

Single equivalent electrical circuit representation of electro-quasistatic problems using CLN method

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The Cauer Ladder Networks (CLN) method has been already applied to reduce a Finite Element (FE) model of an electro-quasistatic problem. The obtained reduced model is based on two equivalent electrical circuits. In this communication, we modify the FE model in order to obtain a reduced model based on only one equivalent electrical circuit when applying the CLN method. The proposed approach is applied on a bushing of a high voltage transformer.

Index Terms—Cauer Ladder Network, Electro-quasistatic

I. INTRODUCTION

THE Cauer Ladder Networks (CLN) method can generate a reduced model of the Finite Element (FE) model as well as an equivalent circuit representation, leading to a natural coupling with an external circuit. For a magneto-quasistatic problem, it has been shown that an equivalent circuit composed of inductances and resistances, can be easily constructed [1]. Recently, authors have proposed to apply the CLN method to electro-quasistatic (EQS) problems discretized by the FE method [2]. The proposed method relies on the solutions of two equivalent circuits made of capacitances and resistances. In this communication, we modify the FE model in order to obtain a reduced model based on only one equivalent electrical circuit when applying the CLN method. The proposed approach is applied on a bushing of a high voltage transformer. In a first part, the CLN approach applied to a EQS problem is described. The second part presents a comparison between results obtained from the CLN and FE models.

II. CAUER LADDER NETWORK METHOD

By neglecting the time variation of magnetic flux density, an electro-quasistatic problem can be derived from the Maxwell equations in the frequency domain as follow:

$$\text{curl } \mathbf{E} = \mathbf{0} \quad (1)$$

$$\text{div}(\mathbf{J} + j\omega\mathbf{D}) = 0 \quad (2)$$

with $\mathbf{E}$ the electric field, $\mathbf{J}$ the current density, $\mathbf{D}$ the electric displacement field and ω the angular frequency. Figure 1 shows an illustration of the studied domain Ω with boundary $\partial\Omega = \Gamma_1 \cup \Gamma_2 \cup \Gamma_D$ and $\Gamma_1 \cap \Gamma_2 = \Gamma_1 \cap \Gamma_D = \Gamma_2 \cap \Gamma_D = \emptyset$. The behavior laws are $\mathbf{J} = \sigma\mathbf{E}$, with σ the conductivity and $\mathbf{D} = \epsilon\mathbf{E}$, with ϵ the permittivity. In the conductive domains Ω_{c1} and Ω_{c2} , the conductivity σ is strictly positive function and equal to zero elsewhere. The permittivity ϵ is a strictly positive function in Ω . The source is either the voltage $\underline{V}$ imposed between Γ_1 and Γ_2 or the current $\underline{I}$ flowing through Γ_1 and Γ_2 . The φ formulation is used to solve the problem. The electric scalar potential φ is defined on the whole domain Ω such as:

$$\mathbf{E} = -\text{grad } \varphi - \underline{V} \text{grad } \alpha, \text{ with } \varphi = 0 \text{ on } \Gamma_1 \cup \Gamma_2 \quad (3)$$

where α is a scalar function and is equal to 1 on Γ_1 and 0 elsewhere. The current $\underline{I}$ can be expressed as:

$$\underline{I} = - \int_{\Omega_{c1}} \text{grad } \alpha \cdot (\mathbf{J} + j\omega\mathbf{D}) d\Omega \quad (4)$$

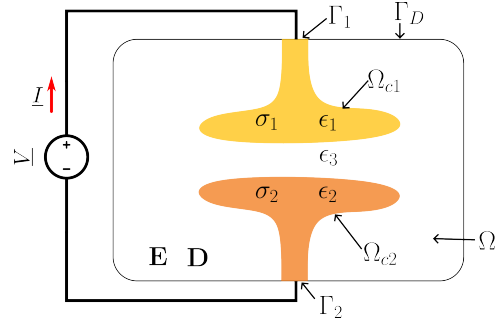


Fig. 1. Definition of the problem.

Applying the FE method, the scalar potential φ and the source term α are discretized with nodal elements. We have then:

$$\varphi = \sum_{i=1}^N X_{\varphi,i} w_i^0 \quad (5)$$

$$\alpha = \sum_{i \in \Gamma_1} w_i^0 \quad (6)$$

with N , the number of nodes, $w_i^0(\mathbf{x})$ the interpolation function associated with the i -th node, $X_{\varphi,i}$ the value of φ at the node i . We denote $\mathbf{X}_{\varphi} \in \mathbb{R}^N$ the vector of entries $X_{\varphi,i}$. If the voltage $\underline{V}$ is known, the following equation system can be obtained:

$$\mathbf{G}\mathbf{X}_{\varphi} + j\omega\mathbf{S}\mathbf{X}_{\varphi} = -\underline{V}\mathbf{F}_1 - j\omega\underline{V}\mathbf{F}_2 \quad (7)$$

with $\mathbf{G}$ and $\mathbf{S}$ two matrices and $\mathbf{F}_1$ and $\mathbf{F}_2$ two source vectors, one depending on ω and the other not. When applying the CLN method, we obtain then two equivalent circuits corresponding to each source term [2]. If $\underline{V}$ is unknown, another equation system is obtained by coupling (4) and (7):

$$\begin{bmatrix} \mathbf{G} & \mathbf{F}_1 \\ \mathbf{F}_2^T & G_{\alpha} \end{bmatrix} \begin{bmatrix} \mathbf{X}_{\varphi} \\ \underline{V} \end{bmatrix} + j\omega \begin{bmatrix} \mathbf{S} & \mathbf{F}_2 \\ \mathbf{F}_1^T & S_{\alpha} \end{bmatrix} \begin{bmatrix} \mathbf{X}_{\varphi} \\ \underline{V} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \underline{I} \end{bmatrix} \quad (8)$$

where G_α and S_α are two scalars. Gathering the vector $\mathbf{X}_\varphi$ and the voltage $\underline{V}$ into one unknown vector:

$$\mathbf{X}_{\varphi V} = \begin{bmatrix} \mathbf{X}_\varphi \\ \underline{V} \end{bmatrix} \quad (9)$$

We can obtain the following system to be solved:

$$\mathbf{N}\mathbf{X}_{\varphi V} + j\omega\mathbf{K}\mathbf{X}_{\varphi V} = \underline{\mathbf{I}}\mathbf{F}_{\varphi V} \quad (10)$$

with the definition of matrices $\mathbf{N}$ and $\mathbf{K}$ and vector $\mathbf{F}_{\varphi V}$:

$$\mathbf{N} = \begin{bmatrix} \mathbf{G} & \mathbf{F}_1 \\ \mathbf{F}_1^T & \mathbf{G}_\alpha \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} \mathbf{S} & \mathbf{F}_2 \\ \mathbf{F}_2^T & \mathbf{S}_\alpha \end{bmatrix}, \quad \mathbf{F}_{\varphi V} = \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix}. \quad (11)$$

In order to apply the CLN method like in [2], a change of variable $s = \frac{1}{j\omega}$, is carried out and we obtain:

$$s\mathbf{N}\mathbf{X}_{\varphi V} + \mathbf{K}\mathbf{X}_{\varphi V} = s\underline{\mathbf{I}}\mathbf{F}_{\varphi V} \quad (12)$$

We can then apply the CLN method (see [2]) on an equation system with only one source term leading to one equivalent circuit with n stages, shown in Fig.2 made of capacitances and resistances. The CLN method leads also to a reduced basis $\mathbf{V} = [\mathbf{v}_0, \dots, \mathbf{v}_{2n}]$ which enables to determine an approximation of the FE solution:

$$\mathbf{X}_\varphi \approx \sum_{i=1}^n v_{2i} \mathbf{v}_{2i} \quad (13)$$

where v_{2i} is the voltage at the terminal of the resistances of the equivalent electrical circuit (see Fig. 2). The number of stages n can be chosen arbitrarily. However, the higher n , the more accurate the reduced model.

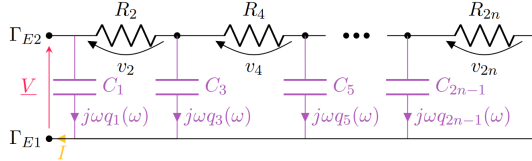


Fig. 2. Equivalent electrical circuit obtained after applying the CLN method.

III. APPLICATION

We apply the proposed method on a bushing (see [2] for more details). The bushing shown in Fig.3 is made of electrodes assumed to be equipotential separated by an insulation made of Resin Impregnated Papers (RIP). We have introduced a defect that locally modifies the conductivity between two electrodes, which is much higher than that of RIP. The problem becomes then 3D. The quantity of interest is the dissipation factor, which can be computed by:

$$\tan \delta(\omega) = \frac{V_R}{V_C}, \quad \text{with } \underline{V} = V_R + jV_C \quad (14)$$

with $\underline{V}$ the voltage when a current $\underline{I} = 1$ A is imposed. In Fig.4, we show the evolution of the dissipation factor $\tan(\delta)$ as a function of frequency calculated using the FE model and the CLN method for a number of stages n equal to 4, 8 and 16. A higher number of stages is needed to increase the accuracy

of the CLN method at low frequency. Table I presents the computational time for:

- 1) The FE model (T_{EQS}) to calculate 20 values of $\tan \delta(\omega)$ in the frequency domain regularly distributed in the interval $[1 \times 10^{-3}, 1 \times 10^3]$ Hz.
- 2) The construction of bases ($n = 16$) of the CLN method during the offline stage ($T_{CLN,offline}$).
- 3) The frequency response of $\tan \delta(\omega)$ from the equivalent circuit during the online stage ($T_{CLN,online}$).

We can see here that the computational time of the FE model is larger than that of the CLN method, accounting for the offline stage while maintaining good accuracy. The CLN method can consequently also be used as a direct solver, provided that an estimation of the error between the FE solution and the approximation (13) can be easily calculated [2].

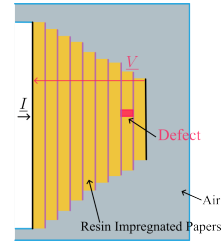


Fig. 3. Cross section of the studied example.

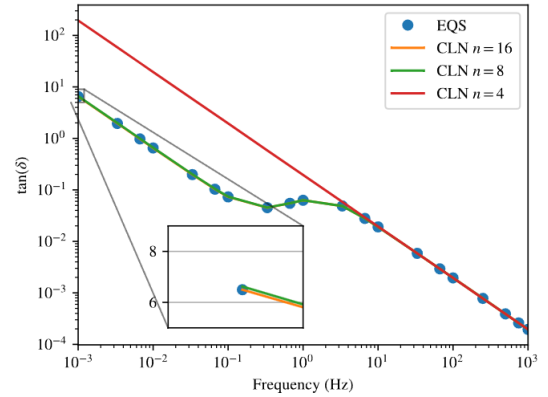


Fig. 4. $\tan \delta(\omega)$ obtained by the FE model (EQS) and by the equivalent circuit with different stages ($n=16$, $n=8$ and $n=4$).

TABLE I
COMPUTATIONAL TIME UNDER DEFECT CASE.

T_{EQS} [s]	$T_{CLN,offline}$ [s]	$T_{CLN,online}$ [s]
13.6	5.8	5.26×10^{-3}

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