

Efficient Surrogate Modeling for High-Dimensional Parametric Problems Based on Multi-Level Fidelity Simulation Combined with Incremental Learning

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For high-dimensional parametric problems, a data-based surrogate model can be generated from simulation or experimental data to quickly evaluate quantities of interest, which can then be applied in optimization or real-time monitoring or control scenarios. Deep neural network (DNN) has become an important approach for surrogate modeling of high-dimensional parametric problems. However, a large amount of data is a prerequisite for training a DNN model. Numerical simulation is an important method for data acquisition. For a DNN model trained by simulation data, an essential problem is how to obtain sufficient data with relatively low computational cost. The fidelity and efficiency of numerical simulation are highly related to the density of meshes. In this paper, we propose an incremental learning (IL) strategy combined with simulation data of different fidelity, which are calculated with meshes of different density, for the efficient training of the DNN-based surrogate model. The proposed method is validated by an example for construction of IC parasitic capacitance pattern library. The results show that a surrogate model with the same accuracy can be developed with lower computational cost from the aspect of data preparation with the presented strategy compared to the traditional DNN.

Index Terms—Finite element method (FEM), parametric problems, incremental learning (IL), surrogate model.

I. INTRODUCTION

In field analysis, a high-dimensional parametric problem needs to be solved repeatedly on a parametric space consisting of geometric parameters, material parameters and operation condition parameters. Surrogate models or macro-models built offline can be applied to a fast evaluation of the quantities of interest online for scenarios like optimization design or real-time monitoring or control. Taking the on-the-fly parasitic extraction in integrated circuits (IC) as an example, parasitic parameters, such as parasitic capacitances of interconnects, should be rapidly evaluated from a pattern model whose variables are geometric parameters of interconnects, including their positions and widths [1]. Deep neural network (DNN) based on deep learning (DL) has become a popular approach for surrogate modeling. Numerical simulation using the field solver is commonly used to prepare data for the supervised training of the DNN. For high-dimensional problems, the computational cost can be large for the data preparation. In this work, we propose to combine training data with different fidelity (which are calculated from the finite element field solver with meshes of different density) for the efficient surrogate modeling of high-dimensional parametric problems. A preliminary DNN model is trained from low fidelity simulation data which are solved on a relatively coarser mesh (referred as low-fidelity data in the context). Then, an incremental learning (IL) [2] procedure is applied to improve the accuracy of the preliminary model with training data from high accuracy simulations with a relatively fine mesh (referred as high-fidelity data in the context). The proposed method is verified by an example for construction of IC parasitic

capacitance pattern library, demonstrating that the accuracy of surrogate model can be guaranteed with lower calculation cost compared to the traditional DNN.

II. METHOD

The proposed approach aims to the efficient training of accurate DNN models for high-dimensional parametric problems. Fig. 1 shows the main design process and the key implementation steps are as follows:

- Data acquisition: First, parameterized variables of the studied problem are sampled within the given range by Latin hypercube sampling (LHS). Then, the simulation model is built for each sample and solved by finite element method (FEM). Among them, x samples are solved with coarse mesh to obtain low-fidelity data and y ($y \ll x$) samples are solved with fine mesh to acquire high-fidelity data. The samples of parameterized variables and their corresponding results of simulation work together as training and testing datasets of neural network model.
- Preliminary DNN model: We predefine the maximum allowed average relative error ϵ_m of test as the error tolerance. The tree-structured parzen estimator (TPE) method is used to determine the optimal hyperparameters of network, including number of hidden layers, neurons in each hidden layer, activation function, batch size and learning rate. The DNN model is then trained with the prepared low-fidelity data until the predicted average relative error is below the error tolerance. During the training process, the model with the least evaluation loss called the best model is saved.
- Incremental learning: Firstly, a small batch of high-fidelity data is used to train the best model. If the prediction error is larger than ϵ_m , the number of high-fidelity data is

increased and the best model is retrained. Until the accuracy of model meets the expectation, the IL process is stopped, that is, a more accurate model is generated.

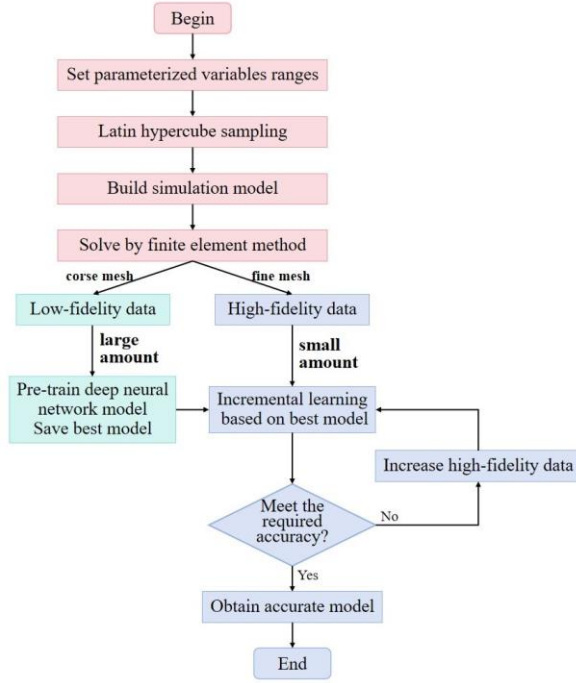


Fig. 1. Main design process of efficient surrogate modeling.

III. EXAMPLE

Construction of IC parasitic capacitance pattern library is taken as an example to validate the proposed method. Fig. 2 shows the studied 2D interconnect pattern. It consists of a substrate ground and five conductors, where the central position of conductor 1 is fixed. The pattern has 9 parameterized variables, including widths and central positions of surrounding conductors. Fig. 3 presents the feature data (input) and label data (output) of DNN. The mutual-capacitances of master conductor 1 with other conductors can be predicted with the trained model. We take the parametric surrogate model for mutual-capacitances here, since from previous experience, more training data are required for the construction of a model for the mutual-capacitance than a model for the self-capacitance [1].

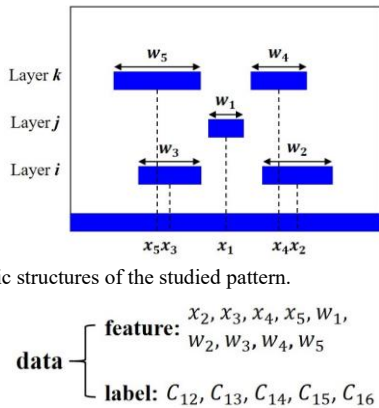


Fig. 2. Geometric structures of the studied pattern.

Fig. 3. Feature data and label data of DNN.

First, the low-fidelity data and high-fidelity data are prepared by the data acquisition method described above. The sizes of coarse mesh and fine mesh are respectively 0.12 and 0.03. The preliminary DNN model is pre-trained with 2500 low-fidelity data and the average relative error of test is 0.45%. The error in high-fidelity testing data using the best model saved during pre-training of the preliminary DNN model is 2.31%. Then, 80 high-fidelity data are incrementally learned based on the best model to make the prediction error below the error tolerance. For the traditional DNN model, at least 2500 high-fidelity data are needed to reach the expected accuracy. Table I compares the FEM solving time for data acquisition between the traditional DNN model and the proposed model. It can be observed that the DNN model based on multi-level fidelity simulation of different density meshes combined with IL greatly reduces the computational cost of preparing data. The radar chart in Fig. 4 shows the average relative error of each mutual-capacitance predicted by the preliminary DNN model and the model based on IL. The blue and pink lines represent the results of the preliminary DNN model and the IL-based model, respectively.

TABLE I
COMPARISON BETWEEN TRADITIONAL DNN MODEL AND PROPOSED MODEL

	Dataset Size		Number of Elements		Solving Time of Each Data		Total Solving Time
	Low Fidelity	High Fidelity	Low Fidelity	High Fidelity	Low Fidelity	High Fidelity	
Traditional DNN Model	0	2500	4000	54000	0.25s	3.5s	2.43h
Proposed Model	2500	80					0.25h

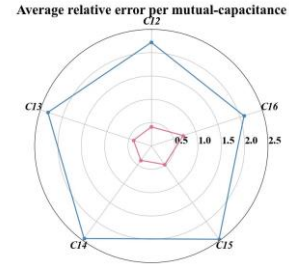


Fig. 4. Average relative errors of different mutual-capacitances.

IV. CONCLUSION

This paper presents an efficient surrogate modeling approach based on multi-level fidelity simulation of different density meshes combined with IL, and it is verified by an example for construction of IC parasitic capacitance pattern library. The results show that compared with the traditional DNN model, this method can greatly reduce the sample calculation amount and time cost of FEM.

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