Learning electromagnetic fields based on finite element basis functions

Merle Backmeyer[1], Michael Wiesheu[1], and Sebastian Schöps[1]

[1]Computational Electromagnetics Group, Technical University of Darmstadt,
Schloßgartenstr. 8, 64289 Darmstadt, merle.backmeyer@tu-darmstadt.de

Parametric surrogate models of electric machines are essential for efficient design optimization and operational monitoring. Addressing geometry variations, spline-based computer-aided design representations play a pivotal role. In this study, we propose a novel approach that integrates isogeometric analysis with deep learning to enable rapid and physically consistent predictions by directly learning spline basis coefficients. The effectiveness of this method is demonstrated using a parametric nonlinear magnetostatic model of a permanent magnet synchronous machine.

***Index Terms*—Isogeometric Analysis, Physics-Informed Neural Networks, Permanent Magnet Synchronous Motor**

I. Introduction

PHYSICS-INFORMED neural networks (PINNs) have emerged as an alternative for solving partial differential equations (PDEs), including applications in electromagnetic problems [1]. This was extended to parametric problems, enabling solutions of entire problem classes rather than instances, [2]. Once the neural network is trained, the solution for a new parameter can be obtained through a single forward pass, requiring only minimal computational effort. However, this efficiency comes with trade-offs, as PINNs may compromise accuracy, lack a rigorous theoretical foundation and are typically more expensive to train than solving the PDE conventionally. In particular, learning tangential continuity is challenging for PINNs, possibly leading to solutions that do not adhere to fundamental physical laws. By contrast, conventional Galerkin-type methods, encode this properties directly via basis functions, ensuring consistency and adherence to physical constraints. Consequently, numerous studies have opted to bypass intermediate field descriptions, focusing instead on learning key performance indicators directly, e.g. [3].

We propose to keep the field description and combine the isogeometric framework [4] with the computational speed of PINNs. The key idea is to train a neural network to learn *coefficients of spline basis functions*. Previous studies have applied this idea to the volumetric Poisson problem as well as to the scattering wave problem [5], [6]. This work proposes application for Permanent Magnet Synchronous Motors (PMSM). Using inputs, e.g. rotation angle, shape representations or electric parameters, the network predicts the electromagnetic field solution via basis functions.

In Section II we introduce the physical problem and proposed method. Section III presents first results. Finally, we outline our next steps in Section IV.

II. Methodology

The geometry of the motor is shown in Fig. 1, details are given in [7]. The model is simplified to a two-dimensional magnetostatic problem by disregarding displacement and eddy currents as well as presuming a sufficient axial length. Given

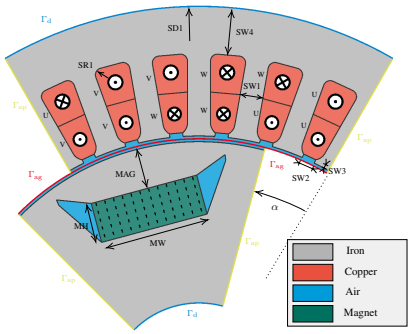


Fig. 1. Parametrization of the PMSM geometry including parameter names, material definitions and boundary conditions, see [7].

computational domains Ω_{R} and Ω_{S}, for rotor and stator respectively, the governing equations are

$$\begin{aligned} \nabla \cdot (\nu \nabla A_{z,\mathrm{R}}) &= \nu \nabla \cdot \mathbf{B}_r^{\perp}, \quad \text{in } \Omega_{\mathrm{R}} \\ \nabla \cdot (\nu \nabla A_{z,\mathrm{S}}) &= -J_{z,\mathrm{source}} \quad \text{in } \Omega_{\mathrm{S}}, \end{aligned} \tag{1}$$

where $A_{z,\mathrm{R}}$ and $A_{z,\mathrm{S}}$ describe the z-components of the magnetic vector potential in rotor and stator, respectively. With $\boldsymbol{B}_r$ we describe the magnetic flux density due to permanent magnets; ν represents the material's (nonlinear) reluctivity. On Γ_{d}, homogeneous Dirichlet boundary conditions and on Γ_{ap} antiperiodic conditions are implemented as the model is reduced to one pole. To ensure the continuity of the magnetic vector potential A_z and the azimuthal magnetic field strength H_ϕ across $\Gamma_{\mathrm{ag}} = \bar{\Omega}_{\mathrm{R}} \cap \bar{\Omega}_{\mathrm{S}}$, the coupling conditions

$$\begin{aligned} A_{z,\mathrm{S}}(\varphi) &= A_{z,\mathrm{R}}(\varphi - \alpha), \\ H_{\varphi,\mathrm{S}}(\varphi) &= H_{\varphi,\mathrm{R}}(\varphi - \alpha), \end{aligned} \tag{2}$$

with the magnetic field strength $\boldsymbol{H} = \nu \boldsymbol{B}$ are enforced on Γ_{ag}. Due to the symmetry of the three-phase current, it is sufficient to consider the rotation angles $\alpha \in [0, 20]$ degrees. For the discretization of (1) we express the magnetic vector potential as $A^h(\boldsymbol{x}) = \sum_k N_k(\boldsymbol{x}) u_k$ with respect to a conforming spline basis [8]. Note that the solution depends implicitly on input parameters, in particular the angle α.

The deep neural network is designed to predict the coefficients u_k of the magnetic vector potential, based on the

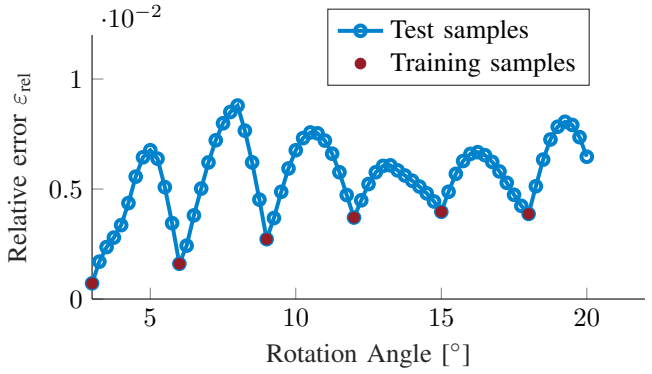


Fig. 2. Relative error of the network's prediction depending on the rotation angle α. Training samples are marked in red.

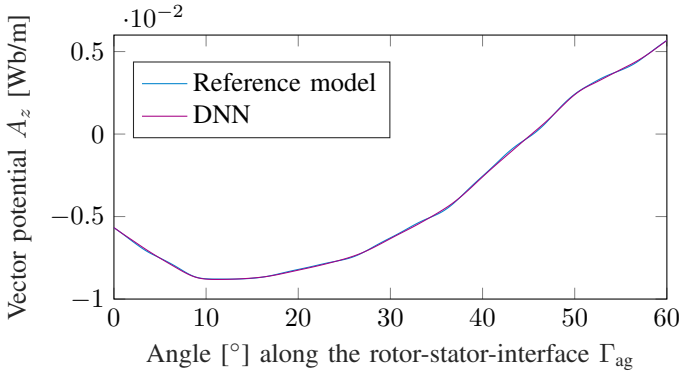


Fig. 3. Comparison of the network's prediction depending on the rotor-stator interface with the high-fidelity for rotor position $\alpha = 10$.

given inputs, e.g. rotation angle. This ensures that the resulting solution resides within the space spanned by the B-Spline basis functions, maintaining physical plausibility such as the correct continuity conditions across interfaces. The network architecture begins with an input layer composed of the rotation angle α, i.e., the vector $\boldsymbol{i} = \left(\cos\left(\frac{2\pi\alpha}{360}\right), \sin\left(\frac{2\pi\alpha}{360}\right), \alpha\right)$. The parametric information is passed through the net by multiplying with the weights θ and applying activation functions σ. It consists of six layers with $40, 80, 160, 320, 640$ neurons respectively. For the generation of training and validation data, we use a high-fidelity (isogeometric) FEM model of the PMSM. The loss function is defined as the relative error in the L^2-norm

$$\varepsilon_{\text{rel}} = \int_\Omega \frac{\sum_k \left(\left(\hat{u}_k - u_{\theta,k}\right) N_k(\boldsymbol{x})\right)^2}{\sum_k \left(\hat{u}_k N_k(\boldsymbol{x})\right)^2} \mathrm{d}\boldsymbol{x} \tag{3}$$

between the coefficients of the training sample $\hat{\boldsymbol{u}}$ and the network's prediction $\boldsymbol{u}_\theta$. During training, the gradient-based optimization algorithms ADAM adapts the weights of the network such that they minimize the sum of ε_{rel} of over all training samples, which incorporates the physical constraint, i.e., the parametric magnetostatic problem. Training is terminated after 2400 epochs. Regularization helps to prevent the network from overfitting.

III. Preliminary Results

We investigate the airgap field. It consists of 462 coefficients. The network was trained using 8 equidistant samples, corresponding to the rotation angles $\alpha = \{0, 3, 6, 9, 12, 15, 18, 21\}°$. The testing domain defined as $[0, 20]°$. Fig. 2 illustrates the relative error ε_{rel} of the network prediction in the airgap. The mean of the relative error of the test samples was $5.5 \cdot 10^{-3}$ and the standard deviation $2.0 \cdot 10^{-3}$. Fig. 3 compares A_z along the rotor-stator interface predicted by the neural network and computed with the reference model for $\alpha = 10$. The model effectively learned the parametric magnetostatic problem, providing an accurate approximation of the system's behavior.

Subsequently, the network was extended to another input parameter, i.e. the geometric parameter MW which represents the width of the magnet, as shown in Fig. 1. The net was trained on 32 training samples for 10.000 epochs. On 50 random test samples, the mean and the standard deviation of the relative error are $8.7 \cdot 10^{-3}$ and $5.9 \cdot 10^{-3}$, respectively, which demonstrates that the model is capable of learning the dependency on multiple parameters.

IV. Summary and Outlook

The results underline that the neural network has successfully learned a parametric magnetostatic problem, providing a reliable approximation for the given inputs. The full paper will present a more advanced network incorporating additional geometric and electric input parameters. This extended network will be applied in an optimization task and its performance will be evaluated in terms of both accuracy and computational efficiency, comparing it to conventional optimization methods.

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References

[1] M. von Tresckow, S. Kurz, H. De Gersem, and D. Loukrezis, "A neural solver for variational problems on CAD geometries with application to electric machine simulation," *J. Mach. Learn. Mod. Comput.*, vol. 3, no. 1, pp. 49–75, 2022.

[2] A. Beltrán-Pulido, I. Bilionis, and D. Aliprantis, "Physics-informed neural networks for solving parametric magnetostatic problems," *IEEE Trans. Energ. Convers.*, vol. 37, no. 4, pp. 2678–2689, 2022.

[3] V. Parekh, D. Flore, and S. Schöps, "Performance analysis of electrical machines using a hybrid data- and physics-driven model," *IEEE Trans. Energ. Convers.*, vol. 38, no. 1, 03 2023.

[4] J. A. Cottrell, T. J. R. Hughes, and Y. Bazilevs, *Isogeometric Analysis: Toward Integration of CAD and FEA*. Wiley, 2009.

[5] M. Möller, D. Toshniwal, and F. van Ruiten, "Physics-informed machine learning embedded into isogeometric analysis," in *Mathematics: Key Enabling Technology for Scientific Machine Learning*, 2021, pp. 57–59.

[6] M. Backmeyer, S. Kurz, M. Möller, and S. Schöps, "Solving electromagnetic scattering problems by isogeometric analysis with deep operator learning," in *2024 Kleinheubach Conference*. IEEE, 2024.

[7] T. Komann, M. Wiesheu, S. Ulbrich, and S. Schöps, "Robust design optimization of electric machines with isogeometric analysis," *Math.*, vol. 12, no. 9, 04 2024.

[8] Z. Bontinck, J. Corno, S. Schöps, and H. De Gersem, "Isogeometric analysis and harmonic stator-rotor coupling for simulating electric machines," *Comput. Meth. Appl. Mech. Eng.*, vol. 334, pp. 40–55, 2018.