

Analytical Formulation for Eddy Current Losses in Conducting Plates under Normal Magnetic Field Excitation

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This work presents an analytical formula derived from Maxwell's equations for calculating eddy current losses induced in conducting plates under a normal (radial) magnetic field excitation. The formula is validated using three-dimensional (3-D) finite element (FE) simulations for a normal magnetic field excitation and different materials utilized in structural and shielding parts. The novelty of this work includes consideration of the eddy current reaction field in the analytical formulation. Further, the proposed expression and the classical eddy current loss formula are used to calculate eddy losses due to magnetic fields with tangential and normal components.

Index Terms—Eddy (stray) loss, electromagnetic shield, finite element simulation, normal excitation, power transformer.

I. INTRODUCTION

Analysis of eddy current (stray) losses are critical to optimize structural elements and shields of electromagnetic devices [1]. Analytical and numerical methods have been employed to compute eddy current losses in structural components [2]-[4]. An eddy current loss curve for a normal magnetic field excitation case is given in [1] using two-dimensional (2-D) FE simulations. Depending on tangential or normal magnetic field excitation, the eddy current losses in a material differ. In the case of tangential magnetic field excitation, the eddy current losses can be calculated using the classical theory of eddy currents [1]. Literature to compute eddy current losses for normal field excitation is scarcely available.

In this work, for normal field excitation, a closed-form expression for eddy current losses induced in structural and shielding plates used in transformers is derived using Maxwell's equations. The proposed formulation is applied to compute the eddy current losses produced by a normal field in plates made of non-magnetic stainless steel (SS), aluminum (Al), copper (Cu), and mild steel (MS), which are common materials employed for structural elements and electromagnetic shields. The analytical formula is validated and compared with 3-D FE simulations. Further, in a magnetostatic simulation, the proposed formulation is used along with the classical eddy current theory to compute losses due to magnetic fields with both tangential and normal components.

II. PROPOSED ANALYTICAL FORMULATION

Many devices commonly have metallic plates under the magnetic field excitation with tangential and normal components on their surface. The configuration of such a problem domain is shown in Fig. 1 (a). The eddy currents due to the normal fields (directed along z-axis) are determined in this work by solving the diffusion equation [3] for the problem domain, see Fig. 1 (b).

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \frac{\partial^2 H_z}{\partial z^2} = j\omega\mu\sigma H_z \quad (1)$$

In (1), ω = angular frequency (rad/s) of excitation, μ and σ are the relative permeability and electrical conductivity of the plate material. The general solution of (1) that corresponds to the variation of fields in z-direction is

$$H_z = C_1 e^{\gamma z} + C_2 e^{-\gamma z}, \gamma = \frac{1+j}{\delta}, \delta = \sqrt{\frac{2}{\omega\mu\sigma}} \quad (2)$$

C_1 and C_2 are constants of integration, determined using the boundary conditions: $H_z = H_m$ at $z = 0$ and $H_z \rightarrow 0$ as $z \rightarrow \infty$. Here, H_m is the maximum magnetic field on the surface of the plate at $z = 0$. The final expression of H_z is

$$H_z = H_m e^{-\gamma z} \quad (3)$$

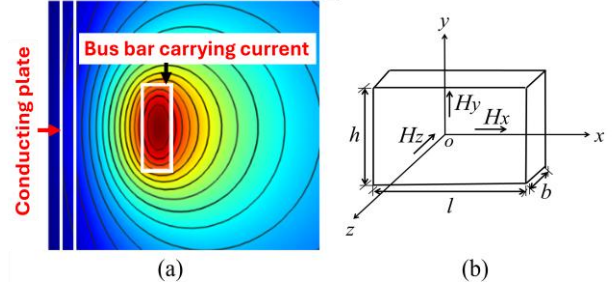


Fig. 1: a) Bus bar-plate configuration with both tangential and normal magnetic field excitation, b) Metallic plate under normal field excitation.

The magnetic flux in the z direction gets diffused into the x and y directions in the plate, which can be derived by solving, $\nabla \cdot \mathbf{H} = 0$

$$\frac{\partial H_z}{\partial x} + \frac{\partial H_z}{\partial y} + \frac{\partial H_z}{\partial z} = 0 \quad (4)$$

Considering the components of $\mathbf{H}$ at the center (origin) as zero (by symmetry), one obtains

$$H_x = \frac{\gamma}{2} H_m x e^{-\gamma z} \text{ and } H_y = \frac{\gamma}{2} H_m y e^{-\gamma z} \quad (5)$$

Using $\nabla \times \mathbf{H} = \mathbf{J}$, the current density components are

$$J_x = -\frac{\partial H_y}{\partial z} = \frac{j\omega\mu\sigma}{2} H_m y e^{-\gamma z} = j \frac{H_m}{\delta^2} y e^{-\frac{(1+j)z}{\delta}} \quad (6a)$$

$$J_y = \frac{\partial H_x}{\partial z} = -\frac{j\omega\mu\sigma}{2} H_m x e^{-\gamma z} = -j \frac{H_m}{\delta^2} x e^{-\frac{(1+j)z}{\delta}} \quad (6b)$$

These circular eddy currents on the surface of the material oppose the incident field. The total eddy current loss is

$$P_e = \frac{1}{2\sigma} \int_0^b \int_{-h/2}^{h/2} \int_{-l/2}^{l/2} (|J_x|^2 + |J_y|^2) dx dy dz \quad (7)$$

$$= lh(l^2 + h^2) \frac{\omega\mu H_m^2}{96\delta} (1 - e^{-2b/\delta})$$

The normal component of $\mathbf{B}$ is continuous at the material interface. So, the loss expression is formulated in terms of $\mathbf{B}$ as

$$P_e = lh(l^2 + h^2) \frac{\omega B_m^2}{96\mu\delta} (1 - e^{-2b/\delta}) \quad (8)$$

Here, B_m is the maximum value of $\mathbf{B}$. The eddy current reaction field, B_e , is responsible for reducing the incident field. These induced eddy currents due to normal field excitation will generate magnetic fields in x , y , and z directions and are determined using Biot–Savart's law.

$$(B_e)_x = \int_{-h/2}^{h/2} \frac{\mu_0 \left(\int_0^b J_z dz \right) dy \hat{a}_x \times \hat{a}_y}{2\pi y} \quad (9a)$$

$$= \hat{a}_z \frac{\mu_0 h}{2\sqrt{2}\pi\mu\delta} (B_{z=0} - B_{z=b}) \angle 45^\circ$$

$$(B_e)_y = \hat{a}_z \frac{\mu_0 l}{2\sqrt{2}\pi\mu\delta} (B_{z=0} - B_{z=b}) \angle 45^\circ \quad (9b)$$

$$B_e = (B_e)_x + (B_e)_y = \hat{a}_z \frac{\mu_0(l+h)}{2\sqrt{2}\pi\mu\delta} (B_{z=0} - B_{z=b}) \angle 45^\circ \quad (9c)$$

The magnetic flux density excitation after considering the eddy current reaction field is

$$B' \angle \phi = B_m - B_e = \frac{B_m}{1 + \frac{\mu_0(l+h)}{2\sqrt{2}\pi\mu\delta} (1 - e^{-\gamma b}) \angle 45^\circ} \quad (10)$$

The final expression of the eddy current loss by considering the effect of the reaction field in (8) is

$$P_e = lh(l^2 + h^2) \frac{\omega B_m^2}{96\mu\delta} \frac{(1 - e^{-2b/\delta})}{\left| 1 + \frac{\mu_0(l+h)}{2\sqrt{2}\pi\mu\delta} (1 - e^{-\gamma b}) \angle 45^\circ \right|^2} \quad (11)$$

III. VALIDATION OF THE PROPOSED FORMULATION USING FEM APPROACH

3-D FE simulations are performed to validate the analytical formula (11). A normal magnetic field is produced using the boundary conditions shown in Fig. 2, and the eddy current losses in a plate with four different materials are evaluated.

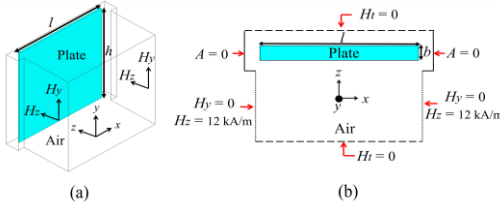


Fig. 2: a) Isometric view of the 3-D FE model, b) top view of the FE model with boundary conditions

The boundary condition of $H_t = 0$ is imposed on the two faces parallel to the xy plane and the tangential field boundary conditions, $H_y = 0$ and $H_z = 12$ kA/m, are applied on the faces parallel to the yz plane. These boundary conditions create a normal magnetic field of $H_z = 12$ kA/m which produces $B_m = 6.6$ mT (RMS value) on the surface of the plate. In addition, homogeneous Dirichlet boundary conditions ($A = 0$) are applied to the remaining faces of the box (as shown in Fig. 2 (b)). Time-harmonic analyses at 50 Hz are performed to compute the eddy current losses. The plate has a length $l = 1.2$ m and height $h = 1.2$ m. The thickness b is varied in the FE simulations for MS, SS, Al, and Cu plates to compute the eddy current losses. Table I shows the properties of these materials. Fig. 3 (a) shows the distribution of flux density in the MS plate

due to the normal field. Fig. 3 (b) shows a comparison of the losses computed using the 3D FE simulations and the proposed formulation (11), confirming its accuracy.

TABLE I: MATERIAL PROPERTIES

Property	Mild Steel (MS)	Aluminum (Al)	Stainless Steel (SS)	Copper (Cu)
μ_r	100	1	1	1
σ (Sm ⁻¹)	7×10^6	29×10^6	1.136×10^6	58×10^6
δ (mm)	2.69	13.2	66.78	9.34

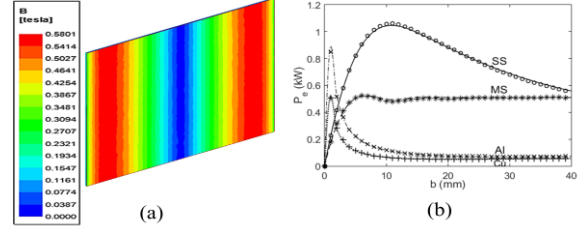


Fig. 3: a) Magnetic flux density in the MS plate, b) eddy current loss curves obtained using (11) (solid curves) and using the 3D FE simulation (markers)

Further, (11) can be used with the classical eddy current loss formula to estimate the losses in conductors when the incident magnetic field has both normal and tangential components. This is demonstrated using the configuration shown in Fig. 1a, in which fields generated by the current-carrying bar have tangential and normal components. The field components on the conductor surface are determined using a magnetostatic FEM simulation. The eddy loss due to the normal field component is calculated by using (11), and the classical eddy loss formula is used for the tangential component. The computed results are compared with the losses determined using a 2D time-harmonic FE simulation for the same problem domain. The comparison in Table II shows good accuracy for the approach employing both methods.

TABLE II: COMPARISON OF EDDY CURRENT LOSSES

Method	Al plate $b = 10$ mm	Al plate $b = 20$ mm	Cu plate $b = 10$ mm	Cu plate $b = 20$ mm
Formula (11) + classical formula	62 W	47 W	37 W	33 W
2-D time harmonic FE simulations	76 W	52 W	41 W	36 W

IV. CONCLUSIONS

An analytical formula to calculate the eddy current losses for normally incident magnetic fields is presented in this work. A closed-form analytical solution is proposed based on Maxwell's equations to study loss variations with plate dimensions and properties. Results of 3-D FE analysis are presented, showing the validation of the proposed formula. Furthermore, this formulation is used along with the classical eddy current theory (for tangential fields) to calculate eddy current losses in structural plates due to an incident field having normal and tangential components. The patterns of the eddy current loss variations with plate thickness match closely with those determined from the 2-D FE analysis.

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