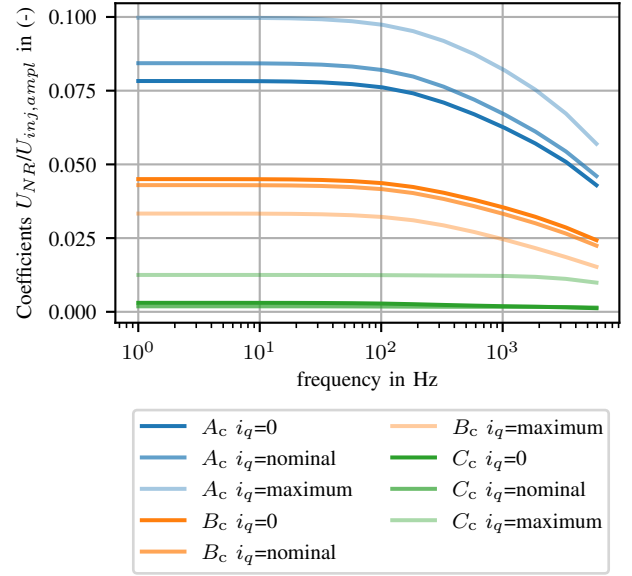


Fig. 3. Coefficients of equation (11) over frequency for different currents/loads for a fractional horsepower outrunner PMSM

full paper this will also be compared to measurement results of the investigated motor. However, selecting the injection frequency for the HF injection also requires considering additional power loss caused by the injection. Lower HF currents at higher frequencies reduce these losses, favoring a higher injection frequency. This trade-off will be discussed in more detail in the full paper.

IV. CONCLUSION

The executed FEA for the PMSM, depending on current in the frequency domain, enables a novel analysis of the injection frequency for a HF-injection method utilizing the neutral point for rotor angle estimation already in the design stage. It is shown that the injection frequency significantly affects the zero-sequence voltage. Furthermore, the analysis for different current levels allows for defining a valid operating range.

The content of this short paper will be extended with measurement results and a trade-off analysis for selecting the injection frequency, taking into account the associated energy loss. The results of this work will enable the identification of an optimal injection frequency that balances additional energy loss and injection voltage requirements for a specific application.

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