

Equivalent Medium Approach for Effective Magnetic Permeability Retrieval of Printed Resonant Metasurfaces in WPT Applications

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A computational method based on Ampere's law and mutual inductance is proposed to calculate the relative permeability of an effective medium between two coupled coils for wireless power transfer with resonant metamaterials. The method's validity is confirmed through comparisons with experimental results obtained using printed coils operating in the MHz range. Results align with full-wave electromagnetic simulation, offering a speedup of nearly 1760 times. This computational efficiency makes the method useful for optimizing WPT system design, facilitating faster iterations, and providing deeper insights into the role of metamaterials in enhancing coupling and overall system performance.

Index Terms—Effective magnetic permeability, near-field coupling, resonant metamaterials, wireless power transfer.

I. INTRODUCTION

WIRELESS POWER TRANSFER (WPT) systems are widely employed in applications such as mobile charging, electric vehicles, and industrial environments, where the effective permeability of a two-coil system defines the interaction between the transmitting coil's (Tx) magnetic field and its surroundings, including the receiving coil (Rx). In systems incorporating resonant metamaterials (MTM), this parameter is crucial for enhancing near-field coupling, concentrating fields, reducing scattering losses, and improving energy transfer efficiency, as well as for frequency tuning and minimizing loss effects to ensure optimal performance [1], [2]. Thus, this paper presents a fast and efficient method for calculating the effective permeability in WPT systems with two coils and an MTM in the magnetic link, combining circuit parameter calculations with magnetic field analysis to significantly reduce computational time and enable tailored optimization strategies for overcoming current challenges in WPT technologies.

II. NUMERICAL APPROACH

A. Extraction of the effective permeability of the medium

A typical WPT system consists of two magnetically coupled coils: a Tx and a Rx. When positioned nearby, the Tx coupled with the Rx coil generates a magnetic field oscillating at a frequency f . In this configuration, the spatial arrangement of the coils often allows for the assumption of a homogeneous magnetic flux between them. The magnetic link in such a system is governed by Ampère's law:

$$H_1(f) \cdot l = n \cdot i_1(f), \quad (1)$$

where n is the turn of the Tx coil, l is the length of a contour passing through the Tx coil along a line of the magnetic field H_1 , and $i_1(f)$ is the current in the Tx coil. The magnetic flux density B_1 is related to the field H_1 through the constitutive relation given by:

$$B_1(f) = H_1(f) \cdot \mu_0 \cdot \mu_r. \quad (2)$$

Since H_1 is considered homogeneous, using (1) and (2), we define a constant C_1 as the ratio between flux density and current in the Tx coil:

$$\frac{B_1(f)}{i_1(f)} = \frac{n}{l} \cdot \mu_0 \cdot \mu_r = C_1. \quad (3)$$

The effective permeability of a medium (μ_{eff}) in a WPT system with an MTM is affected by the presence of the MTM in the magnetic coupling region. The modified magnetic link equations in the presence of an MTM are derived analogously to the case of two coils without the MTM. However, in this case, the ratio between $B_2(f)$ and $i_2(f)$ is represented by the constant C_2 . Since l and n remain constant for both the standard two-coil system and the case with the MTM, and although l is not directly known, the assumption of an equal contour length across the two measurements allows it to cancel out when forming a ratio. Thus, the ratio between C_2 and C_1 is directly the μ_{eff} of the system, as expressed by:

$$\mu_{eff} = \frac{C_2}{C_1}. \quad (4)$$

B. System modeling

The modeling of the two-coil WPT system with and without MTM can be performed using the circuit model (CM) method proposed in [3]. Fig. 1(a) illustrates the coils' dimensions. This method provides an approach based on the calculation of circuit parameters, allowing for the rapid computation of currents in each system element by solving the linear system $Z \cdot I = V$, where I is the current matrix, V is the voltage matrix, and Z is the impedance matrix. The Tx and Rx coils are considered an equivalent RL circuits, and the MTM cell is an equivalent RLC circuit as shown in Fig. 1(b). The mutual inductance between two adjacent loops 1 and 2 is calculated using Neumann's formula:

$$M = \frac{\mu_0}{4\pi} \oint_{\Gamma_1} \oint_{\Gamma_2} \frac{d\vec{r}_i \cdot d\vec{r}_j}{r_{ij}}. \quad (5)$$

The integrals in (5) are evaluated through the closed contours Γ_1 and Γ_2 along the current flow in the loops 1 and 2,

respectively. Besides, r_{ij} is the distance between the differential elements dr_i and dr_j . It should be noted that parasitic capacitances and the influence of the FR4 substrate were disregarded.

The calculation of the magnetic field $\vec{B}$ in this work was performed using the Biot-Savart law as in (6). The total field was determined as the summation of contributions from each turn of the Tx coil. It is further assumed that the current distribution across each coil turn is uniform, and the computed magnetic field represents an average value considered homogeneous across the transverse cross-section of the effective medium analyzed in this study.

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I_1 d\vec{r}_i \times \hat{r}}{r^2} \quad (6)$$

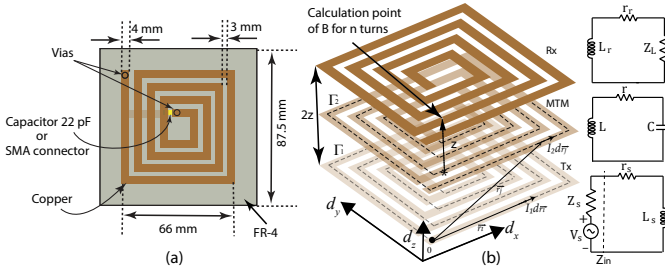


Fig. 1. Problem under analysis: (a) coil dimensions; (b) calculation - field B origin (*) and mutual inductance origin (•) in a WPT system and CM.

III. METHOD VALIDATION

Fig. 2 demonstrates the validation of the proposed method through experimental results. The CM method, implemented in Matlab®, was used to compute the currents in the Tx coil and the MTM cell, as well as the magnetic field $\vec{B}$. From these values, the parameters C_1 and C_2 were derived to calculate μ_{eff} using (4) at $z = 10$ mm. The experimental setup comprises a Tx coil, an MTM cell positioned parallel and axially at a distance of 60 mm, and a 22 pF lumped capacitor (C). Measurements of current and magnetic field were obtained using probes connected to a VNA, as shown in Fig. 2.

The results obtained with the method proposed in this work were compared with the impedance-based permeability extraction approach proposed in [4]. This formulation is based on the input impedance measurement of a loop without ($Z_{in}^{loop} = R_{in}^{loop} + j\omega X_{in}^{loop}$) and with the MTM ($Z_{in}^{loop+MTM} = R_{in}^{loop+MTM} + j\omega X_{in}^{loop+MTM}$):

$$\mu_{eff} = \Re - j\Im = \frac{X_{in}^{loop+MTM}}{X_{in}^{loop}} - j \frac{R_{in}^{loop+MTM}}{X_{in}^{loop}}. \quad (7)$$

The input impedance for both cases was evaluated through simulations in CST® Studio Suite using a frequency domain solver. The (7) was implemented in Matlab®, and the result shown in Fig. 2.

The overall results demonstrate the effectiveness of the proposed method in characterizing the system despite its simplifications. The close agreement between experimental and simulated data validates the method, particularly for predicting the operational frequency range and effective permeability. Minor discrepancies, mainly due to disregarded effects such as parasitic capacitance, dielectric losses, component tolerances,

and measurement probe influence, contributed to the slight displacement in the experimental results. Nonetheless, the method provides a reliable and computationally efficient approach for analyzing metamaterial-based systems.

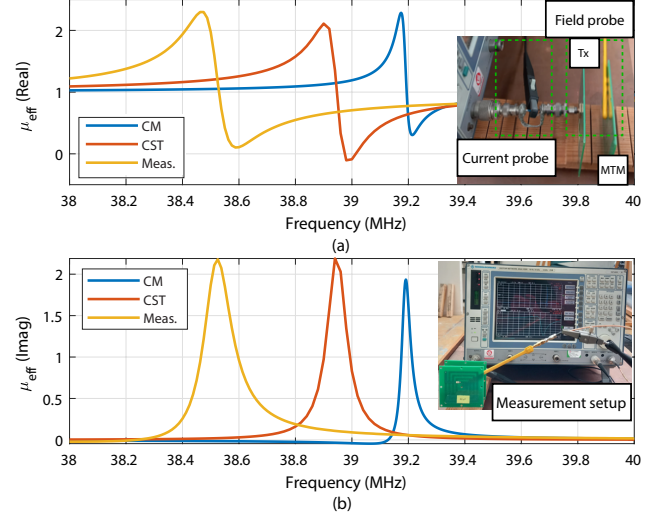


Fig. 2. Validation of the effective magnetic permeability of the Tx coil with the MTM: (a) real part and (b) imaginary part.

In terms of computational efficiency, the full-wave electromagnetic simulation on CST required approximately 2640 seconds for the WPT system with MTM, not including post-processing for effective permeability extraction. In contrast, the CM was solved in just 1.5 seconds, including postprocessing, achieving a speedup of nearly 1760 times. Both computations were performed on a computer with an Intel® i5-9400F processor, 16 GB of RAM, and a 2.90 GHz clock.

IV. CONCLUSION

This work proposes a model to reduce the computational cost of analyzing the effective permeability in WPT systems. By calculating circuit parameters, the flux density is determined using the Biot-Savart law, which otherwise demands significant computational resources in electromagnetic simulation software. Applied to a WPT system with a single metamaterial cell, the proposed approach accurately reproduces the system's behavior, yielding numerical results closely matching those from commercial simulations while reducing computational time by approximately 1760 orders of magnitude. Furthermore, the final version of this paper will present a complete WPT system with a more complex metamaterial and the implementation of substrate characteristics in the CM, demonstrating the model's capability for more advanced and realistic scenarios.

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