

Finite Element Adaptive Mesh Density Predictor Based on Deep Neural Network

Yufeng Niu^{1,2}, Shuhong Wang¹, and Youpeng Huangfu¹

¹ School of Electrical Engineering, Xi'an Jiaotong University, 710100, Xi'an, China

² Dipartimento di Elettronica, Informazione e Bioingegneria, Politecnico di Milano, 20133 Milan, Italy

This paper introduces a novel approach to automatic unstructured mesh generation, leveraging deep neural network to predict an optimal finite element mesh distribution. A posteriori error estimation method is employed to generate training samples. Compared to traditional mesh adaptation techniques, this approach generates an optimal mesh in a fraction of the time. We demonstrate its performance using standard test problems, such as Poisson's equation. The results show that, in comparison to conventional adaptive mesh generation methods, the neural network approach is highly competitive, striking an excellent balance between accuracy and computational cost while effectively predicting the mesh density distribution across the solution domain.

Index Terms—mesh generation, error estimation, artificial neural networks, mesh optimization.

I. INTRODUCTION

Physical phenomena in real life are often described by partial differential equations (PDEs), and the finite element method (FEM) is a widely used approach for solving them. The accuracy of a finite element solution largely depends on the quality of the computational mesh. Ideally, the error in the finite element solution should be uniformly distributed across all elements of the mesh, ensuring that the error between the finite element and actual solutions remains within an acceptable tolerance [1]. When the solution exhibits strong local variations or discontinuities, adaptive mesh refinement in these regions is especially beneficial.

Traditional mesh refinement strategies typically start by solving the problem with an initial uniform, coarse mesh. Then, a posteriori error estimation is used to determine the error distribution across the solution domain [2]. The mesh is iteratively refined based on this error distribution to improve accuracy. In this paper, we propose using an artificial neural network to guide the mesh generation process, eliminating the need for expensive error estimation in each refinement iteration [3-4]. Once trained on the provided data, the neural network can predict the optimal mesh density distribution without relying on the finite element solution or error estimation [6-8]. The aim of this paper is to develop a method that provides an efficient and reasonable mesh density distribution. Experimental results show that this approach strikes an effective balance between computational cost and accuracy, and it has been successfully validated using Poisson's problem.

II. CONSTRUCTION FINITE ELEMENT ERROR ESTIMATION

To better capture the phenomenon of field concentration in the solution domain, we define an L-shaped domain Ω to solve Poisson's equation ($\nabla^2 u = 0$) and apply two different boundary conditions on the boundary ($\partial\Omega$) to simulate the charge-free potential distribution. The two red boundaries are set with Dirichlet boundary conditions ($\partial\Omega = 0$ and $\partial\Omega = 10$), while the remaining black boundaries are assigned Neumann boundary conditions ($\partial\Omega/\partial n = 0$), as shown in Fig.1.

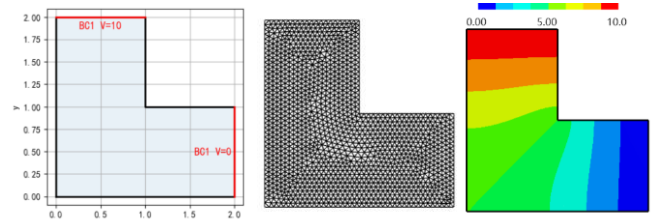


Fig. 1. L-shape domain, finest mesh and potential distribution

We use error estimates to obtain the error distribution across the entire solution domain [5]. Based on the weak form of the finite element method and the first-order interpolation function, the norm of the partial differential decomposition is multiplied by the mesh size (e.g., the diameter h of the circumcircle of a triangular element) to serve as the error indicator. We refer to this approach as the simple estimation method.

$$error_{indicator} = \|\nabla u\| * h \quad (1)$$

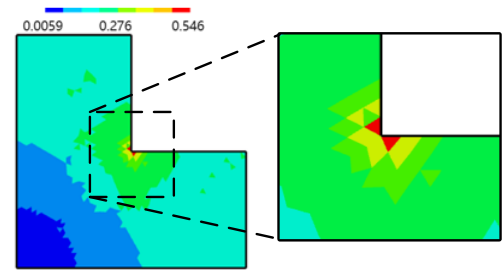


Fig. 2. Error distribution based on simple estimate method

The method maps the unit error to the node, ensuring continuity in the error estimation and treating it as the true error distribution.

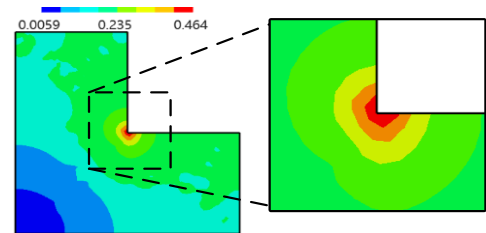


Fig.3. Maps element error to the node

III. NEURAL NETWORK

This paper develops a deep neural network (DNN) to predict the optimal mesh density distribution. The structure of the network is shown in Fig.4.

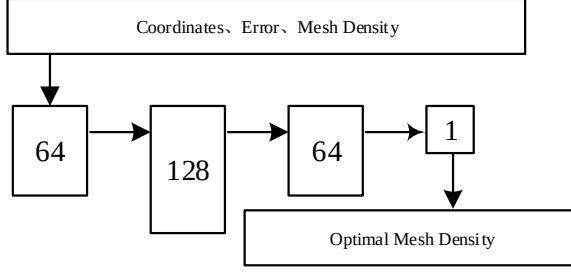


Fig. 4. Structure of neural network

The original error between adjacent elements is discontinuous. By mapping the errors to the nodes, each node is assigned a unique error, ensuring that the error is continuous across the solution domain.

The activation function used between the hidden layers of the network is the ReLU function, while the activation function between the hidden layer and the output layer is the sigmoid function. The optimizer used is Adam, the loss function is Mean Squared Error (MSE), and the evaluation metric is Mean Absolute Error (MAE), with a learning rate set to 0.001.

The maximum mesh sizes selected were 0.1, 0.05, 0.025, and 0.0125 for the uniform meshes, and their corresponding errors were calculated as samples. We used 80% of the data for training and 20% for validation. The final results and the training process are shown in Figs. 5 and 6.

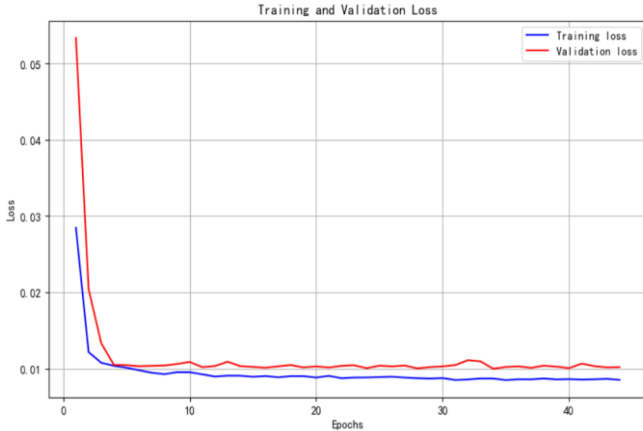


Fig 5. training_history

It is important to note that the color bar in Fig. 6 does not represent the absolute mesh density; rather, it indicates the density relative to the sparsest region.

By training on the mesh data, the neural network refines the element errors and computes the loss function. If an element lies in more than one subdomain, the predicted average is used. Fig. 6 demonstrates that the ANN method can efficiently and reliably generate optimal meshes. However, the refinement appears somewhat random in certain areas. Despite these minor imperfections, the model is generally successful in accurately learning the error distribution across the entire solution domain and providing a density distribution based on the energy error. This offers a valuable reference for generating optimal meshes.

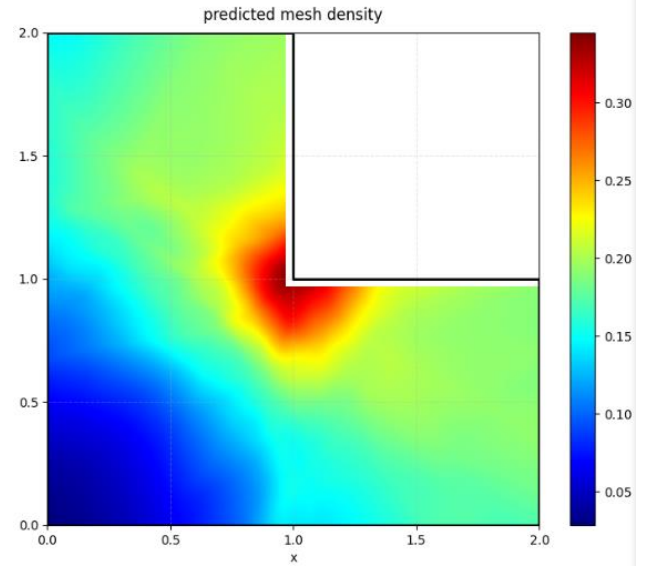


Fig. 6. Mesh density distribution prediction

IV. CONCLUSION

The key to non-uniform mesh generation is determining an appropriate mesh density distribution in space. Instead of using expensive error estimators at each step, we compute and store high-quality mesh density distributions obtained through FEM, followed by accurate error estimation, as a pre-processing step. This approach replaces the traditional iterative refinement process with a neural network. Compared to conventional global mesh refinement, this method does not require specialized knowledge, provides a reasonable mesh distribution, and achieves a faster error reduction rate, significantly improving the efficiency of the mesh optimization process.

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