

Electrical resistance tomography of thin conductive materials with wavelet regularization

Alessandro Arduino, Alessandro Cultrera, Luca Zilberti

Istituto Nazionale di Ricerca Metrologica (INRiM), Torino, 10135 Italy, a.arduino@inrim.it

This work presents a wavelet regularization for electrical resistance tomography applied to thin conductive materials. The proposed method, implemented using the fast iterative shrinkage thresholding algorithm, is successfully applied to test measurements of a fluorinated tin oxide film with a known defect and compared with the results obtained by a traditional Tikhonov regularization.

Index Terms—Cell method, electrical resistance tomography, inverse problem, Tikhonov regularization, wavelet transform

I. INTRODUCTION

Electrical resistance tomography (ERT) is a non-destructive, minimally invasive technique that retrieves a map of the conductivity of the sample on the basis of a series of transresistance measurements. ERT originated (along with its alternating current counterpart, electrical impedance tomography, EIT) for clinical and geological applications [1]. It was later applied to verify the homogeneity of doping in silicon wafers for electronics and recently found applications in process engineering. ERT can be successfully applied also in material science. It has been used to map the conductivity of chemical vapour deposition monolayer graphene [2] and of other nanostructured materials [3]. Here, a wavelet regularization for ERT applied to thin conductive materials is proposed and tested on experimental data of a fluorinated tin oxide (FTO) thin film on which a known defect was introduced.

II. FORWARD MODEL

Let us denote by $\Omega \subset \mathbb{R}^2$ the surface of the investigated sample. A current density j_z is injected in the sample through a set of electrodes applied on Ω . According to Kirchhoff's first law (i.e., the conservation of currents) applied to any generic area $\mathcal{A} \subset \Omega$, this leads to the generation of a planar current density $\mathbf{j}_{xy}$ within the sample thickness, as illustrated in Fig. 1. By introducing the electric scalar potential φ and the sample electrical conductivity σ , it is possible to write $\mathbf{j}_{xy} = -\sigma \nabla \varphi$. Finally, taking advantage of the divergence theorem and the generality of $\mathcal{A}$, we get the partial differential equation (PDE)

$$\begin{cases} -\nabla \cdot (\tilde{\sigma} \nabla \varphi) = j_z, & \text{in } \Omega \\ \tilde{\sigma} \nabla \varphi \cdot \mathbf{n} = 0, & \text{on } \partial\Omega \end{cases}, \quad (1)$$

being $\tilde{\sigma} = d\sigma$ the surface conductivity and d the thickness.

III. INVERSE MODEL

N electrodes, whose tips are assumed to be infinitesimal, are used for the measurement procedure, consisting in a stimulation pattern. Here, the stimulation pattern involves currents injected through pairs of selected electrodes and transresistance measurements performed between pairs of the other electrodes. For each injected current, denoted by the index s , the measured values are collected in $\mathbf{R}_{\text{meas}}^s$ [2].

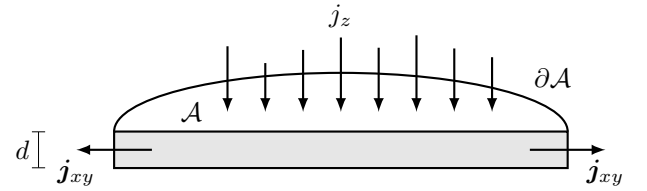


Fig. 1. Graphical representation of Kirchhoff's first law.

Assuming a guess distribution $\tilde{\sigma}$, the PDE (1) can be solved for any injected current and the corresponding transresistances can be evaluated from the resulting electric potential, leading to the set of virtual measurements $\mathbf{R}^s(\tilde{\sigma})$.

If N_s injected currents are considered, the inverse problem of ERT can be written as the optimization problem [2]

$$\min_{\tilde{\sigma}} \sum_{s=1}^{N_s} \frac{1}{2} \|\mathbf{R}^s(\tilde{\sigma}) - \mathbf{R}_{\text{meas}}^s\|_2^2. \quad (2)$$

Being ERT an ill-posed problem, a regularization cost is needed to obtain a robust estimator of the target distribution $\tilde{\sigma}^*$.

One common regularization is the Tikhonov one [1], which modifies the cost functional of the optimization problem (2) by adding the term $\lambda_T \|\nabla \tilde{\sigma}\|_2^2$. Such a penalty on the gradient of $\tilde{\sigma}$ formalizes the *a priori* knowledge of the lack of large and frequent variations (like ripples) in the surface conductivity. Tikhonov regularization can be easily integrated in a minimization algorithm based on the Gauss–Newton (GN) method.

Being based on L^2 -norm, Tikhonov regularization tends to smooth the solution. On the contrary, thin film materials, like graphene, may involve sharp discontinuous conductivity distributions. For this reason, the total variation (TV) regularization, consisting in adding the term $\lambda_{TV} \|\nabla \tilde{\sigma}\|_1$ to (2), is often preferred. It has been proved that the TV regularization is related to the Haar wavelet transform [4]. Moving from this, we propose here a wavelet regularization term for ERT, $\lambda_W \|W\tilde{\sigma}\|_1$, where W denotes the Haar wavelet transform [5].

The Haar wavelet transform describes $\tilde{\sigma}$ in terms of local averages and gradients, usually called details. Penalizing its L^1 -norm means fostering its sparsity, so looking for a solution with few and small details, namely a solution as much homogeneous as possible and potentially with sharp discontinuities.

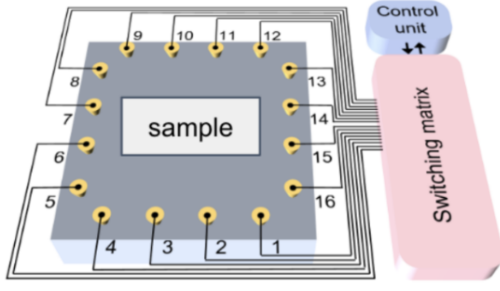


Fig. 2. Diagram of the acquisition system with 16 electrodes controlled in groups of four by means of a switching matrix.

To take advantage of the fast iterative shrinkage-thresholding algorithm (FISTA) [6], the whole problem is reformulated in the Haar wavelet space,

$$\min_w \sum_{s=1}^{N_s} \frac{1}{2} \| \mathbf{R}^s (W^{-1}w) - \mathbf{R}_{\text{meas}}^s \|_2^2 + \lambda_W \|w\|_1, \quad (3)$$

and the conductivity is provided by $\tilde{\sigma} = W^{-1}w$.

IV. TEST CASE

The developed method is applied to a test measurement performed on a FTO thin film whose surface had been voluntarily damaged with a known thin straight cut [2]. The measurement set-up, depicted in Fig. 2, has $N = 16$ electrodes arranged to touch the sample near its boundary. The transresistances are measured using groups of four electrodes selected by means of a switching matrix. Two electrodes are connected to a current source, whilst the other two to a voltmeter. The switching matrix is driven to select the electrodes according to a stimulation pattern. In our case, the adopted stimulation pattern is the pairwise adjacent (or Sheffield) one [1].

Both the GN with Tikhonov regularization and the FISTA with wavelet regularization methods have been implemented in Python. In both cases, the square surface of the sample with sides of 1 cm has been discretized with a structured uniform Cartesian mesh with 40 pixels per side. For each guess surface conductivity, the forward model (1) is approximated on this mesh according to the cell method [7] and is solved to compute both the residual and its Fréchet differential.

The surface conductivity maps reported in Fig. 3 have been obtained after 100 iterations of the minimization methods, at which convergence is reached, starting from an initial guess with a uniform surface conductivity of 150 mS. The used regularization weights are $\lambda_T = \eta \cdot 5 \times 10^{-4} \text{ m}^2 \text{ S}^{-2}$ and $\lambda_W = \eta \cdot 5 \times 10^{-4} \text{ m S}^{-1}$, where $\eta = \sum_{s=1}^{N_s} \frac{1}{2} \| \mathbf{R}_{\text{meas}}^s \|_2^2$ is used to balance the two terms of the cost functional. GN with Tikhonov regularization run in about 8 s, whereas FISTA with wavelet regularization in about 3 s. Similar results were obtained by using a finer mesh with 80 pixels per side, but in a longer computational time and in a greater number of iterations.

Both reconstruction approaches identify an almost homogeneous sample far from the electrodes with a drop in conductivity along a diagonal in the bottom left part of the surface, where

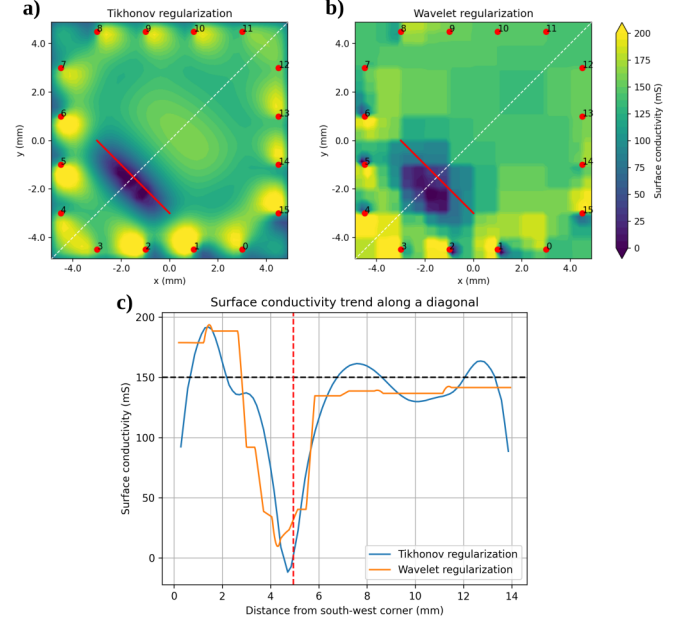


Fig. 3. Surface conductivity of a cutted FTO sample (the red line denotes the location of the cut) estimated by (a) GN with Tikhonov regularization and by (b) FISTA with wavelet regularization. (c) Trend of the estimated surface conductivity along the diagonal of the sample passing through the cut (white dashed line). The black dashed line denotes the expected value for a sample without the cut; the red dashed line denotes the location of the cut.

the cut had been made. In case of Tikhonov regularization, the retrieved map is smooth and a bit blurred, as it is typical of the L^2 -norm regularization. The wavelet regularization, instead, is blocky and clearly shows the appearance of macro-pixels, which reflect the nature of Haar wavelets. By forcing the sparsity in the Haar wavelet transform, we are gathering together nearby pixels limiting the gradient among them unless the fidelity term of the cost functional, namely the first addendum of (3), is too large. As a consequence, the method automatically determines the local resolution needed to capture the physical and geometrical features of the measurements. Indeed, far from the cut, where the sample is homogeneous, large macro-pixels of about 8 pixels per side are used, whereas smaller and smaller macro-pixels are used moving towards the cut.

REFERENCES

- [1] C. Dimas, V. Alimisis, N. Uzunoglu, P. P. Sotiridis, "Advances in electrical impedance tomography inverse problem solution methods: from traditional regularization to deep learning," *IEEE Access*, vol. 12, 2024.
- [2] A. Cultrera, *et al*, "Mapping the conductivity of graphene with electrical resistance tomography," *Sci. Rep.*, vol. 9, 2019.
- [3] G. Milano, *et al*, "Tomography of memory engrams in self-organizing nanowire connectomes," *Nat. Commun.*, vol. 14, 2023.
- [4] U. S. Kamilov, "A parallel proximal algorithm for anisotropic total variation minimization," *IEEE Trans. Image Process.*, vol. 26, 2017.
- [5] S. Mallat, *A wavelet tour of signal processing: the sparse way*, 3rd ed. San Diego, CA, USA: Academic Press, 2009.
- [6] A. Beck, M. Teboulle, "A fast iterative shrinkage-thresholding algorithm for linear inverse problems," *SIAM J. Imaging Sci.*, vol. 2, 2009.
- [7] P. Alotto, F. Freschi, M. Repetto, C. Rosso, "The cell method for electrical engineering and multiphysics problems: an introduction," in *Lecture Notes in Electrical Engineering*, vol. 230. Berlin, Heidelberg: Springer Berlin Heidelberg, 2013.