

Slot Effects in 2D Axial Flux motors via Conformal Mapping

Giovanni Landi¹, Luca Sani¹, and Antonino Musolino¹

¹University of Pisa, 56122 Pisa, Italy, giovanni.landini@phdunipi.it

¹University of Pisa, 56122 Pisa, Italy, luca.sani@unipi.it

¹University of Pisa, 56122 Pisa, Italy, antonino.musolino@unipi.it

This paper investigates the magnetic field distribution in the air gap of a single-stator, single-rotor axial flux permanent magnet (AFPM) machine. The analysis simplifies the system by linearizing the device at the machine's mean radius. A quasi-static analytical no-load model is developed using the Fourier series method to evaluate the magnetic field generated by a traditional North-South array. Furthermore, the influence of slotting effects on the magnetic field waveform is analyzed by implementing the Schwarz-Christoffel conformal transformation. This approach provides a systematic methodology to quantify and assess the distortions caused by slotting, contributing to a more accurate characterization of the phenomena within the context of AFPM machines.

Index Terms—Axial flux, no-load, quasi-static, Schwarz-Christoffel, scalar potential

I. INTRODUCTION

AIR-GAP magnetic field is a fundamental parameter for enhancing the performance of electrical machines. Analytical methods allow for a deeper and more immediate understanding of electromagnetic phenomena while adopting necessary simplifications compared to finite element (FEM) models.

Several studies provide analytical approaches based on vector and scalar magnetic potentials [1], [2]. A relevant contribution in [3] investigates a 2D linear machine with segmented Halbach arrays, providing a general framework for the evaluation of magnetic fields. This methodology can be adapted to axial flux machines, such as the single-stator, single-rotor Torus topology, through a simplified quasi-3D modeling approach. This approach divides the disk machine into multiple linear slices, applying suitable boundary conditions to ensure field continuity across the model [4].

Under no-load, quasi-static conditions, Fourier-based analytical methods are particularly effective in evaluating the magnetic fields, even for configurations with complex magnet arrangements such as Halbach arrays [5]. In this analysis a traditional north-south (NS) pole configuration is considered, where a single axially magnetized permanent magnet is placed along each pole pitch τ . The proposed analytical model simplifies the analysis of a Torus-type axial flux machine by linearizing the system at the mean radius and imposing appropriate boundary conditions [6].

However, the existing framework in [3], while comprehensive with respect to magnetization, does not account for the effects of slotting—a critical phenomenon in the analysis of practical electrical machines [7]. To address this limitation, a complex permeability function λ is introduced, which is derived analytically through conformal mapping, specifically employing the Schwarz-Christoffel transformation, as proposed in [8]. This extension significantly enhances the analytical model by incorporating slot effects and improving its agreement with real-world physical conditions.

II. ANALYTICAL MODELING

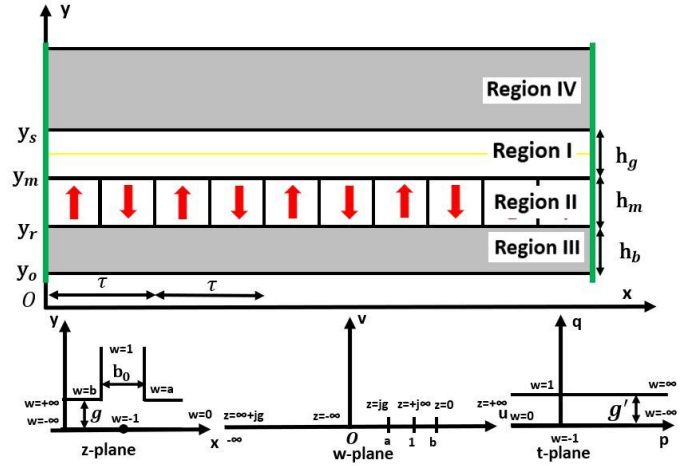


Fig. 1. Analytical model for NS AFPM:(top) no-load slotless model; (bottom) conformal mapping for slotting

A. No-load slotless model

Fig. 1(top) shows the no-load slotless model for a NS AFPM, highlighting the different subdomains within the system, the periodic boundary conditions (in green), and the line in the air-gap (in yellow) where the fields are evaluated. For each subdomain, assuming a Laplacian solution form, material linearity, homogeneous magnetization, and neglecting hysteresis and eddy current losses due to the quasi-static condition, the scalar potential φ can be written as follows:

$$\varphi_n = \sum_{k=1,3,5,\dots}^{\infty} (A_{nk} e^{k \frac{\pi}{\tau} y} + B_{nk} e^{-k \frac{\pi}{\tau} y}) \cos \left(k \frac{\pi}{\tau} x \right) \quad (1)$$

where n is the region number, A_{nk} and B_{nk} are constants determined by applying the Neumann and Dirichlet boundary conditions for each region. The fields B_x and B_y are then

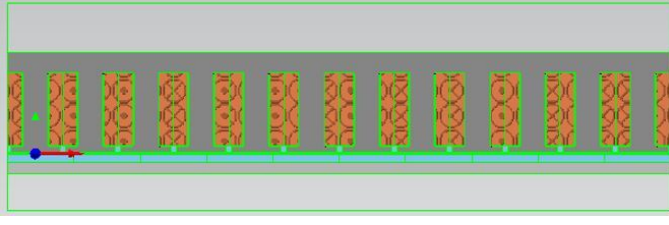


Fig. 2. FEM linear model for Torus machine

determined by solving the corresponding six-by-six system of equations, which assumes:

$$H = -\nabla\varphi \quad (2)$$

B. No-load slotted model

To incorporate the slotting effect, the Schwarz-Christoffel conformal transformation is employed. Fig.1(bottom) depicts the transformations needed to convert a slotless domain into a slotted domain, under the assumptions of infinitely deep slots and no mutual interaction between adjacent slots. These assumptions allow for an analytical approach to coordinate transformations, mapping the physical domain into a canonical domain where the solution is already established in Sec.II-A. More in detail:

$$z = -\frac{2g}{\pi} \left(\operatorname{atanh}\left(\frac{p}{b}\right) - \operatorname{atanh}(p) + \frac{\operatorname{atan}\left(\frac{p}{\sqrt{b}}\right)(b-1)}{\sqrt{b}} \right) \quad (3)$$

$$t = \frac{g \log(w)}{\pi} \quad (4)$$

where:

$$p^2 = \frac{b-w}{a-w} \quad (5)$$

$$a = \frac{1}{b} \quad (6)$$

$$b = \left(\frac{b_0}{2g} + \sqrt{\frac{b_0^2}{4g^2} + 1} \right)^2 \quad (7)$$

For the field conversion from the domain t to the z -plane, it is necessary to use the complex permeability λ , defined as:

$$\lambda = \frac{\partial t}{\partial z} = \frac{\partial t}{\partial w} \frac{\partial w}{\partial z} \quad (8)$$

By periodicizing the function λ with respect to the number of slots N_s , the field values B_z in complex plain z can be obtained as follows:

$$B_z = B_t \lambda^* = (B_{x,t} + jB_{y,t})(\lambda_a - j\lambda_b) \quad (9)$$

where:

$$B_{z,x} = B_{x,t}\lambda_a + B_{y,t}\lambda_b \quad (10)$$

$$B_{z,y} = B_{x,t}\lambda_a - B_{y,t}\lambda_b \quad (11)$$

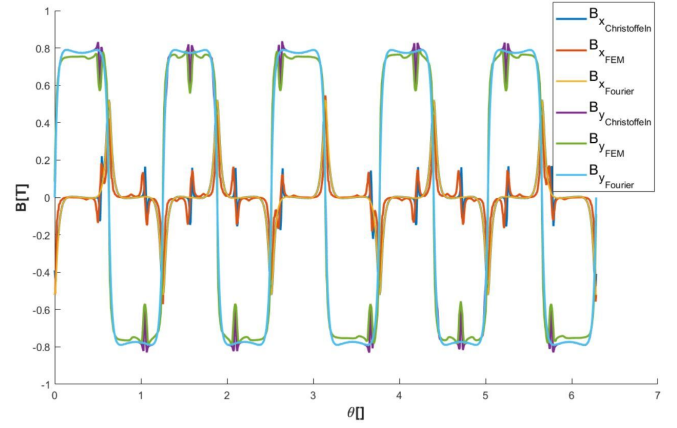


Fig. 3. B comparison in terms of FEM, no-load Fourier based and no-load Christoffeln model

TABLE I
TORUS MAIN MODEL PARAMETER

$R_{in}[\text{mm}]$	30	$R_{out}[\text{mm}]$	45
p	10	N_s	12
$h_m[\text{mm}]$	2.5	$h_b[\text{mm}]$	4
$h_g[\text{mm}]$	1	$b_0[\text{mm}]$	2
Magnet material	N_{48}	Stator and yoke material	Fe_{370}

III. RESULTS

Fig. 2 presents the FEM model used to linearize the Torus machine at the mean radius, with the geometric parameters detailed in Table III. Fig.3 compares the flux density waveform at middle gap, revealing "dips" in the analytical model incorporating the conformal transformation. These dips, which are absent in the initial model, accurately represent the slot ripple and are consistent with the equivalent FEM model. Moreover, the presence of these dips contributes to eddy current losses in the rotor, highlighting their impact on machine performance.

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