

# Investigation of Function of Current Vector Potential $T$ in Edge FEM with Magnetic Vector Potential $A$ and the Convergence of ICCG

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In the edge element model for solving eddy current fields, the continuity of the current density is maintained by introducing the current vector potential  $T$  as an intermediate variable, which enhances the convergence of the ICCG method. In this paper, numerical comparisons reveal that the curl of  $T$  does not strictly correspond to the current density  $J$ , which contradicts the theoretical definition of  $T$ . To address this, the equivalence is studied between the numerical solution of the current vector potential  $T$  and its definition. By introducing the electric scalar potential  $u$ , the underlying cause of the discrepancy between the curl of  $T$  and the current density  $J$  is thoroughly analyzed. Furthermore, the role of the electric scalar potential  $u$  in ensuring the current density continuity and its impact on the ICCG solution process are examined, along with its effect on the field distribution. The validity of the numerical method is confirmed by means of the TEAM 21b benchmark model.

**Index Terms**—Edge element, finite element method, current vector potential, ICCG, scalar potential.

## I. INTRODUCTION

THE formulation based on the magnetic vector potential (MVP) is widely used in solving electromagnetic field problems because it can deal with multi-connected domain problems naturally. The process of introducing magnetic vector potential only defines the curl of MVP but not the divergence of MVP, thus the uniqueness of MVP cannot be guaranteed. Theoretically, since the uniqueness of magnetic flux density is strictly satisfied, any set of magnetic vector potential solutions will yield a unique magnetic flux density. Numerical experiences show that ungauged MVP often leads to the poor convergence of solving the algebraic equation system.

In each finite element, the edge element interpolation function automatically satisfies the condition of zero divergence. However, the jump of normal component at the interfaces can induce a global weak divergence, even when the divergence is piecewise zero. As a result, it becomes infeasible to directly apply the Coulomb gauge to the field equations. To improve the convergence of solving the edge element equation, techniques such as tree gauge and acceleration factors are commonly employed [1,2]. However, selecting the optimal tree remains a challenging issue and the application of acceleration factors is limited in scope from the perspective of system equation solutions. It has been found that the compatibility of edge element equation is strictly satisfied and the convergence of solving edge element equation gets better when the continuity of current density is strictly guaranteed. A more widely used approach is to introduce the current vector potential  $T$  as the intermediate variable to enhance current density continuity numerically and improve the convergence [3-7].

For a single coil with a simple geometry, the current vector potential  $T$  can be expressed analytically. However, for multiple coils or coils with complex geometries,  $T$  must be computed numerically. When  $T$  is calculated numerically in a multiple coil system as [3], an evident discrepancy is observed between the curl of  $T$  and the given current density. This discrepancy

contradicts the theoretical definition of  $T$ , yet the normal field distribution can still be accurately determined. The article introduces the electric scalar potential through mathematical derivation, explains the cause of the discrepancy, and analyzes its impact on current density continuity, ICCG convergence, and field distribution. Finally, the validity of the theory is confirmed through the TEAM 21b problem.

## II. FEM FOR 3D EDDY CURRENT PROBLEM

The computational region  $\Omega$  is divided into two parts,  $\Omega_1$  is the conductor region and  $\Omega_2$  is the non-conducting region containing the source current region. The boundary of  $\Omega$  is consist of  $\Gamma_B$  and  $\Gamma_H$ . The field equation of eddy current problem with modified magnetic vector potential  $A$  can be given as

$$\begin{aligned} \nabla \times v \nabla \times A + \sigma \frac{\partial A}{\partial t} &= J \\ n \times A &= 0 \quad \text{on } \Gamma_B \\ v \nabla \times A \times n &= 0 \quad \text{on } \Gamma_H \end{aligned} \quad (1)$$

where  $v$  is the magnetic reluctivity,  $\sigma$  the electric conductivity,  $J$  the given current density.

$$A = \sum_{i \in I} N_i A_i \quad (2)$$

As shown in (2),  $A$  in (1) can be approximated by edge element where  $i$  is the edge number,  $I$  a set of edges,  $A_i$  the line integral of magnetic vector potential along the edge,  $N_i$  the edge element interpolation function at edge  $i$ .

### A. Governing equation with current vector potential $T$

According to [3], the compatibility of the edge element equation needs to strictly satisfy the current density continuity. The continuity of the current density at the corner of coils is difficult to guarantee when the current density is expressed numerically. To overcome the problem, the current vector potential  $T$  can be introduced as

$$\nabla \times T = J \quad (3)$$

thus, the governing equation can be expressed as below:

$$\nabla \times \nabla \times \mathbf{A} + \sigma \frac{\partial \mathbf{A}}{\partial t} = \nabla \times \mathbf{T} \quad (4)$$

For multiple coils or coils with complex geometries,  $\mathbf{T}$  can be calculated as:

$$\int_{\Omega_c} \nabla \times \mathbf{N} \cdot \nabla \times \mathbf{T} d\Omega = \int_{\Omega_c} \nabla \times \mathbf{N} \cdot \mathbf{J} d\Omega \quad (5)$$

where  $\Omega_c$  should be a simply connected region.

### B. Function of current vector potential $\mathbf{T}$ and electric scalar potential $u$

By solving (5), we obtain the following distribution of current density in Fig.1, which is compared with the excitation current density distribution.

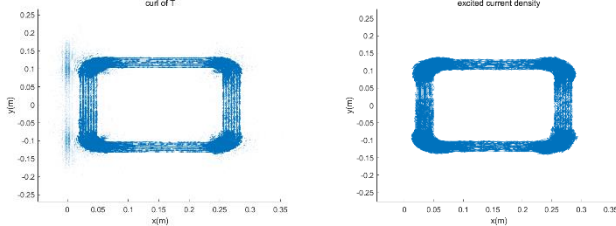


Fig. 1. Comparison between curl of  $\mathbf{T}$  and excited current density  $\mathbf{J}$ .

It can be seen that the curl of  $\mathbf{T}$  is obviously different from  $\mathbf{J}$ , which contradicts to the definition  $\nabla \times \mathbf{T} = \mathbf{J}$ . Theoretically, this discrepancy could potentially affect the field distribution; however, in practice, the method produces reasonable magnetic field distributions. The reason for the phenomenon can be explained here.

According to Galerkin finite element correlation theory, the equation (5) to numerically solve  $\mathbf{T}$  is actually equivalent to (6):

$$\nabla \times \nabla \times \mathbf{T} = \nabla \times \mathbf{J} \quad (6)$$

which is necessary but not sufficient condition for the definition (4). Owing to vector identities  $\nabla \times \nabla u = \mathbf{0}$ , an electric scalar potential  $u$  is described here as:

$$\nabla \times (\nabla \times \mathbf{T} - \mathbf{J}) = \nabla \times \nabla u = \mathbf{0} \quad (7)$$

It can be seen that the current vector potential  $\mathbf{T}$  solved by (5) only satisfies  $\nabla \times \mathbf{T} = \mathbf{J} + \nabla u$  in the light of (7).  $\nabla u$  explained the differences in Fig.1. Since the divergence of  $\nabla u$  is non-zero and its curl is zero, it can be inferred that  $\nabla u$  represents the Coulomb electric field generated by free charges and it does not affect the magnetic field distribution. The analysis of the normal component of the current density reveals that:

$$\mathbf{J}_{1n} + \left( \frac{\partial u}{\partial n} \right)_1 = \mathbf{J}_{2n} + \left( \frac{\partial u}{\partial n} \right)_2 \quad (8)$$

As can be seen from the above equation, the introduction of free charges compensates for the current density discontinuity caused by the numerical discretization of  $\mathbf{J}$ , thereby improving the convergence of the ICCG method in solving the edge finite element equations.

### III. NUMERICAL RESULTS

TEAM workshop problem 21b is used to test the accuracy of 3D nonlinear eddy current analysis method. This problem consists one magnetic steel plate and two exciting coils. The structure is shown in Fig.2.

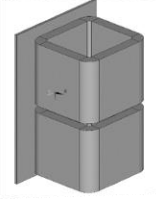


Fig. 2. Structure of TEAM 21B.

The calculated results are compared with the experimental results, as shown in Figure 3 and Table 1.

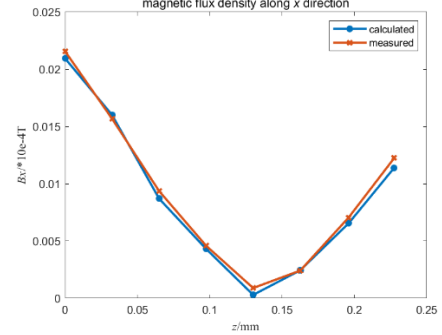


Fig. 3. Magnetic flux density along x direction.

TABLE I  
LOSS COMPARISON BETWEEN CALCULATED AND MEASURED RESULTS

| Losses       | Calculated(W) | Measured(W) |
|--------------|---------------|-------------|
| Eddy current | 8.19          | 8.35        |
| Hysteresis   | 3.88          | 3.78        |
| Total        | 12.07         | 12.13       |

The results show that although there is a difference between the curl of  $\mathbf{T}$  and  $\mathbf{J}$ , the correct field distribution can still be obtained through (4) and (5).

### IV. CONCLUSION

The paper addresses the observed discrepancy between the curl of  $\mathbf{T}$  and  $\mathbf{J}$  derived from the numerical solutions of multiple coil systems. Through derivation and proof, it is demonstrated that this discrepancy corresponds to the Coulomb electric field generated by free charges. This approach effectively addresses the issue of current density discontinuity while having no impact on the magnetic field distribution. The proposed theoretical framework is validated through a detailed analysis of the TEAM 21b benchmark problem.

### V. ACKNOWLEDGEMENT

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