

Applying FEM-DBCI to 3D Skin Effect Problems

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This paper describes how the hybrid FEM-DBCI method can be used to solve time-harmonic skin effect problems in three-dimensions. The domain of analysis is partitioned into two subdomains: conductors and free space. The conductors are meshed by means of tetrahedral edge elements and the electrical field is assumed as unknown. The free space is truncated by means of a fictitious boundary and is meshed by means of tetrahedral nodal elements where the reduced magnetic potential is the relative unknown. The two domains are treated separately by two FEM analyses which are applied alternatively in an iterative scheme.

Index Terms—Finite element method, integral equations, open-boundary problems, skin effect.

I. INTRODUCTION

THE skin effect problem has attracted the attention of numerous researchers in the field of computational electromagnetics, both in two and in three dimensions.

In [1] Konrad derives a single integrodifferential equation in term of the magnetic vector potential to solve 2D time-harmonic skin effect problems by means of the FEM. In [2] the authors combine the integrodifferential equation in [1] with the hybrid FEM-DBCI (Dirichlet Boundary Condition Iteration) to deal with the unbounded free space around the conductors.

In [3] Mayergoyz solves 3D time-harmonic skin effect problems by means of a boundary integral method which is derived by considering the electric field in the conductors and the reduced magnetic scalar potential in free space. In [4] Biro et alii compute 3D current driven skin effect using a current vector potential. In [5] Coperich et alii use the Partial-Element Equivalent-Circuit (PEEC) to compute skin effect currents.

In this paper we present a method to solve three dimensional skin effect problems in open boundaries. The method adopts the same mathematical formulation of [3], based on the adoption of the electric field in the conductor and of the reduced magnetic scalar potential in free space. Two separated meshes of tetrahedra are used to discretize the conductor and the free space around it, which has been truncated by means of a fictitious boundary. Edge elements are used in the conductor and nodal elements in the free space. The unknown Dirichlet condition on the truncation boundary is expressed by means of an integral equation, involving the free-space Green function. The major advantage of this method lies in its simplicity of implementation and in the avoidance of singular integrals in the integral equation, since the truncation boundary and the integration surface are geometrically separated.

The paper is organized as follows: in Sect. 2 the mathematical formulation is described. In Sect. 3 the numerical equations are derived based on FEM and FEM-DBCI. The solving iterative procedure is described in Sect 4, whereas its software implementation is highlighted in Sect. 5. A validation example is given in Sect. 6.

II. MATHEMATICAL FORMULATION

Consider a conductor which carries a time-harmonic total current $i(t)$, represented symbolically by the phasor I . The conductor has electric conductivity σ and magnetic permeability μ . Let V indicates the domain of the conductor, Γ its doubly connected surface and L a closed line passing (approximatively) through the centers of the various cross sections of the conductor (see Fig. 1).

The conductor is surrounded by an unbounded free space, extending to infinity. In order to apply the FEM, we introduce a closed truncation surface Γ_T which strictly includes the conductor. The free space inside Γ_T is denoted by V_0 , so that the interface between V and V_0 is the surface Γ .

At a point P_0 in V_0 , we define the magnetic field:

$$\bar{H}_0(P_0) = \frac{I}{4\pi} \oint_L \frac{d\bar{s} \times \bar{r}}{r^3} \quad (1)$$

where r is the distance of P_0 from the generic point P along L .

Then, the total magnetic field in free space can be written as:

$$\bar{H}(P_0) = \bar{H}_0(P_0) - \nabla\varphi(P_0) \quad (2)$$

where φ is the reduced magnetic scalar potential, satisfying the Laplace equation:

$$\nabla^2\varphi = 0 \quad (3)$$

By virtue of (1) and (2) the H field has zero rotation in V_0 , as appropriate. Inside the conductor the electric field E satisfies:

$$\nabla \times \nabla \times \bar{E} + j\omega\sigma\mu\bar{E} = 0 \quad (4)$$

The two partial differential equations (3) and (4) are linked by the following boundary conditions on Γ :

$$\hat{n} \cdot \nabla\varphi = \hat{n} \cdot \left(\bar{H}_0 + \frac{1}{j\omega\mu_0} \nabla \times \bar{E} \right) \quad (5)$$

$$\hat{n} \times \nabla \times \bar{E} = j\omega\mu\hat{n} \times (-\bar{H}_0 + \nabla\varphi) \quad (6)$$

which express the continuity of the normal component of the magnetic flux density and of the tangent component the magnetic field, respectively, through the surface Γ .

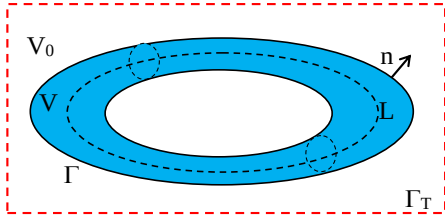


Fig. 1 – A conductor embedded in a free space truncated by the fictitious boundary Γ_T (dashed red line). L is the mean line in the conductor.

However, equations (3)-(6) are not sufficient to solve the problem, because the Dirichlet boundary condition on Γ_T is not known. This condition can be expressed by means of the integral:

$$\varphi(P_T) = \iint_{\Gamma} \left(\varphi(P) \frac{\partial G(P, P_T)}{\partial n} - G(P, P_T) \frac{\partial \varphi(P)}{\partial n} \right) dS \quad (7)$$

where P_T and P are points on Γ_T and Γ , respectively, G is the free-space Green function and n is the outward normal to Γ .

III. NUMERICAL IMPLEMENTATION

In order to solve the problem (3)-(7), the two subdomains V_0 and V are meshed by means tetrahedral nodal and edge elements, respectively. The two meshes share the same triangular faces on Γ . By applying FEM to (4) and (6), we obtain:

$$\mathbf{A}\mathbf{e} = \mathbf{b}_0 + \mathbf{F}\boldsymbol{\varphi} \quad (8)$$

where $\mathbf{A}$ is a square sparse matrix, $\mathbf{e}$ is the vector of the edge expansion coefficients of the electric field, $\mathbf{b}_0$ is a known vector due to field H_0 in (6), $\mathbf{F}$ is a sparse rectangular matrix due to potential φ in (6), and $\boldsymbol{\varphi}$ is the vector of the nodal values of φ in free space, except for those lying on Γ_T .

For the hybrid problem (3), (5) and (7) in free space we use the FEM-DBCI [2], so that:

$$\mathbf{M}\boldsymbol{\varphi} = \mathbf{k}_0 + \mathbf{C}\mathbf{e} + \mathbf{D}\boldsymbol{\varphi}_T \quad (9)$$

$$\boldsymbol{\varphi}_T = \mathbf{G}\boldsymbol{\varphi} \quad (10)$$

where $\mathbf{M}$ is a square sparse matrix, $\mathbf{k}_0$ is a known vector due to H_0 in (5), $\mathbf{C}$ is a matrix due to the field $\mathbf{E}$ in (5), $\mathbf{D}$ is a matrix due to the unknown Dirichlet boundary condition on Γ_T , $\boldsymbol{\varphi}_T$ is the vector of the nodal values of potential φ on this boundary and $\mathbf{G}$ is a dense rectangular matrix.

IV. ITERATIVE SOLUTION

The linear system (8), (9) and (10) can be solved iteratively as follow:

- evaluate the field $\bar{H}_0$ by means of (1) at the centers of the triangular faces lying on Γ ;
- select null first guesses for $\boldsymbol{\varphi}_T$ and $\mathbf{e}$;
- solve the FEM-DBCI subsystem (9)-(10) for $\boldsymbol{\varphi}$ and $\boldsymbol{\varphi}_T$;
- check convergence by computing the difference between the new and the old vectors $\boldsymbol{\varphi}_T$; if convergence is reached, halt the procedure; otherwise continue;
- solve the FEM subsystem (8) for $\mathbf{e}$, to obtain:
$$\mathbf{e} = \mathbf{A}^{-1}(\mathbf{b}_0 + \mathbf{F}\boldsymbol{\varphi}) \quad (11)$$
- update the known term $\mathbf{k}_0 + \mathbf{C}\mathbf{e}$ in (9) and go back to c).

V. SOFTWARE IMPLEMENTATION

The algorithm described in the previous section could be implemented in various ways, as for example by developing a computer code that deals with sub-meshes with different type of unknowns relative to edges and nodes.

However, we have opted for another implementation, where two separate main FEM programs solve the two subproblems: one operates on the interior problem by considering only the edge elements in the conductor, and the other operates on the exterior problem by considering only the nodal elements in the free space. The two main programs interact by means of (internal) files and semaphores. The data exchanged between the programs are the values of $\hat{n} \cdot \nabla \times \bar{\mathbf{E}}$ and $\hat{n} \times \nabla \varphi$ evaluated at the centers of the triangular faces of the tetrahedra lying on the interface Γ between conductor and free space.

VI. VALIDATION EXAMPLE.

An example is provided to validate the method previously described. It consists of a toroid copper conductor of square cross section 2×2 cm, internal radius $R_1 = 10$ cm. The torus center is the origin of the reference system and its axis is the z -axis. The torus carries a total current $I = 1$ A at a frequency $f = 1$ kHz.

The system exhibits axial symmetry, so that the results can be compared with those of a 2D analysis. The truncation boundary is selected homologous to the outer surface of the conductor at a distance of 2 mm. Due to symmetries, only one octant of the system is discretized by imposing homogeneous Dirichlet boundary conditions on the edges lying on the xz and yz planes. A natural homogeneous Neumann condition holds on the xy plane. The mesh consists of 43200 tetrahedra (13200 in air) and 56010 edges. The iterative procedure converges in 5 iterations with an end-iteration tolerance of 0.1 per cent.

The same problem is solved by taking into account its axial symmetry. A 2D mesh of 576 triangular elements is built which coincides with the tetrahedra faces on the xz plane. FEM-DBCI is used to solve this problem [2]. The values of the current density are computed at the centers of the triangles and compared. An average difference of 0.91 per cent is observed.

The method described in this paper has been implemented in ELFIN, a FEM code developed by the authors for scientific research [6].

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