

A Methodology of Evaluation the Magnetic Vector Potential Around the Transmission Lines Using FEM and Adjoint Sensitivity

A. L. Paganotti ^{1,2}, R. R. Saldanha ¹, M. M. Afonso ², B. A. M. Duane ² and A. C. Lisboa¹

¹Graduate Program in Electrical Engineering, *Universidade Federal de Minas Gerais*, Belo Horizonte, Brazil,

²Electrical Engineering Postgraduate Program, *UFSJ-CEFET-MG*, Belo Horizonte, Brazil,

paganotti@cefetmg.br, rodney@cpdee.ufmg.br, marciomatias@cefetmg.br, bellduani@gmail.com, adrianochaves@cefetmg.br

In this paper, a new methodology is proposed for the sensitivity analysis of the magnetic vector potential near the transmission lines and the associated magnetic flux density. This potential is modeled using the finite element method and the adjoint method, which is concerned with the geometric positions of the cables. Methods based on gradient information have been used for this sensitivity of the objective function during the optimization process with reduced computational efforts. The adjoint sensitivity is not dependent on the number of interesting design variables. The numeric model of the transmission lines can be adopted during the optimization process without compromising the required computational processing time using the adjoint method. This new approach based on adjoint sensitivity of the magnetic vector potential near the overhead transmission lines can achieve new geometries with high surge impedance loading (HSIL).

Index Terms—Adjoint method, Magnetic vector potential, Finite element method, Sensitivity analysis.

I. INTRODUCTION

THE adjoint method has been used in the optimization of electrical engineering problems since the 1970s. It is a very useful tool for sensitivity analysis and the optimization of electromagnetic problems with high dimensions [1].

The optimization of transmission line (TL) geometries has been investigated by researchers and energy companies around the world. The objective has been to achieve TL's with high surge impedance loading (HSIL) while reducing dimensions and minimizing electromagnetic field imbalance. This problem has been addressed using analytical models of TL's based on symmetric geometries. Deterministic methods, such as those based on gradient information and evolutionary strategies, have been employed in these studies, with the focus on improving SIL, reducing electromagnetic fields, and achieving other objectives [2], [3].

The finite element method (FEM) modeling makes it possible to explore new configurations of TL's that could not be obtained using analytical models. The use of the adjoint method in low-frequency electromagnetic problems to obtain the sensitivity of the magnetic vector potential with respect to conductor positions is not available in the literature. This approach aims to explore new possibilities by using other state variables in low-frequency electromagnetic problems through the adjoint method.

In this work, a new sensitivity of the magnetic vector potential using the adjoint-analytic method with respect to the cable coordinates is obtained. A numerical model of the TL's is developed, and the magnetic vector potential around it is calculated. The adjoint sensitivity methodology enables methods based on gradient information to obtain the sensitivity of the objective function with reduced computational costs compared to the central finite difference (CFD) method. New geometries with high surge impedance loading (HSIL) are achieved.

II. TRANSMISSION LINE MODELING

The scalar complex electric potential and the magnetic vector potential around the TL are calculated using the Ampère-Maxwell law [4]. For the 2D problem, the equations that describe these electromagnetic fields are generalized by [4]:

$$\frac{\partial}{\partial x} \beta \frac{\partial \phi}{\partial x} + \frac{\partial}{\partial y} \beta \frac{\partial \phi}{\partial y} = b \quad (1)$$

where for the quasi-magnetostatic problem, ϕ is the magnetic vector potential A ($V.m/s$) and $\beta = (1/\mu)$ is the magnetic reluctance of the domain and $b = J$ is given by the source currents involved [4]. Similarly, the equation (1) is used for the quasi-electrostatic problem, where ϕ is the electric scalar potential (V) and $\beta = (\sigma + j\omega)$ are the dielectric properties of the domain (electric conductivity and electric permittivity) and $b = 0$. A circular domain truncated at a radius with five times the highest height of the bundles is adopted [4]. The boundary conditions are defined in the borders Γ_1 , Γ_d , and Γ_{int} , as shown in Fig. 1.

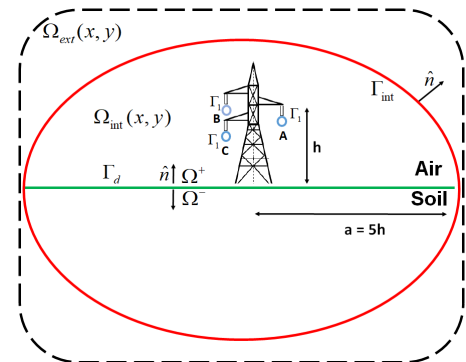


Fig. 1. Boundary value problem: Γ_1 Dirichlet boundary on the cable nodes, Γ_{int} Neumann boundary on the upper limit of the domain, Γ_d interface between air and soil, Ω^+ domain formed by air and Ω^- domain formed by the soil, h the highest height of the conductor.

The Γ_{int} is a homogeneous Neumann condition imposed on the upper limit of the domain. The electric potentials are imposed on the nodes of the cables using Dirichlet boundary conditions (Γ_1). For the solution of the quasi-magnetostatic problem, the frontiers of the domain and the inner parts of the cables are considered during the FEM discretization. The homogeneous current densities of the conductors are given by $J_{cable_k} = |I_{phase_k}| \angle \delta_k^\circ / S_{cond_k}$, where I_{phase_k} (A) is the current in the cable k , and δ_k° is the angle of the three-phase equilibrated system and S_{cond_k} (m^2) is the section area of the cables [4]. For the sake of simplicity, the shield wires are not evaluated.

The magnetic vector potential is calculated and the soil is taken as a homogeneous medium with a constant resistivity. The Galerkin method is used to get the weak form of the partial differential equations that describe the magnetic vector potential. The global matrix equations system obtained after 2D discretization by the FEM yields the magnetic potential vector [4]:

$$[K_A][A] = [b_A] \quad (2)$$

where $K_A(x, y)$ is the $(N \times N)$ matrix, N is the number of the nodes in the domain, $A(x, y)$ is the N -vector of state variables and b_A is the excitation vector size N . The sensitivity of $A(x, y)$ is given by the Adjoint sensitivity.

III. ADJOINT SENSITIVITY

The $K_A(x, y)$ matrix elements are functions of the coordinates of the nodes, thereby the exact sensitivity related with x and y spatial coordinates can be obtained [1]. The objective function adopted is given by:

$$f(x, y) = \frac{1}{2} \left(\sum_{i=1}^n A_i(x, y) \right)^2 \quad (3)$$

where n are all nodes of the domain. The sensitivity of the objective function on x direction is given by [5]:

$$\frac{df(x, y)}{dx} = \frac{\partial f(x, y)}{\partial x} + \hat{A}^T \left(\frac{\partial [b_A]}{\partial x} - \frac{\partial [K_A]}{\partial x} [\bar{A}] \right) \quad (4)$$

where $\bar{A}$ is obtained solving (2) and the adjoint variable $\hat{A}$ is given by [5]:

$$K_A^T \hat{A} = \left(\frac{\partial f}{\partial A^T} \right) \quad (5)$$

where K_A^T is the transpose of stiffness matrix. In the adjoint system given by (5), the excitation vector $(\partial f / \partial A)$ must have an analytical derivative with respect to the state variable A .

IV. RESULTS

To show the effectiveness of the proposed methodology, a TL with 2 cables per phase 345 kV is optimized using an enhanced ellipsoidal method with multiple cuts strategy associated with the adjoint sensitivity for minimizing the objective function (3). In Fig. 2 is shown that the magnetic flux density is reduced significantly by the proposed methodology. The geometries given by Fig. 3 show new geometries with reduced dimensions of the bundles. The surge impedance

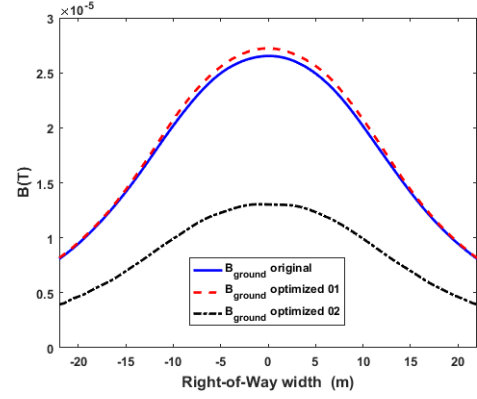


Fig. 2. Magnetic flux density at the ground level - TL 02 cables per phase 345 kV, optimized 01 using the sensitivity of the electric potential scalar [3] and optimized 02 using the sensitivity of the magnetic potential vector.

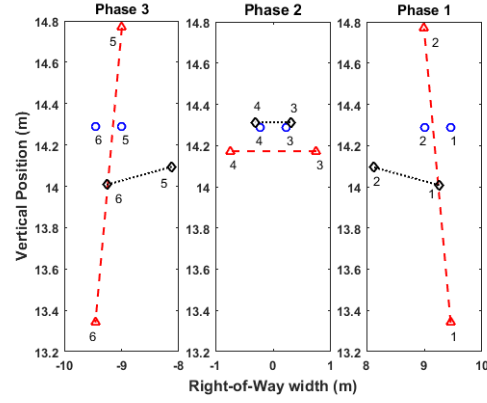


Fig. 3. Original (○), optimized using adjoint sensitivity of electric potential scalar [3] (△) and optimized using adjoint sensitivity of magnetic potential vector (◇) configuration - TL 2 cables per phase - 345 kV.

loading gain is near 10% with lower magnetic field intensity at ground level on the right-of-way width of the TL.

The results validate the proposed methodology and suggest that it can be effectively applied to more complex cases, such as those involving more than two conductors per phase and other constraints.

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