

Physics-Informed Neural Networks for Modelling Magnetostatic Problems

Sina Toghranegar, Hussain Kazmi, Geert Deconinck, Ruth V. Sabariego

Department of Electrical Engineering (ESAT), KU Leuven, EnergyVille, 3001 Leuven, Belgium

sina.toghranegar@kuleuven.be; hussain.kazmi@kuleuven.be; geert.deconinck@kuleuven.be; ruth.sabariego@kuleuven.be

This paper explores the application of Physics-Informed Neural Networks (PINNs) for addressing a magnetostatic problem with particular attention to material interfaces. A representative test case featuring an inductor composed of two different materials is employed to solve Poisson's equation. The results generated by the PINN are compared to those obtained using the Finite Element Method (FEM).

Index Terms—Magnetostatic, Physics-Informed Neural Networks.

I. INTRODUCTION

IN recent years, deep learning and neural networks (NNs) have emerged as leading approaches across diverse fields. A prominent area of modern research explores the application of NNs in scientific computing, particularly for solving partial differential equations that form the foundation of numerous scientific and engineering challenges [1]. A hybrid approach is being explored as a promising solution to address the limitations of traditional methods by integrating the strengths of both physics-based and data-driven techniques. Among such approaches, Physics-Informed Neural Networks (PINNs) stand out as a notable example. PINNs solve differential equations by minimizing loss terms that incorporate both data-driven and physics-informed components, with the physics contribution being regularized through the residual form of the governing physical laws [2].

PINNs have proven effective for electromagnetic problems. Their flexibility and ability to embed physical laws into the training process make them well-suited for complex electromagnetic simulations. The research presented in [3] explores the utilization of PINNs to address magnetostatic boundary value problems. The study emphasizes the significant impact of domain similarity on the effectiveness of PINNs, highlighting the necessity for additional investigations to enhance their efficiency. The study by [4] proposes deep learning models for magnetostatic simulations, including a PINN. These models were tested on scenarios such as a current-carrying coil in linear and nonlinear magnetic materials, as well as a C-core actuator configuration.

Accurate electromagnetic modeling requires ensuring field continuity at interfaces between materials with different properties. Predicting magnetic field values at these interfaces is essential for maintaining simulation accuracy and understanding the performance of electromagnetic devices. The material interfaces are automatically taken into account in methods like Finite Element method (FEM), but this consideration is lost when working solely with points, as in PINNs.

This paper proposes a PINN for solving a magnetostatic problem involving an inductor with two distinct materials and

an interface between them, providing an interesting test case. In this study, no data were utilized to train the interface or other aspects; instead, the PINNs were trained exclusively using embedded physical laws and transmission conditions.

II. PROBLEM DESCRIPTION

Let us consider the Poisson's equation characterizing a 2D magnetostatic problem, as

$$\begin{aligned} -\nabla \cdot (\mu^{-1} \nabla a_z(\mathbf{x})) &= j_z, \quad \mathbf{x} \in \Omega, \\ a_z(\mathbf{x}) &= 0, \quad \mathbf{x} \in \Gamma_D, \\ \frac{\partial a_z(\mathbf{x})}{\partial n} &= 0, \quad \mathbf{x} \in \Gamma_N. \end{aligned} \quad (1)$$

where a_z and j_z represent the z-component of the magnetic vector potential and the excitation current density, respectively. Homogeneous Dirichlet and Neumann boundary conditions (Γ_D and Γ_N , respectively) are applied at the boundary of the computational domain ($\Gamma = \Gamma_D \cup \Gamma_N$), where Ω represents the computational domain. The excitation current density, j_z , is $10^7 \frac{\text{A}}{\text{m}^2}$. The geometry of the inductor consists of a core and a coil. The dimensions of the core are 10×10 cm and the coil has dimensions of 4×5 cm. Each domain is made of a different material, resulting in distinct relative permeability, μ_r , values for each. The coil has a relative permeability of $\mu_{r_{Coil}} = 1$. In contrast, the core exhibits a higher relative permeability of $\mu_{r_{Core}} = 2$. The permeability of free space (μ_0) is $4\pi \times 10^{-7} \frac{\text{H}}{\text{m}}$.

III. PROBLEM IMPLEMENTATION

Each domain is assigned a separate PINN for training. Each PINN incorporates several loss terms, which are outlined as follows:

$$L^{Core} = \lambda_1 L_{\Omega}^{Core} + \lambda_2 L_{\Gamma}^{Core} + \lambda_3 L_{\gamma_{CB}}^{Core} \quad (2)$$

$$L^{Coil} = \lambda_4 L_{\Omega}^{Coil} + \lambda_5 L_{\Gamma}^{Coil} + \lambda_6 L_{\gamma_{CB}}^{Coil} \quad (3)$$

For each region, a physics-based loss term, denoted by Ω , is defined based on (1). The core total loss includes loss

terms corresponding to Dirichlet and Neumann boundary condition, collectively denoted by Γ , while the coil area has only Neumann boundary conditions. Additionally, each region incorporates an interface loss term, denoted by γ , that couple all the PINNs together. The regions are labeled as follows: C for core and B for coil. Note that, in this study, no data were used to train the interface or other points; instead, the PINNs relied solely on the embedded physical laws. The loss terms in each region is weighted by factors denoted as λ , which are multiplied by their respective loss terms and predetermined prior to training. The transmission conditions at the interface (γ) are described below.

$$\hat{n} \times (\vec{h}_2 - \vec{h}_1) \Big|_{\gamma_{12}} = 0, \quad \hat{n} \cdot (\vec{b}_2 - \vec{b}_1) \Big|_{\gamma_{12}} = 0 \quad (4)$$

Where $\hat{n}$ represents the normal, while h and b denote the magnetic field intensity and magnetic flux density, respectively. The subscripts 1 and 2 refer to different materials. The conditions incorporated into the interface loss terms are derived from the primary conditions described in equations (4), and are expressed as follows:

$$\mu_2^{-1} \frac{\partial a_{z_2}}{\partial n} \Big|_{\gamma_{12}} = \mu_1^{-1} \frac{\partial a_{z_1}}{\partial n} \Big|_{\gamma_{12}}, \quad a_{z_2} \Big|_{\gamma_{12}} = a_{z_1} \Big|_{\gamma_{12}} \quad (5)$$

The PINN architectures employed are fully connected feedforward neural networks. The input layer is designed to accept two features: the coordinates x and y . During the final stage, which is the testing phase of the PINN, the predictions are compared with FEM results.

IV. RESULTS

Figure 1 compares the z -component of the magnetic vector potential, a_z , obtained using the FEM and PINN. Fig. 1-(a) displays the FEM results, which serve as the reference solution, with a_z values ranging from approximately 0 to 0.012 $\frac{\text{Wb}}{\text{m}}$. Fig. 1-(b) depicts the error distribution between the FEM and PINN results, showing absolute error values in the range of 0.001 to 0.0015 $\frac{\text{Wb}}{\text{m}}$. The relative L^2 error is 9.46%.

Figure 2 illustrates the PINN predictions for the interface conditions between each region, compared to the FEM results. The predictions show a close alignment with the reference values, highlighting the PINN effectiveness in accurately capturing the magnetic vector potential across interfaces.

V. CONCLUSION

This study has demonstrated the effectiveness of PINNs in modeling magnetostatic problems with attention to the material interface, specifically for an inductor. By embedding physical laws directly into the training process, the PINN captured the behavior of the magnetic vector potential across different materials, achieving a maximum absolute error of 0.0015 $\frac{\text{Wb}}{\text{m}}$ compared to FEM results. Further results will be presented, including higher contrast for the material characteristics and analysis of the accuracy regarding the data sampling of the domain.

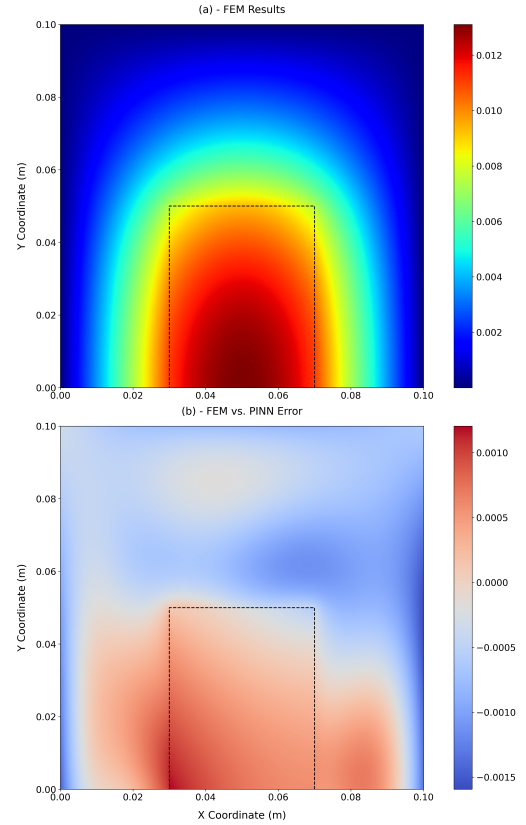


Fig. 1. Comparison of the z -component of the magnetic vector potential ($\frac{\text{Wb}}{\text{m}}$) results: (a) FEM results and (b) error between FEM and PINN solutions.

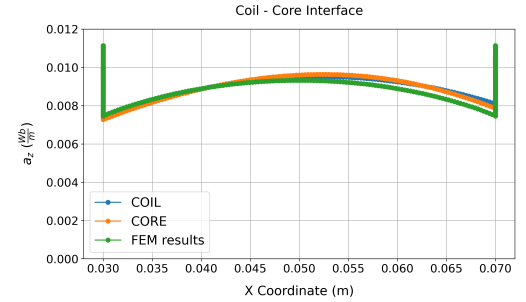


Fig. 2. Comparison of core-coil interface predictions with FEM results.

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