

Numerical Analysis of Grounding Systems with Counterpoise Wires Using IEFGM Meshless Method

Ursula C. Resende¹, Jean L. M. Vieira¹, and Victor G. L. C. Amorim¹

¹Department of Electrical Engineering, CEFET-MG, Belo Horizonte, Minas Gerais, Brasil,
resendeursula@cefetmg.br, jeanlmv@gmail.com, vglca8@gmail.com

This study performs a numerical analysis of an electrical grounding system comprising Counterpoise Grounding Wires buried in homogeneous soil and subjected to surge currents under low-frequency conditions. Since traditional mesh-based methods encounter significant computational challenges when modeling 3D systems, the adoption of the Interpolant Element-Free Galerkin Method (IEFGM), a meshless approach is proposed. The proposed solution incorporates innovative strategies, including domain size reduction and optimized node distribution, to enhance computational efficiency. Furthermore, a novel mathematical formulation for direct surge current injection improves both the accuracy and the realism of the analysis. Numerical results obtained validate the proposed approach, demonstrating its potential for the analysis of large-scale three-dimensional problems.

Index Terms—Counterpoise wire, grounding systems, interpolant element-free Galerkin method, meshless methods, surge current.

I. INTRODUCTION

THE NUMERICAL analysis of electrical grounding systems under low-frequency conditions is fundamental to ensuring the operational safety and reliability of electrical infrastructures. Accurate prediction of the electric potential distribution and leakage current behavior in the soil is essential for designing efficient grounding networks. At low frequencies, electric currents penetrate deeply into the soil, resulting in a broader charge distribution that can influence large areas around the grounding point. Therefore, detailed three-dimensional (3D) modeling is required to accurately represent current dispersion and the resulting electric field in the vicinity of the grounding system.

Traditional mesh-based methods face significant challenges in complex 3D grounding problems, as generating a sufficiently precise mesh can be computationally expensive and time-consuming, especially when high mesh density is required. Additionally, constructing a mesh that adapts to diverse soil layers or irregular geometries can be both challenging and computationally restrictive. In this context, meshless methods offer a robust alternative by eliminating the need for a structured mesh and operating directly on a set of distributed points within the domain. This approach enhances flexibility for complex geometry and large soil volumes, significantly reducing computational demands in scenarios requiring high resolution to capture detailed electric field behavior near the grounding structure.

In this work, the Interpolant Element-Free Galerkin Method (IEFGM) [1]-[2], a meshless approach, is employed to analyze a low-frequency grounding problem involving a Counterpoise Grounding Wire (CGW), a widely used configuration for grounding transmission lines. The horizontal orientation of the CGW and the injection of surge current, I_s , at only one end of the CGW prevent the use of symmetry conditions, demanding a full three-dimensional (3D) analysis. Additionally, the CGW's length, significantly greater than its radius, requires a large computational domain and refined discretization to ensure accuracy. To address these computational challenges,

this work proposes two cost-reduction strategies: (1) limiting the domain size by imposing a specific electric potential, V_D , at the soil boundary [3], and (2) optimizing the distribution of nodes and Gauss integration points within the domain. Moreover, we introduce a novel mathematical formulation that enables the direct injection of I_s into the numerical model, providing a more precise and realistic analysis of the problem.

II. MATHEMATICAL MODELING

The problem under analysis is a CGW, with length L_C , radius R_C and electrical conductivity σ_c , buried at a depth P_C in a homogeneous soil with electrical conductivity σ_s , subjected to a surge current, I_s , as illustrated in Fig. 1. The 3D domain of the problem, Ω_3 , is bounded by soil surface SS (dimensions d_1 , d_2 and d_3) and air surface SA (dimensions d_1 and d_2). The cartesian 3D Laplace equation is used to solve the problem, with the weak form derived using the Method of Weighted Residuals (MWR) and the test function $T(x,y,z)$:

$$\sigma \int_{\Omega_3} (\nabla T \cdot \nabla T) dv = \sigma \int_{\Omega_3} \left(\frac{\partial T}{\partial x} \frac{\partial V}{\partial x} + \frac{\partial T}{\partial y} \frac{\partial V}{\partial y} + \frac{\partial T}{\partial z} \frac{\partial V}{\partial z} \right) dv = 0, \quad (1)$$

where $dv = dx dy dz$, $\sigma = \sigma_s$ or $\sigma = \sigma_c$ and $V(x,y,z)$ is the electric potential. Since $\partial V / \partial \mathbf{n} = 0$ on SA , and $V = V_D = I_s / (2\pi \sigma_s D_C)$ [3] on SS (where the shape functions are zero), representing the Neumann and Dirichlet boundary conditions, respectively, as result, the surface integral arising the MWR is zero. Given that the CGW is a good conductor, the electric field on the surface where current is injected ($S_C = 2\pi R_C$) is normal to the surface, $\mathbf{E} = E_x \hat{x}$. Therefore, (1) can be rewritten as:

$$\sigma \int_{\Omega_3 - S_C} \left(\frac{\partial T}{\partial x} \frac{\partial V}{\partial x} + \frac{\partial T}{\partial y} \frac{\partial V}{\partial y} + \frac{\partial T}{\partial z} \frac{\partial V}{\partial z} \right) dv = -\sigma_c \int_{S_C} \left(\frac{\partial T}{\partial x} \frac{\partial V}{\partial x} \right) dv. \quad (2)$$

Using (2) the injection of I_s into the mathematical model is:

$$\sigma_c \int_{S_C} \left(\frac{\partial T}{\partial x} \frac{\partial V}{\partial x} \right) dv = \int_{S_C} \left(\frac{\partial T}{\partial x} E_x \right) dv - \int_{S_C} \frac{\partial T}{\partial x} \left(\frac{I_s}{2\pi R_C} \right) dv. \quad (3)$$

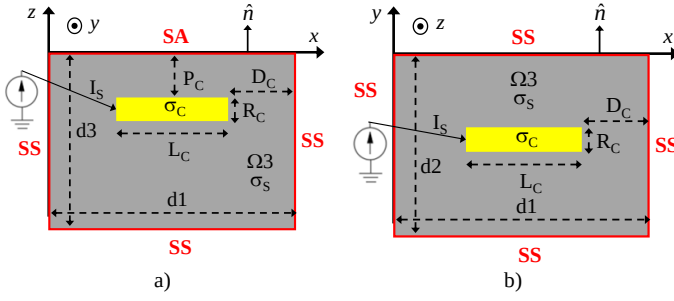


Fig. 1. Grounding system: a) Side view and b) Top view.

III. IEFGM FORMULATION

A set of N nodes is placed in Ω_3 . Each node, I , is a point $\mathbf{x}_I(x, y, z) \in \Omega_3$ for which a shape function, $\Phi_I = (\mathbf{x})$ is associated, $\mathbf{x}(x, y, z) \cdot \Phi_I = 0$ over the whole domain, except near the corresponding node. Therefore, the unknown function can be approximated by the following trial function [4]:

$$V(\mathbf{x}) \approx V^h(\mathbf{x}) = \sum_{I=1}^N \Phi_I(\mathbf{x}) v_I, \quad (4)$$

where v_I is the unknown coefficient of node I . Using de Galerkin Method the linear system $[\mathbf{K}][\mathbf{U}] = [\mathbf{F}]$ is obtained, with: $K_{ij} = \sigma \int_{\Omega_3 - S_C} (\nabla \Phi_i \cdot \nabla \Phi_j) dv$ and $F_j = - \int_{S_C} \nabla \Phi_j (I_s / 2\pi R_C) dv$.

In the proposed MLS approximation, v_I are determined by minimizing a weighted discrete L^2 norm. The weight function used is singular, defined as $W(r_I) = 1/(r_I^n + \beta^n)$, where β is a small constant to avoid division by zero, and n is an adjustable parameter used to enhance result accuracy. Additionally, $r_I = |\mathbf{x} - \mathbf{x}_I|/d_I$, $d_I = \alpha d_c$, α is a scaling factor for the influence domain, and d_c corresponds to the nodal distance [4].

The proposed Gauss point distribution grid was designed to include a point at the center of the S_C , where the current I_s is injected. To achieve this, the integration cells are arranged so that the cross-section of the CGW is centered within a integration cell. An odd number of Gauss Integration Points (GIP) is used within each integration cell to ensure precise alignment. For the node distribution, a rectangular uniform grid is used, with nodes spaced at a distance equal to nc in the soil region. For the CGW, nodes are placed exclusively on its lateral surface. Along the CGW's length, the nodes are also spaced nc apart, while its perimeter is represented by four nodes. In this work, nc is adjusted independent of CGW radius, enabling the use of significantly fewer nodes. The distinction between the soil and CGW regions was made by employing the visibility criterion.

IV. NUMERICAL RESULTS

The system investigated in this work consists of a CGW with $L_C = 50$ m, $R_C = 0.01$ m, and $\sigma_C = 5.7 \times 10^7$ S/m (copper), buried at a depth of $P_C = 5$ m in homogeneous soil with a resistivity of $1 \Omega \cdot m$, and subjected to a current $I_s = 1000$ A. The computational domain dimensions are $d1 = 290$ m, $d2 = 250$ m,

and $d3 = 150$ m, with $nc = 10$ m, resulting in 24 nodes on the CGW surface. Each integration cell consists of $3 \times 3 \times 3$ GIP. This odd number ensures that there is a line of GIP at the center of the CGW. $\alpha = 6$ for the nodes on the CGW and $\alpha = 3$ for the nodes in the soil, ensuring proper connectivity between the CGW nodes. For the calculation of V_D , the average distance $D_C = 120$ m was considered. The results obtained are presented in Table I, where V_H is the constant electrical potential obtained on the CGW nodes, the grounding resistance $R_A = (V_H - V_D)/I_s$, and $E_r(\%) = 100 \times |R_A^A - R_A^{IEFGM}|/R_A^A$ is the error calculated considering the analytical approximation presented in (5). NN represents the total number of nodes, and NPI is the total number of GIP. The obtained distribution of electric potential at the ground level is shown in Fig. 2. As can be observed, the incorporation of the proposed innovative strategies led to accurate and computationally efficient results.

$$R_A^A = \frac{1}{2\pi\sigma_s L_C} \left[\ln \left(\frac{2L_C^2}{R_C P_C} \right) - 2 + \frac{2P_C}{L_C} - \left(\frac{P_C}{L_C} \right)^2 + \frac{1}{2} \left(\frac{P_C}{L_C} \right)^4 \right] \quad (5)$$

TABLE I
NUMERICAL RESULTS

V_D (V)	V_H (V)	R_A^{IEFGM} (Ω)	R_A^A (Ω)	E_r (%)	NN	NPI	Time (s)
1,326	27,010	0,027	0,030	10	11.025	254.474	8,672

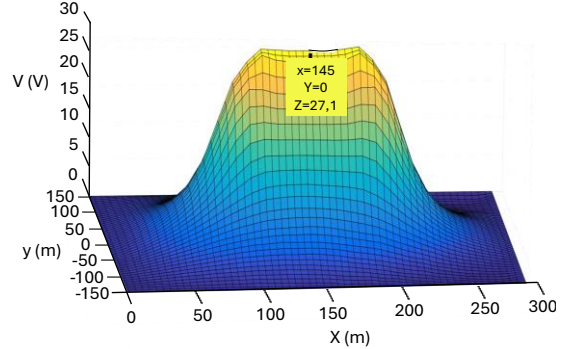


Fig. 2. Electric potential at the ground level.

ACKNOWLEDGMENT

This work was supported in part by FAPEMIG, in part by CAPES, in part by CNPq, and in part by CEFET-MG

REFERENCES

- [1] T. Belytschko, Y. Y. Lu, and L. Gu, "Element-free Galerkin methods," *Int. J. Numer. Methods Eng.*, vol. 37, no. 2, pp. 229–256, 1994.
- [2] B. F. Zwick, G. Bourantas, and A. Wittek, "A meshless approach using EFG interpolating moving least-squares method for electromagnetic scattering," *IEEE Trans. Antennas Propag.*, vol. 68, no. 5, pp. 400–410, May 2020.
- [3] M. L. Oliveira, R. B. Macedo, U. C. Resende, and R. S. Alípio, "Analysis of grounding problems using interpolation element-free Galerkin method with reduction of computational domain," in *Proc. Int. Microw. Optoelectron. Conf. (IMOC)*, Águas de Lindóia, São Paulo, Brazil, 2017.
- [4] U. C. Resende, R. Alípio, "Proposals for Inclusion of the Electrode Radius in Grounding Systems Analysis Using Interpolating Element-Free Galerkin Method," *IEEE Trans. Magn.*, vol. 54, no. 1, pp. 1–4, 2018.