

Physics-Informed Neural Networks for Time-Domain Electromagnetic Field Computations with an Embedded Metamaterial

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Metamaterials are artificially structured materials with unique properties not found in nature, offering significant potential in manipulating electromagnetic fields. However, when applying numerical methods to solve metamaterial-related problems, challenges arise in balancing the computational efficiency and the accuracy. A physics-informed neural network based method is thus proposed in this paper to address the low-frequency electromagnetic fields including metamaterials in the time domain. The proposed method demonstrates a comparable accuracy to the time-domain finite-difference method.

Index Terms—Finite difference time domain method, metamaterial, physics-informed neural network, residual loss.

I. INTRODUCTION

METAMATERIALS (MMs) are artificial materials with a structured arrangement which exhibit electromagnetic properties that are not found in natural materials, such as negative permeability and negative permittivity. Current methods for solving MM problems typically rely on numerical computation techniques such as finite element methods (FEM) [1] and finite difference methods (FDM) [2], [3]. In contrast, physics-informed neural networks (PINNs) offer a different approach to numerically solving the governing partial differential equations (PDEs) by directly incorporating the physical laws into the neural network framework, providing an alternative to the FEM and FDM [4], [5]. Fang *et al.* proposed a PINN-based approach for designing the MM by solving the frequency domain Maxwell's equation [6]. He *et al.* applied PINNs to topology optimization for designing 2-dimensional MM unit cells [7]. However, the application of PINNs to time-domain MM problems remains unexplored. A key challenge lies in accurately addressing the governing equations within the MM region due to its dispersion characteristics. This paper develops a PINN method based on the Lorentz model as a viable alternative to traditional numerical methods approaches for solving low-frequency MM problems in the time domain.

II. METAMATERIAL EMBEDDED PROBLEM

In this study, a MM is embedded within a free space, as illustrated in Fig. 1.

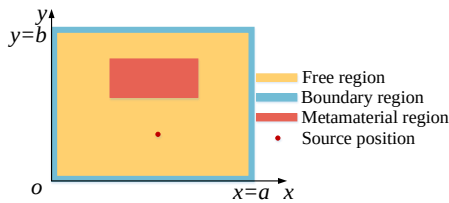


Fig. 1. Configuration of MM embedded region.

The entire domain is divided into three distinct regions: the free region, the MM region, and the boundary region. A sinusoidal source node is positioned within the free region. The physical residual of governing equation is formulated as a

combination of three components: the free region residual, the MM region residual, and the boundary region residual.

A. Residual in Free Region

The electromagnetic field equation in free space when working at TE_z mode is:

$$\begin{aligned} \frac{\partial H_z}{\partial y} &= \varepsilon \frac{\partial E_x}{\partial t} + \sigma E_x \\ -\frac{\partial H_z}{\partial x} &= \varepsilon \frac{\partial E_y}{\partial t} + \sigma E_y \\ \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} &= -\mu \frac{\partial H_z}{\partial t} \end{aligned} \quad (1)$$

where E_x and E_y are electrical field in the x -axis direction and in the y -axis direction, respectively, H_z is the magnetic field in the z -axis direction, ε is the permittivity, μ is the permeability, and σ is the conductivity. The residual of electromagnetic field equation in the free region includes three parts:

$$\begin{aligned} loss_{free}^1 &= \frac{1}{N_f} \sum_{i=1}^{N_f} \left(\varepsilon \frac{\partial E_x}{\partial t} - \frac{\partial H_z}{\partial y} + \sigma E_x \right)^2 \\ loss_{free}^2 &= \frac{1}{N_f} \sum_{i=1}^{N_f} \left(\varepsilon \frac{\partial E_y}{\partial t} + \frac{\partial H_z}{\partial x} + \sigma E_y \right)^2 \\ loss_{free}^3 &= \frac{1}{N_f} \sum_{i=1}^{N_f} \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} + \mu \frac{\partial H_z}{\partial t} \right)^2 \end{aligned} \quad (2)$$

where N_f is the number of calculations required in the free region. Based on (2), the total physical loss in the free region is:

$$loss_{free} = loss_{free}^1 + loss_{free}^2 + loss_{free}^3 \quad (3)$$

B. Residual in the MM Region

As previously mentioned, MMs are dispersive media, requiring the frequency-dependent relationship between frequency and intrinsic parameters to be considered when updating E_x , E_y , and H_z . Classical dispersion models can be employed to represent ε and μ in the time domain. The Lorentz model is commonly used to describe the response near the resonant frequency, which expressed as [2]:

$$\begin{aligned}\varepsilon_r(\omega) &= 1 + \frac{\omega_{pe}^2}{\omega_{0e}^2 - \omega^2 + j\Gamma_e\omega} \\ \mu_r(\omega) &= 1 + \frac{\omega_{pm}^2}{\omega_{0m}^2 - \omega^2 + j\Gamma_m\omega}\end{aligned}\quad (4)$$

where ω_{pe} and ω_{pm} are the plasma frequencies, ω_{0e} and ω_{0m} are the resonance frequencies, Γ_e and Γ_m are the damping coefficients, respectively; ε_r is the relative permittivity, μ_r is the relative permeability. Substituting (4) into (1) yields:

$$\begin{aligned}\frac{\partial H_z}{\partial y} &= \varepsilon_0 \left[1 + \frac{\omega_{pe}^2}{\omega_{0e}^2 - \omega^2} \right] \frac{\partial E_x}{\partial t} \\ -\frac{\partial H_z}{\partial x} &= \varepsilon_0 \left[1 + \frac{\omega_{pe}^2}{\omega_{0e}^2 - \omega^2} \right] \frac{\partial E_y}{\partial t} \\ -\mu_0 \left[1 + \frac{\omega_{pm}^2}{\omega_{0m}^2 - \omega^2} \right] \frac{\partial H_z}{\partial t} &= \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\end{aligned}\quad (5)$$

Thus, the residual loss in the MM region can be obtained from (5) as follows:

$$\begin{aligned}loss_{mm}^1 &= \frac{1}{N_m} \sum_{i=1}^{N_m} \left(\varepsilon_0 \left[1 + \frac{\omega_{pe}^2}{\omega_{0e}^2 - \omega^2} \right] \frac{\partial E_x}{\partial t} - \frac{\partial H_z}{\partial y} \right)^2 \\ loss_{mm}^2 &= \frac{1}{N_m} \sum_{i=1}^{N_m} \left(\varepsilon_0 \left[1 + \frac{\omega_{pe}^2}{\omega_{0e}^2 - \omega^2} \right] \frac{\partial E_y}{\partial t} + \frac{\partial H_z}{\partial x} \right)^2 \\ loss_{mm}^3 &= \frac{1}{N_m} \sum_{i=1}^{N_m} \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} + \mu_0 \left[1 + \frac{\omega_{pm}^2}{\omega_{0m}^2 - \omega^2} \right] \frac{\partial H_z}{\partial t} \right)^2\end{aligned}\quad (6)$$

where N_m represents the number of calculations required for the metamaterial region. Based on (6), the total physical loss in the MM region is:

$$loss_{mm} = loss_{mm}^1 + loss_{mm}^2 + loss_{mm}^3 \quad (7)$$

C. Residual Loss in Boundaries

Assuming the x -range is $[0, a]$ and the y -range is $[0, b]$, the second-order absorption boundary conditions at the boundaries $x = 0$, $x = a$, $y = 0$, and $y = b$ in the TE_z mode are given as:

$$\begin{aligned}\frac{1}{c} \frac{\partial^2 H_z}{\partial x \partial t} - \frac{1}{c^2} \frac{\partial^2 H_z}{\partial t^2} + \frac{1}{2} \frac{\partial^2 H_z}{\partial y^2} \Big|_{x=0} &= 0 \\ \frac{1}{c} \frac{\partial^2 H_z}{\partial x \partial t} + \frac{1}{c^2} \frac{\partial^2 H_z}{\partial t^2} - \frac{1}{2} \frac{\partial^2 H_z}{\partial y^2} \Big|_{x=a} &= 0 \\ \frac{1}{c} \frac{\partial^2 H_z}{\partial y \partial t} - \frac{1}{c^2} \frac{\partial^2 H_z}{\partial t^2} + \frac{1}{2} \frac{\partial^2 H_z}{\partial x^2} \Big|_{y=0} &= 0 \\ \frac{1}{c} \frac{\partial^2 H_z}{\partial y \partial t} + \frac{1}{c^2} \frac{\partial^2 H_z}{\partial t^2} - \frac{1}{2} \frac{\partial^2 H_z}{\partial x^2} \Big|_{y=b} &= 0\end{aligned}\quad (8)$$

The physics loss at the boundary can be derived as:

$$\begin{aligned}loss_{boundary}^{x=0} &= \sum_{i=1}^{N_{b1}} \left(\frac{1}{c} \frac{\partial^2 H_z}{\partial x \partial t} - \frac{1}{c^2} \frac{\partial^2 H_z}{\partial t^2} + \frac{1}{2} \frac{\partial^2 H_z}{\partial y^2} \right)^2 \\ loss_{boundary}^{x=a} &= \sum_{i=1}^{N_{b1}} \left(\frac{1}{c} \frac{\partial^2 H_z}{\partial x \partial t} + \frac{1}{c^2} \frac{\partial^2 H_z}{\partial t^2} - \frac{1}{2} \frac{\partial^2 H_z}{\partial y^2} \right)^2 \\ loss_{boundary}^{y=0} &= \sum_{i=1}^{N_{b2}} \left(\frac{1}{c} \frac{\partial^2 H_z}{\partial y \partial t} - \frac{1}{c^2} \frac{\partial^2 H_z}{\partial t^2} + \frac{1}{2} \frac{\partial^2 H_z}{\partial x^2} \right)^2 \\ loss_{boundary}^{y=b} &= \sum_{i=1}^{N_{b2}} \left(\frac{1}{c} \frac{\partial^2 H_z}{\partial y \partial t} + \frac{1}{c^2} \frac{\partial^2 H_z}{\partial t^2} - \frac{1}{2} \frac{\partial^2 H_z}{\partial x^2} \right)^2\end{aligned}\quad (9)$$

where N_{b1} is the number of calculation points along $x = 0$ and $x = a$, N_{b2} is the number of calculation points along $y = 0$ and $y = b$. The total boundary loss is then given as:

$$loss_{boundary} = \frac{(loss_{boundary}^{x=0} + loss_{boundary}^{x=a} + loss_{boundary}^{y=0} + loss_{boundary}^{y=b})}{2(N_{b1} + N_{b2})} \quad (10)$$

III. EXPERIMENTS AND RESULTS

A sinusoidal source excitation is located at $x = 50, y = 25$ with $H_z = \sin(2\pi f_0 \cdot t)$, where $f_0 = 6\text{MHz}$. The parameter settings for metamaterials are consistent with those in [2], with a refractive index of -2. In the FDTD method, a grid consisting of 100×100 spatial cells and 400-time steps is used. Fig. 2 compares the H_z component obtained using the PINN-based method and the FDTD method [2] at four different time steps (100, 200, 300, and 400). The black dashed rectangle highlights the MM region, while the surrounding area represents the free region. As shown in Fig. 2, the overall patterns and wave propagation characteristics between the two methods are consistent, with slight differences observed. A more detailed analysis and additional results will be presented in the full paper.

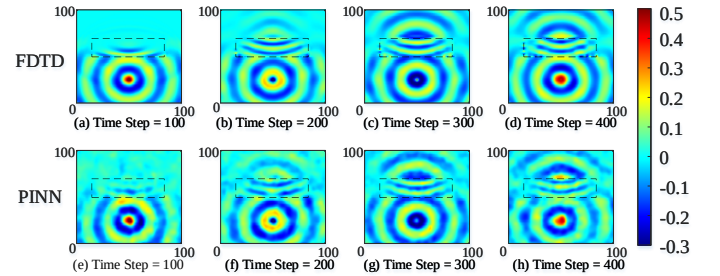


Fig. 2. H_z component obtained by (a)FDTD, time step=100, (b)FDTD, time step=200, (c)FDTD, time step=300, (d)FDTD, time step=400, (e)PINN, time step=100, (f)PINN, time step=200, (g)PINN, time step=300, (h)PINN, time step=400.

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