

2.5D Modeling of Twisted Submarine Cable

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Submarine power cables exhibit complex geometries designed to minimize electromagnetic losses, particularly through the counter-helical arrangement of core conductors and armor wires. Traditional standards, such as the IEC, often overestimate these losses, necessitating more precise modeling techniques. This paper introduces a novel 2.5D finite element method (FEM) model for analyzing the electromagnetic field profile and accurately estimating losses in submarine cables

Index Terms—Armor losses, Cable twisting, Finite element, Three-core submarine cable, 2.5D model

I. INTRODUCTION

In submarine cables, the conductors' pitch helps minimize losses in the armor, usually oriented in opposition to the armor's laying pitch. The helical lay of the armor wires and core conductors performs a canceling effect of the induced current in the armor. Because of the complex composition of submarine cables, many authors have shown that the IEC Standard greatly overestimates losses on submarine cables [1] [2] [3]. This work proposes a 2.5D FEM model for calculating electromagnetic field profile and estimating submarine cable losses, that is a two-dimensional FEA where some aspects of the three-dimensional model are incorporated.

II. THE MAGNETIC VECTOR POTENTIAL FORMULATION

The solution of quasi-static magneto dynamic problems through the method of finite elements is generally accomplished through a magnetic vector potential formulation of the Maxwell Equations. This formulation is mainly favored because of its capability to bypass field discontinuities at the interface of two electromagnetic domains. The magnetic vector potential $\mathbf{A}$ is defined in terms of the magnetic induction $\mathbf{B}$ as

$$\mathbf{B} = \vec{\nabla} \times \mathbf{A} \quad (1)$$

Both quasi-static Ampère law and Faraday's law can be combined to obtain the general formulation for the magnetic vector potential as

$$\vec{\nabla} \times \mu^{-1} \vec{\nabla} \times \mathbf{A} - j\omega_e \sigma \mathbf{A} = \mathbf{J}_s \quad (2)$$

Where $\mathbf{J}_s$ is the source current density, μ is the permeability, σ is the conductivity and ω_e is the electric frequency. The zero-divergent vector potential is able to be obtained by a gauged transformation, where, in this case, the equations are decoupled and can be easily solved through the Finite Element Method using scalar basis functions. The PDE system is presented as

$$\begin{cases} -\frac{\partial^2 A_x}{\partial x^2} - \frac{\partial^2 A_x}{\partial y^2} - \frac{\partial^2 A_x}{\partial z^2} - \sigma j\omega_e \mu A_x = J_{s_x}, \\ -\frac{\partial^2 A_y}{\partial x^2} - \frac{\partial^2 A_y}{\partial y^2} - \frac{\partial^2 A_y}{\partial z^2} - \sigma j\omega_e \mu A_y = J_{s_y}, \\ -\frac{\partial^2 A_z}{\partial x^2} - \frac{\partial^2 A_z}{\partial y^2} - \frac{\partial^2 A_z}{\partial z^2} - \sigma j\omega_e \mu A_z = J_{s_z}. \end{cases} \quad (3)$$

III. PERFECT HELICAL SYMMETRY

Firstly, the equations are developed for a perfect helical symmetry (a single laying pitch). Given the cartesian coordinate system $\{x, y, z\}$, a new auxiliary coordinate system $\{r, \theta, \tau\}$ is used to represent the helical movement, such that we have

$$x = r \cos(\theta + \omega \tau), \quad y = r \sin(\theta + \omega \tau), \quad z = \tau. \quad (4)$$

Where ω is the twist angular frequency.

A. Helical Symmetry

For a helically twisted, infinitely long cable, the problem presents a particular symmetry. This means the electromagnetic field at any point in the three-dimensional space can be related to the field solution in a single two-dimensional cross-section of the cable. This is translated to

$$A_x(r, \theta, \tau) = A_x(r, \theta, 0) \cos(\omega \tau) - A_y(r, \theta, 0) \sin(\omega \tau), \quad (5)$$

$$A_y(r, \theta, \tau) = A_x(r, \theta, 0) \sin(\omega \tau) + A_y(r, \theta, 0) \cos(\omega \tau), \quad (6)$$

$$A_z(r, \theta, \tau) = A_z(r, \theta, 0). \quad (7)$$

and, consequently,

$$\frac{\partial A_x}{\partial \tau} = -\omega A_y, \quad \frac{\partial A_z}{\partial \tau} = 0, \quad \text{and} \quad \frac{\partial A_y}{\partial \tau} = \omega A_x. \quad (8)$$

From multi-variable calculus theory, the derivatives in relation to τ may be rewritten in cartesian coordinates and grouped to obtain the second-order derivatives in relation to z . For the x , y and z component, we have

$$\begin{aligned} \frac{\partial^2 A_x}{\partial z^2} &= \omega^2 y^2 \frac{\partial^2 A_x}{\partial x^2} + \omega^2 x^2 \frac{\partial^2 A_x}{\partial y^2} - 2\omega^2 xy \frac{\partial^2 A_x}{\partial xy} - \omega^2 x \frac{\partial A_x}{\partial x} - \omega^2 y \frac{\partial A_x}{\partial y} \\ &\quad - \omega^2 A_x - 2\omega^2 y \frac{\partial A_y}{\partial x} + 2\omega^2 x \frac{\partial A_y}{\partial y}, \end{aligned} \quad (9)$$

$$\begin{aligned} \frac{\partial^2 A_y}{\partial z^2} &= \omega^2 y^2 \frac{\partial^2 A_y}{\partial x^2} + \omega^2 x^2 \frac{\partial^2 A_y}{\partial y^2} - 2\omega^2 xy \frac{\partial^2 A_y}{\partial xy} - \omega^2 x \frac{\partial A_y}{\partial x} - \omega^2 y \frac{\partial A_y}{\partial y} \\ &\quad - \omega^2 A_y + 2\omega^2 y \frac{\partial A_x}{\partial x} - 2\omega^2 x \frac{\partial A_x}{\partial y}, \end{aligned} \quad (10)$$

$$\frac{\partial^2 A_z}{\partial z^2} = \omega^2 y^2 \frac{\partial^2 A_z}{\partial x^2} + \omega^2 x^2 \frac{\partial^2 A_z}{\partial y^2} - 2\omega^2 xy \frac{\partial^2 A_z}{\partial xy} - \omega^2 x \frac{\partial A_z}{\partial x} - \omega^2 y \frac{\partial A_z}{\partial y}. \quad (11)$$

The equations 9, 10 and 11 are substituted in the PDE system 3 to remove all dependencies of the variable z .

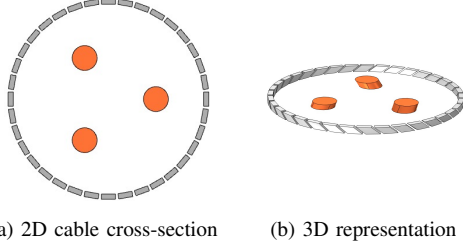


Fig. 1: Representation of a submarine cable, showing three phase conductors and an outer armor.

IV. CONSIDERING BOTH THE ARMOR AND THREE-PHASE TWIST

As mentioned, in reality, the armor and three-phase conductors are twisted in opposite directions to one another. The work done in [4] has shown that, for this condition, a very short periodicity exists due to the constant alignment of phase conductors and armor filaments. Because of this, even if the relations 8 are not exactly met, on average, they still remain true.

A. Weak Formulation

Using internal product notation $\langle \cdot, \cdot \rangle$ where $\langle A, B \rangle = \iint_{\Omega} AB d\Omega$, the weak formulations are

$$\begin{aligned} & \langle (1 + \omega^2 y^2) \frac{\partial A_x}{\partial x}, \frac{\partial \phi}{\partial x} \rangle + \langle (1 + \omega^2 x^2) \frac{\partial A_x}{\partial y}, \frac{\partial \phi}{\partial y} \rangle - 2\omega^2 \langle y \frac{\partial A_x}{\partial y}, \frac{\phi + x \partial \phi}{\partial x} \rangle \\ & - \omega^2 \langle x \frac{\partial A_x}{\partial x}, \phi \rangle - \omega^2 \langle y \frac{\partial A_x}{\partial y}, \phi \rangle - 2\omega^2 \langle y \frac{\partial A_y}{\partial x}, \phi \rangle + 2\omega^2 \langle x \frac{\partial A_y}{\partial y}, \phi \rangle \\ & + j\mu\omega_e \sigma \langle A_x, \phi \rangle = \langle \mu J_{sx}, \phi \rangle, \end{aligned} \quad (12)$$

$$\begin{aligned} & \langle (1 + \omega^2 y^2) \frac{\partial A_y}{\partial x}, \frac{\partial \phi}{\partial x} \rangle + \langle (1 + \omega^2 x^2) \frac{\partial A_y}{\partial y}, \frac{\partial \phi}{\partial y} \rangle - 2\omega^2 \langle y \frac{\partial A_y}{\partial y}, \frac{\phi + x \partial \phi}{\partial x} \rangle \\ & - \omega^2 \langle x \frac{\partial A_y}{\partial x}, \phi \rangle - \omega^2 \langle y \frac{\partial A_y}{\partial y}, \phi \rangle + 2\omega^2 \langle y \frac{\partial A_x}{\partial x}, \phi \rangle - 2\omega^2 \langle x \frac{\partial A_x}{\partial y}, \phi \rangle \\ & + j\mu\omega_e \sigma \langle A_y, \phi \rangle = \langle \mu J_{sy}, \phi \rangle, \end{aligned} \quad (13)$$

$$\begin{aligned} & \langle (1 + \omega^2 y^2) \frac{\partial A_z}{\partial x}, \frac{\partial \phi}{\partial x} \rangle + \langle (1 + \omega^2 x^2) \frac{\partial A_z}{\partial y}, \frac{\partial \phi}{\partial y} \rangle - 2\omega^2 \langle y \frac{\partial A_z}{\partial y}, \frac{\phi + x \partial \phi}{\partial x} \rangle \\ & + \omega^2 \langle x \frac{\partial A_z}{\partial x}, \phi \rangle - \omega^2 \langle y \frac{\partial A_z}{\partial y}, \phi \rangle + j\mu\omega_e \sigma \langle A_z, \phi \rangle = \langle \mu J_{sz}, \phi \rangle, \end{aligned} \quad (14)$$

The weak formulation of the newly proposed method depends only on a single rotating frequency ω , which is the rotation of the three-phase power conductors, as they are the main source of electromagnetic fields and their angular twist creates the sporadic helical symmetry.

V. LOSS COMPARISON

The armor losses were obtained through a 2D model, a short periodic 3D model, and the proposed model for the cable shown in Figure 1, with geometric and electromagnetic parameters shown in Table I. The losses per cable meter are presented in Table II. The magnitude of A_z obtained from the 3D model and the proposed model are shown in Figure 2.

TABLE I: Geometric and electromagnetic parameters

Parameter	
Phase conductor radius	13.1 mm
Phase conductor axial distance	50 mm
Phase conductor conductivity	1E7 S/m
Phase conductor permeability	1
Phase Conductor lay pitch	0.5 m
Current excitation	90/0° A , 90/-120° A, 90/120° A
Armor filaments	36
Armor radius	102.8 mm
Armor filament dimension	5.6 mm x 15 mm
Armor conductivity	5E6 S/m
Armor permeability	1
Armor lay pitch	-0.5 m

TABLE II: Armor loss and deviation from 3D model.

Model	Losses (W/m)
2D model	0.22871
2.5D model	8.6644
3D short periodic model	9.1984

VI. CONCLUSION

For a twisted submarine cable, a 2.5D model was developed that accurately models the 3D electromagnetic behavior in two dimensions. The equations were obtained from an average model with perfect helical symmetry because of the short equivalent periodicity. The model was validated through the electromagnetic profile and armor loss calculation and showed great accuracy.

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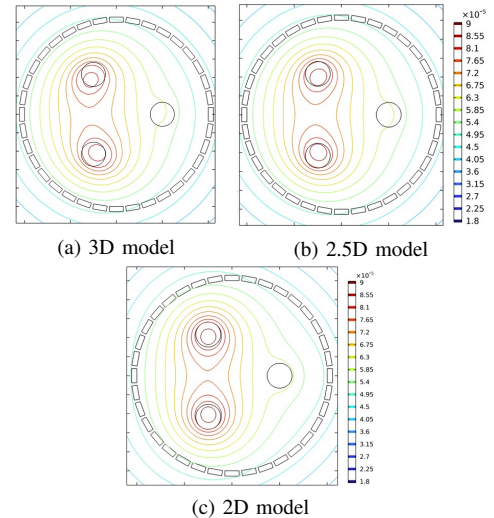


Fig. 2: Magnitude of A_z