

Periodic Boundary Conditions in Vector Nodal Meshless Method

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The purpose of this paper is to introduce the periodic boundary conditions in the Vector Nodal Meshless Method (VNMM). The VNMM constructs its approximations using shape functions based on the $H(curl)$ space and its projection verifies the delta Kronecker property such that the imposition of the periodic boundary conditions can be easily done. However, the imposition of the periodic condition in vector nodal modeling is different from that one well known in the scalar nodal analyses due to the vector nature of the problem. So, in this work a new approach which can take into account the vector nature of the functions and based in edge elements concept is proposed. The periodic boundary conditions allow take account the periodic symmetry of the electromagnetic devices in order to reduce the study domain. For validation, the TEAM 13 problem is solved with and a good performance of the VNMM is observed.

Index Terms— Periodic boundary condition, Periodic structures, Meshless method, Numerical methods, Electromagnetic fields.

I. INTRODUCTION

THE VECTOR NODAL MESHLESS METHOD (VNMM) is a numerical method proposed for approximation of vector fields in $H(curl)$ space. In VNMM, a set of nodes is distributed on the problem domain and its boundaries, and a unit vector is associated to each node. Like any meshless method, its shape functions are constructed without taking into account connectivity relationships between nodes. Besides, the VNMM shape functions verify the Kronecker delta property which facilitates the imposition of boundary conditions.

The characteristics of VNMM are attractive to solve problems with moving parts such as electrical machines. Furthermore, the inherent periodic symmetry of those machines allows the use of the periodic boundary conditions in order to reduce substantially the size of the computational domain. In 3-D analysis, this condition is especially important to reduce memory requirements and processing time.

The VNMM shape functions have been used in 2-D and 3-D problems [1]. However, the periodic boundary conditions for use in the vector nodal analyses have not been implemented yet. In this sense, the aim of this paper is to introduce the periodic boundary conditions in the VNMM context.

II. VECTOR NODAL MESHLESS METHOD

This section presents the VNMM approximation and its tri-dimensional shape functions. Given a set of nodes distributed on the problem domain Ω in $\mathbb{R}^3$ and its boundaries $\partial\Omega$, the approximation $\vec{u}^h$ of a given vector field $\vec{u}$ at point $\mathbf{x}$, belonging to Ω , is performed using an arbitrary subset of the nodes. These nodes, called support nodes, and their associated directions will be used to construct the VNMM shape functions. When Ω is composed of more than one material, the support nodes must belong to the same material or be located at the interface between the materials. The approximation of the $\vec{u}^h$ is written as:

$$\vec{u}^h(\mathbf{x}) = \sum_{i=1}^n \vec{N}_i(\mathbf{x}) c_i \quad (1)$$

where n is the number of support nodes, $\vec{N}_i$ are the vector shape functions and c_i are the coefficients of the projection of $\vec{u}^h$ in the directions of the respective nodes. The vector shape function $\vec{N}_i$ associated to the i -th support node is given by [1]:

$$\vec{N}_i = \beta_{1i} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \beta_{2i} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \beta_{3i} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \beta_{4i} \begin{bmatrix} 0 \\ -z \\ y \end{bmatrix} + \beta_{5i} \begin{bmatrix} z \\ 0 \\ -x \end{bmatrix} + \beta_{6i} \begin{bmatrix} -y \\ x \\ 0 \end{bmatrix} \quad (2)$$

where β_{ji} , with $j = 1, \dots, 6$ are the coefficients to be found. The coefficients are found forcing the projection of the i -th shape function in the vector direction associated to the k -th support node, such as [1]:

$$\vec{N}_i \cdot \vec{t}_k = \delta_{ik} \quad (3)$$

where $\vec{t}_k$ is the unit vector associated with the k th node and δ_{ik} is the Kronecker delta. If the shape functions satisfy (3), it is possible to impose the Dirichlet boundary conditions and the interface conditions directly.

III. PROBLEM FORMULATION

In order to validate the technique presented in this paper the TEAM problem 13 is solved. The behavior of the steel material is nonlinear and defined through a $\vec{B}(\vec{H})$ curve shown in [2]. The domain of interest, given by Ω bounded by the surface $\Gamma = \Gamma_g \cup \Gamma_h \cup \Gamma_p$, where Γ_g is the boundary with Dirichlet condition, Γ_h is the symmetrical boundary with Neumann condition and Γ_p are the boundaries with periodic conditions. The variational form is: given $v(\vec{B})$ find $\vec{A}$ such that:

$$\int_{\Omega} (\nabla \times \vec{A}) v(\vec{B}) (\nabla \times \vec{F}) d\Omega + \epsilon \int_{\Omega} \vec{A} \cdot \vec{F} d\Omega = \int_{\Omega} \vec{J} \cdot \vec{F} d\Omega \quad (4)$$

where $\vec{A}$ is the magnetic vector potential, $\vec{J}$ is the current density, $\vec{F}$ is the test function and v is the magnetic reluctivity. The term $\epsilon \int_{\Omega} \vec{A} \cdot \vec{F} d\Omega$ is a regularization technique for curl problems with divergence-free constraint [3].

IV. PERIODIC BOUNDARY CONDITION

To introduce the periodic boundary conditions in VNMM, consider the domain Ω in $\mathbb{R}^3$, represented in Fig. 1, where the nodes n_I, n_J, n_K and their respective vectors on the pair of periodic boundaries are highlighted. The geometric position of the nodes on the periodic surface Γ'_p is obtained by rotating the position of the nodes located in Γ_p around the z axis. This rotation defines an angle α between the surfaces so that the directions of the degrees of freedom of the nodes are rotationally symmetric between the periodic boundaries. In other words, the vector direction of the nodes on surface Γ'_p is obtained by mirroring the vector direction of the nodes in Γ_p .

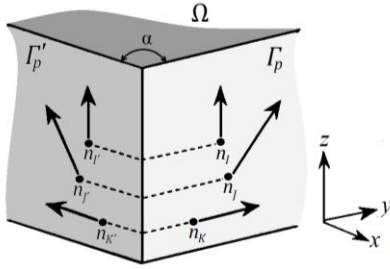


Fig.1. Three-dimensional domain with its periodic boundaries Γ_p ($n_I, n_J \in n_K$) and Γ'_p ($n'_I, n'_J \in n'_K$). Representation of the symmetry between the boundary nodes for application of the periodic boundary condition.

The magnitudes of the degrees of freedom along the symmetric nodes are the same, that is: $|A_I| = |A_{I'}|$, $|A_J| = |A_{J'}|$, $|A_K| = |A_{K'}|$, where $A_I, A_{I'}, A_J, A_{J'}, A_K, A_{K'}$ are projections of the potential vector $\vec{A}$ along the symmetric pair directions $n_I - n_{I'}, n_J - n_{J'}, n_K - n_{K'}$. The signs are assigned to each symmetric pair such that a degree of freedom located on one boundary point in the direction identical to that indicated by its symmetric partner while rotating about the z -axis through the angle α [4].

The imposition of periodic boundary conditions is done after all local contributions have been assembled into the global matrix. For this, the contributions related to the nodes on the boundary Γ_p ($n_I, n_J \in n_K$) must be associated with those on the boundary Γ'_p ($n'_I, n'_J \in n'_K$) and duly assembled into the global matrix. For an antiperiodic structure the procedure is similar, however, the contributions related to node $n_{I'}$ must be transferred to node n_I with opposite signs, since, in this case, nodes $n_{I'}$ and n_I are in a subdomain whose sources have opposite signs [5].

V. RESULTS

Considering both the symmetry and the periodicity of the TEAM 13 problem only $1/4$ of the geometry needs to be modeled. The problem is solved using the VNMM, with 46,293 nodes distributed in the domain Ω and six support nodes in the approximation. Successive approximations are applied to solve nonlinearity and it converges after 12 iterations with a criterion of 10^{-6} .

The magnetic flux density is evaluated in the three regions along the steel. Fig. 2 plots the results obtained by VNMM method approximation and the measured data provided by [2]. It is observed a good agreement between measurements and VNMM approximation.

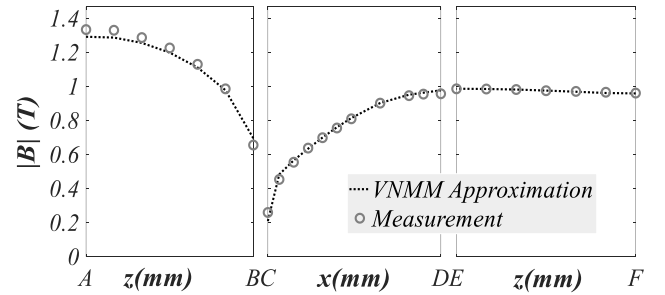


Fig. 2. Numerical results of TEAM 13 with 1,000-AT coil current excitation.

VI. CONCLUSION

The obtained results show that the periodic boundary conditions can be successfully incorporated by VNMM method. More details about the technique and the algorithm for implementing the periodic boundary conditions in VNMM will be presented in the extended version.

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