

Magnetic Circuit Parameter Extraction by Dual Finite Element Formulations

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Abstract—This paper presents dual finite element formulations for extraction of magnetic circuit (MC) parameter in electromagnetic devices. Similar with electric circuit (EC), lumped parameters in MC namely magnetic-resistance and “magnetic-inductance” can be extracted by dual formulations using scalar and vector potential variables. The dual formulations used in MC parameter extraction also shows the properties in complementary bounds. By introducing a magnetic port, we show the formulations to consider the case when the topology of both permeability domain and conductivity domain is nontrivial.

Keywords—magnetic circuit, dual formulation, magnetic-resistance, magnetic-inductance.

I. INTRODUCTION

Circuit simulation offers fast computational speed but requires appropriate lumped parameters for circuit model. EC and MC provide different perspectives for analyzing electromagnetic problems. Lumped parameters in MC can be used purely for circuit analysis (for instance, initial design analysis for devices like motor and power transformer), or for field-circuit coupled simulation [1] to obtain an optimal trade-off between simulation accuracy and problem size.

Besides the commonly used MC parameter magnetic-resistance (Rm), the concept of “magnetic-inductance” (Lm) [2] is introduced recently to consider effect of induced current in the MC. In conventional MC method, Rm is calculated in an analytical way. It takes a short calculation time but with some simplifications in the modeling process which reduces the accuracy in calculation with complex structure or large leakage flux. This limits the application of MC method in some case such as special motor designs. Furthermore, Lm cannot be calculated generally in an analytical way.

Similar with EC parameter, MC can be extracted in numerical way to obtain a better accuracy while received few attentions. This paper presents dual finite element formulations for extraction of the MC parameters. One of the important properties of dual formulations is the complementary energy bounds, which has been used for the fast and accurate determination of EC parameters [3]. In fact, the parameters of EC and MC are defined with same problem type but with different port, as shown in table 1, where, R is resistance, L is inductance. Therefore, the dual formulations used in MC

parameter extraction could also keep the properties in complementary bounds. Note that generally the parameters of EC and MC cannot be converted to each other due to the fact that the ports in EC and MC are not one-to-one correspondence.

Table 1. Lumped parameter from the view of different port.

	Direct-current	Magneto-static
Electric port	R	L
Magnetic port	Lm	Rm

In this paper, the authors seek to extract MC parameter by introducing the magnetic port. Since the electric port exist in electromagnetic problems naturally in most cases, we must consider both the electric port and the magnetic port at the same time, and consequently solve the problem in multiply connected regions. Here, a cutting domain [4] is introduced to deal with the case when both topology of domain of permeability and conducting are nontrivial.

II. THEORY AND METHOD

Let us consider a problem with confined flows of the magnetic flux as well as electric current, as shown in Fig.1. There are two domains without intersection, MC domain in Ω_μ and EC in Ω_σ , and both topology of domain Ω_μ and Ω_σ are nontrivial. The magnetic ports and the electric ports are defined on a cutting surface where the flux flows through respectively. We setup a space to build a connection between Ω_μ and Ω_σ . In theory of differential geometry, such a space is the discrete space of de Rham’s co-homology group in the cutting domain C_μ and C_σ [3]. When considering the case with no confined flow of the magnetic flux (the leakage flux), Ω_μ will be defined in the whole domain including Ω_σ .

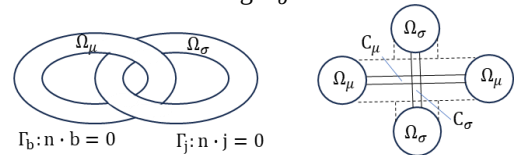


Fig.1. Problem definition with model of electric circuit and magnetic circuit.

A. Reluctance Extraction

We consider the magneto-static problem shown in Fig.1, which can be described by the following equations:

$$\text{curl } \mathbf{h} = \mathbf{j}, \text{div } \mathbf{b} = 0, \mathbf{b} = \mu \mathbf{h} \quad (1)$$

where $\mathbf{h}$ is the magnetic field, $\mathbf{b}$ is the magnetic flux density, μ is the permeability, $\mathbf{j}$ is current density.

In terms of vector potential, (1) can be written as:

$$\text{curl } v \text{ curl } \mathbf{a} = 0 \quad (2)$$

where v is magnetic reluctivity, $\mathbf{a}$ is magnetic vector potential.

In finite element method the formulation states: find $\mathbf{a} \in \{\mathbf{a} \in W^1 \mid \mathbf{n} \times \mathbf{a} = 0 \text{ on } \Gamma_b / \Gamma_{\Lambda\mu}\}$, such that

$$\int_{\Omega_\mu} v \text{ curl } \mathbf{a}' \cdot \text{curl } \mathbf{a} \, d\Omega - \int_{\Omega_\mu \cap C_\mu} v \text{ curl } \mathbf{a}' \cdot \text{curl } \mathbf{a}_s \, d\Omega = 0 \quad (3)$$

$$\forall \mathbf{a}' \in \{\mathbf{a} \in W^1 \mid \mathbf{n} \times \mathbf{a} = 0 \text{ on } \Gamma_b\}$$

where, $\mathbf{a}_s \in W_H(C_\mu)$ is precalculated for a given flux, W^1 denotes the Whitney edge element space, W_H denotes the Whitney element space for the cutting domain, $\Gamma_{\Lambda\mu} = \Omega_\mu \cap C_\mu$.

In terms of scalar potential, (1) can be written as:

$$\text{div } \mu (\text{grad } \varphi + \mathbf{t}_s) = 0 \quad (4)$$

Let W^0 denotes the Whitney node element space, the dual formulation states: find $\varphi \in \{\varphi \in W^0 \mid \varphi = c \text{ on } \Gamma_h\}$, such that

$$\int_{\Omega_\mu} \mu \text{ grad } \varphi' \cdot \text{grad } \varphi \, d\Omega - \int_{\Omega_\mu \cap C_\sigma} \mu \text{ grad } \varphi' \cdot \mathbf{t}_s \, d\Omega = 0 \quad (5)$$

$$\forall \varphi' \in \{\varphi \in W^0 \mid \varphi = 0 \text{ on } \Gamma_h\}$$

where, $\mathbf{t}_s \in W_H(C_\sigma)$ is precalculated for a given magnetomotive force.

(3) and (5) are solved respectively and the magnetic reluctance is calculated with both formulations.

B. Magnetic-Inductance Extraction

We consider the direct-current problem which can be described by the following equations:

$$\text{curl } \mathbf{e} = -\dot{\mathbf{b}}, \text{div } \mathbf{j} = 0, \mathbf{j} = \sigma \mathbf{e} \quad (6)$$

where, $\mathbf{e}$ is electric field, $\dot{\mathbf{b}}$ is the time derivative of $\mathbf{b}$ which has the voltage per square meter unit, σ is the conductivity.

In terms of vector potential, (6) can be written as:

$$\text{curl } \frac{1}{\sigma} \text{ curl } \mathbf{t} = -\text{curl } \dot{\mathbf{a}}_s \quad (7)$$

where, $\mathbf{t}$ is the current vector potential, $\dot{\mathbf{a}}_s$ is time derivative of $\mathbf{a}_s$ and is precalculated for a given induced voltage.

The formulation states: find $\mathbf{t} \in \{\mathbf{t} \in W^1 \mid \mathbf{n} \times \mathbf{t} = 0 \text{ on } \Gamma_j / \Gamma_{\Lambda\sigma}\}$, $\mathbf{t}_s \in W_H(C_\sigma)$, such that

$$\begin{aligned} \int_{\Omega_\sigma} \frac{1}{\sigma} \text{ curl } \mathbf{t}' \cdot \text{curl } \mathbf{t} \, d\Omega + \int_{\Omega_\sigma \cap C_\sigma} \frac{1}{\sigma} \text{ curl } \mathbf{t}' \cdot \text{curl } \mathbf{t}_s \, d\Omega \\ + \int_{\Omega_\sigma \cap C_\mu} \text{ curl } \mathbf{t}' \cdot \dot{\mathbf{a}}_s \, d\Omega = 0 \\ \int_{C_\sigma} \frac{1}{\sigma} \text{ curl } \mathbf{t}_s' \cdot \text{curl } \mathbf{t} \, d\Omega + \int_{C_\sigma} \frac{1}{\sigma} \text{ curl } \mathbf{t}_s' \cdot \text{curl } \mathbf{t}_s \, d\Omega \\ + \int_{C_\sigma \cap C_\mu} \text{ curl } \mathbf{t}_s' \cdot \dot{\mathbf{a}}_s \, d\Omega = 0 \end{aligned} \quad (8)$$

$$\forall \mathbf{t}' \in \{\mathbf{t} \in W^1 \mid \mathbf{n} \times \mathbf{t} = 0 \text{ on } \Gamma_j\}, \forall \mathbf{t}_s' \in W_H(C_\sigma)$$

where, $\Gamma_{\Lambda\sigma} = \Omega_\sigma \cap C_\sigma$.

In terms of scalar potential, (6) can be written as:

$$\text{div } \sigma (\text{grad } v + \dot{\mathbf{a}}_s) = 0 \quad (9)$$

The dual formulation states: Find $v \in \{v \in W^0 \mid v = c \text{ on } \Gamma_e\}$, such that

$$\int_{\Omega_\sigma} \sigma \text{ grad } v' \cdot \text{grad } v \, d\Omega - \int_{\Omega_\sigma \cap C_\mu} \sigma \text{ grad } v' \cdot \dot{\mathbf{a}}_s \, d\Omega = 0 \quad (10)$$

$$\forall v' \in \{v \in W^0 \mid v = 0 \text{ on } \Gamma_e\}$$

We can solve (8) and (10), respectively, and calculate the magnetic-inductance further.

III. NUMERICAL EXAMPLE

The test case is a transformer with primary and secondary winding coil, as shown in Fig.2. There are two electric circuits but only one magnetic circuit. An example of the cutting domains of C_μ and C_σ are also shown in Fig.2. The variation of magnetic-resistance and magnetic-inductance with mesh fineness are shown in Fig.3 and Fig.4 respectively. It can be noticed that the dual formulations used in MC parameter extraction also keeps the properties of complementary bounds.

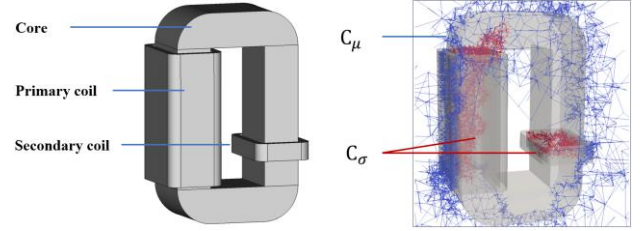


Fig.2. Model of transformer and example of the cutting domains.

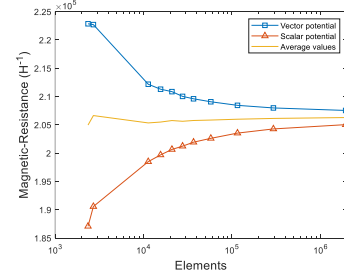


Fig.3. Variation of magnetic-resistance with mesh fineness.

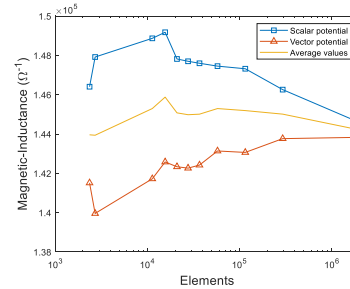


Fig.4. Variation of magnetic-inductance with mesh fineness.

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