

Adaptive regularization boundary informed alone network for electromagnetic inverse problem

Kegan Li¹, and Feng Chen²

¹School of Electrical Engineering, Xi'an Jiaotong University, Xi'an, Shaanxi, China, lkg818@stu.xjtu.edu.cn

²School of Electrical Engineering, Xi'an Jiaotong University, Xi'an, Shaanxi, China, fchen@outlook.com

Over the past years, inverse problems in partial differential equations have garnered increasing interest in electromagnetics. However, due to the lack of conventional stability, nonlinearity and non-convexity, these problems are quite challenging and difficult to solve. In this paper, a new kind of neural network with adaptive regularization has been proposed to dealing with electromagnetic inverse problems. Compared with other neural network methods, the advance of the proposed method originates from the need of training data, in which the boundary conditions are the only requirement. To mitigate the instability arising from the reliance on boundary information in the analysis of internal media, an adaptive regularization approach is proposed. Compared to other neural network approaches under the same data conditions, this method demonstrates significantly superior performance. Furthermore, the introduction of the adaptive regularization term further enhances the solution accuracy.

Index Terms—adaptive regularization, neural network, inverse problem, boundary element method

I. INTRODUCTION

COMPUTATIONAL Electromagnetic (CEM) focuses on simulating the interaction between electromagnetic fields and power equipment through numerical methods such as the finite element method (FEM) and time-domain finite-difference (FDTD)[1]. It serves as a core technological pillar for smart grids, high-energy physics, and aerospace engineering. The key research directions in CEM include electromagnetic modeling of complex structures (e.g., superconducting cables, transformers), multi-physics coupled analysis of electromechanical-thermal-stress interactions, and parameter optimization for fault diagnosis. However, the field faces three major challenges:

- 1) **High computational complexity of nonlinear problems:** Material characteristics (e.g., the B-H hysteresis curves of ferromagnetic materials) and multi-physics coupling effects create highly nonlinear systems, rendering traditional numerical methods inefficient for large-scale simulations.
- 2) **The curse of dimensionality in parameter space exploration:** Optimizing numerous discrete variables (e.g., defect dimensions, material properties) to characterize internal equipment states often leads to combinatorial explosion in parameter space search.
- 3) **The trade-off between real-time responsiveness and accuracy:** Online monitoring of power equipment demands millisecond-scale computational analysis for fault detection. Conventional full-domain forward simulations cannot meet this requirement due to their intensive grid discretization and iterative solving processes.

However, the inverse problem is among the most challenging research topics in electromagnetic engineering, aiming to identify unobservable internal failure sources (e.g., insulation layer damage, partial discharge locations) within devices through measurable external data such as magnetic field or temperature distributions. Compared to forward analysis, inverse problems face three core challenges[2]:

- 1) **Ill-posedness:** Many inverse problems violate Hadamard's criteria of well-posedness, rendering solutions highly sen-

sitive to measurement noise. For instance, in temperature inversion of gas-insulated switchgear (GIS) equipment, errors in infrared thermography may misinterpret contact resistance faults as dielectric loss anomalies due to noise amplification.

- 2) **Non-uniqueness:** A single observed response can correspond to multiple internal fault configurations. For example, "thermal source displacement" and "changes in conductivity" may produce similar temperature field distributions, leading to ambiguous interpretations of the underlying physical mechanism.
- 3) **Data dependency:** Traditional approaches, such as finite-element-based inverse iteration, require global mesh discretization, demanding high-precision sensor configurations and comprehensive measurement coverage. Current research attempts to address these limitations through regularization techniques[3] (e.g., Tikhonov regularization) or Bayesian inference frameworks. However, these methods still suffer from critical trade-offs between computational tractability and the enforcement of physical constraints, often sacrificing solution accuracy for reduced complexity.

Over the past decade, statistical learning methods and deep neural network architectures have demonstrated exceptional potential in addressing high-dimensional data processing and nonlinear feature extraction[4]-[5]. These methods leverage inherent capabilities for approximating complex nonlinear relationships and efficiently handling multi-modal data, making them particularly suitable for resolving ill-posed physical inverse problems[6]. Among existing approaches, physics-informed neural networks (PINN)[7], the Deep Ritz Method (DRM)[8], and Weak Adversarial Networks (WAN)[9] have been extensively applied across diverse inverse problem domains. However, these methods fundamentally rely on governing equations defined within the entire solution domain, necessitating internal-domain measurements for both parameter inversion and forward computation of physical fields. This

requirement poses significant practical challenges, especially for systems where invasive internal sensing is either technically infeasible or prohibitively costly such as in non-destructive testing of complex industrial components or in vivo biomedical imaging.

To address this limitation, we propose a novel neural network architecture termed the Boundary-Informed Alone Neural Network (BIAN)[10], which exclusively utilizes boundary condition measurements to reconstruct unknown internal fields or parameters. By eliminating the dependency on interior data acquisition, BIAN significantly enhances the applicability of inverse problem solutions in scenarios where internal sampling is restricted. However, we observed that the absence of internal physical constraints introduces interpolation instability across material interfaces in heterogeneous media, leading to accuracy degradation in piecewise-continuous domains. To mitigate this issue, we further developed an adaptive regularization framework that dynamically adjusts penalty coefficients based on residual gradients at material boundaries. This strategy stabilizes solution convergence while preserving the boundary-only data requirement, thereby extending the robustness of BIAN to complex multi-physics and multi-material systems.

In this paper, we first introduce the mathematical principles of BIAN and the adaptive regularization. Then, we present a comparison of the accuracy under complex media conditions between the PINN using internal samples, the BIAN relying on boundary information, and the BIAN with adaptive regularization.

II. MATHEMATICAL PRINCIPLE

A. Boundary-informed alone network

Consider a two-dimensional boundary value problem governed by an elliptic differential equation with indeterminate medium distribution. The mathematical formulation of this system is shown as follows:

$$\begin{aligned}\mathcal{L}_\epsilon[u(\mathbf{x})] &= f(\mathbf{x}) \\ u(\mathbf{x}) &= \hat{u}(\mathbf{x}) \\ \frac{\partial u(\mathbf{x})}{\partial \vec{n}} &= q(\mathbf{x})\end{aligned}\quad (1)$$

where $\mathcal{L}_\epsilon$ donates the elliptic differential operator incorporating indeterminate medium distribution, explicitly expressed as $\mathcal{L}_\epsilon[u(\mathbf{x})] = \nabla \cdot (\epsilon(\mathbf{x}) \nabla u(\mathbf{x}))$.

Equation 1 forms a well-posed boundary value problem with indeterminate medium distribution $\epsilon(\mathbf{x})$. To tackle this problem, we employ a neural network approach to approximate $\epsilon(\mathbf{x})$. However, directly solving the indeterminate medium distribution typically requires a substantial amount of internal information. While acquiring such data is often extremely challenging. Therefore, we propose a neural network solving inverse problems utilizing boundary information.

In addressing such problems, reinterpreting the terms containing indeterminate medium distribution as the equivalent source term, the new governing equation is expressed as:

$$-\nabla \cdot (\nabla u(\mathbf{x})) = g(\mathbf{x}) \quad (2)$$

where $g(\mathbf{x})$ is determined by $u(\mathbf{x})$, $\epsilon(\mathbf{x})$ and $f(\mathbf{x})$. To establish the relationship between the energy distribution within the solution domain and the energy flux at the boundary, we propose the weighted residual approach that utilizes the Green's function as the weight function. The weight residual function is shown as:

$$\int_{\Omega} G \cdot (\nabla^2 u + g) \cdot d\Omega = 0 \quad (3)$$

Through integration by parts and the Gauss's theorem, we can obtain the relationship between the energy distribution within the solution domain and the energy flux at the boundary.

$$\begin{aligned}c(\mathbf{x}'_b)u(\mathbf{x}'_b) &= \int_{\Gamma} (G(\mathbf{x}_b, \mathbf{x}'_b) \nabla u(\mathbf{x}_b) - u(\mathbf{x}_b) \nabla G(\mathbf{x}_b, \mathbf{x}'_b)) \cdot d\Gamma \\ &+ \int_{\Omega} G(\mathbf{x}_i, \mathbf{x}'_b) g(\mathbf{x}_i) \cdot d\Omega\end{aligned}\quad (4)$$

As the neural network is utilized as an approximator of the indeterminate medium distribution, how to train such the network to learn the unknown physical information is the key problem. Taking 4 into consideration, the indeterminate medium distribution $\epsilon(\mathbf{x})$ is entirely determined by the source term $g(\mathbf{x})$, which can be represented by essential boundary condition and natural boundary condition solely. Substituting the essential boundary condition and the natural boundary condition with 4, we can get the residual for a given boundary source point $\mathbf{x}'_b$.

$$\begin{aligned}R(\mathbf{x}'_b) &= c(\mathbf{x}'_b)u(\mathbf{x}'_b) + \int_{\Gamma} u(\mathbf{x}_b) \nabla G(\mathbf{x}_b, \mathbf{x}'_b) d\Gamma(\mathbf{x}_b) \\ &- \int_{\Gamma} G(\mathbf{x}_b, \mathbf{x}'_b) \nabla u(\mathbf{x}_b) d\Gamma(\mathbf{x}_b) - \int_{\Omega} G(\mathbf{x}_i, \mathbf{x}'_b) g(\mathbf{x}_i) d\Omega(\mathbf{x}_i)\end{aligned}\quad (5)$$

Utilizing the neural network $NN(\mathbf{x}; \theta)$ to replace the source term $g(\mathbf{x})$, we can define the loss function that guides the training of the network. Suppose N_b boundary source points are allocated, so that the loss function is taken as the average of the residuals among the source points.

$$Loss = \frac{1}{N_b} \sum_{i=1}^{N_b} ||R(\mathbf{x}'_b; \theta)||^2 \quad (6)$$

B. Adaptive regularization

However, in cases involving piecewise medium distributions, the abrupt variations in medium properties at the interface can cause a decrease in the solution precision of BIAN. To address this challenge, we present a neural-network-output-based adaptive regularization approach.

Specifically, to reconcile the sharp variations at interface boundaries with the smooth transitions within individual media, we developed a network-output-driven, total-variation-based adaptive regularization strategy.

Total variation, in the form as 7, facilitates effective edge preservation by retaining sharp discontinuities while removing small-scale noise.

$$TV(u) = \int_{\Omega} |\nabla u| \cdot d\Omega \quad (7)$$

To effectively reduce noise in smooth regions while preserving sharp transitions in areas of discontinuity, we propose

a novel adaptive total variation regularization term. The loss function of the network with adaptive regularization is expressed as:

$$\begin{aligned} Loss_r &= Loss + \int_{\Omega} \lambda(\mathbf{x}) |\nabla NN(\mathbf{x}; \theta)| d\mathbf{x} \\ &= (A \cdot NN(\mathbf{x}; \theta) + b(\mathbf{x})) + \int_{\Omega} \lambda(\mathbf{x}) |\nabla NN(\mathbf{x}; \theta)| d\mathbf{x} \end{aligned} \quad (8)$$

where $\lambda(\mathbf{x}) = \lambda_0(\mathbf{x}) / (1 + |\nabla NN(\mathbf{x}; \theta)|^a)$. $\lambda_0(\mathbf{x})$ refers to the initial value of the regularization coefficient, a is an adjustable parameter that controls the sensitivity of the regularization. The optimality conditions for this method are:

$$\begin{aligned} \nabla \cdot (\lambda(\mathbf{x}) \frac{\nabla NN(\mathbf{x}; \theta)}{|\nabla NN(\mathbf{x}; \theta)|}) + \nabla \lambda(\mathbf{x}) \cdot \frac{\nabla NN(\mathbf{x}; \theta)}{|\nabla NN(\mathbf{x}; \theta)|} \\ - A^T (A \cdot NN(\mathbf{x}; \theta) - b(\mathbf{x})) = 0 \end{aligned} \quad (9)$$

When the gradient of the equivalent source field is minimal and the medium distribution exhibits spatial homogeneity, the regularization coefficient asymptotically approaches a maximal value, thereby enforcing the neural network to prioritize global smoothness and continuity of the solution function. Conversely, at regions where the equivalent source gradient exhibits pronounced magnitude typically corresponding to material interface the regularization coefficient dynamically diminishes toward zero, enabling the network to emphasize local feature preservation and capture abrupt property variations.

Numerical experiments in the subsequent section demonstrate that the integration of this gradient-driven adaptive regularization framework significantly enhances both the convergence stability and solution accuracy of the BIAN method for piecewise heterogeneous media problems, particularly in resolving interfacial discontinuities without compromising sub-domain smoothness.

III. RESULTS AND ANALYSIS

To rigorously validate the efficacy of the gradient-driven adaptive regularization approach incorporating Total Variation (TV) regularization, this section presents a systematic numerical experiment across two benchmark scenarios with escalating complexity in medium heterogeneity. The first experiment evaluates the performance of the BIAN method in homogeneous medium and compare its accuracy against the physics-informed neural network (PINN). The second investigates the impact of TV regularization on solution fidelity in piecewise uniform medium through quantitative comparisons of pre- and post-regularization results. There are two hidden layers in the network we utilized in this experiment, each layer of the residual blocks has $m = 10$ neurons. There are a total of 400 parameters in this model.

A. The behavior of BIAN and PINN in homogeneous medium

Consider the Laplace equation with indeterminate spatially varying medium distribution:

$$\begin{aligned} -\nabla \cdot (\varepsilon(\mathbf{x}) \nabla u(\mathbf{x})) &= f(\mathbf{x}), & \mathbf{x} \in \Omega \\ u(\mathbf{x}) &= \sin(\pi x_2), & \mathbf{x} \in \Gamma_1 \\ u(\mathbf{x}) &= 0, & \mathbf{x} \in \Gamma_2 \end{aligned} \quad (10)$$

Here $\Omega = \{(x_1, x_2) | (x_1, x_2) \in (0, 1) \times (0, 1)\}$, $\Gamma_1 = \{(x_1, x_2) | (x_1, x_2) = \{0, 1\} \times (0, 1)\}$, $\Gamma_2 = \partial\Omega \setminus \Gamma_1$, $\varepsilon(\mathbf{x}) = e^{\pi^2(x_1 - x_1^2)/2} / \pi$ and $f(\mathbf{x}) = 0$, the solution of this problem $u(\mathbf{x}) = e^{\pi^2(x_1^2 - x_1)/2} \cdot \sin(\pi x_2)$. The natural boundary condition can be obtained from the analytical expression of the solution. The solution and the medium distribution of the Laplace problem is shown as Fig. 1, and we can get the analytical solution of the natural boundary condition. For BIAN, We select 10 points on each boundary as the boundary source points $\{\mathbf{x}_b\}$ and 100 random points inside the domain as the internal source point $\{\mathbf{x}_i\}$.

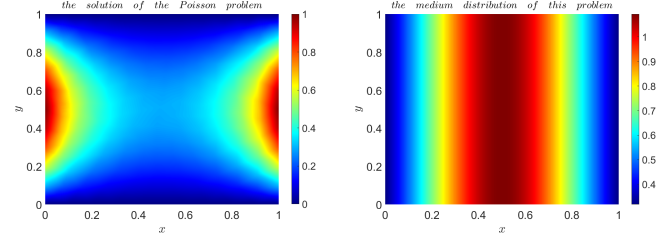


Fig. 1. The solution and the medium distribution of the Laplace problem.

The solution and the medium distribution obtained from BIAN and PINN are shown as follows and the L_2 error of each method is shown in Table 1. By comparing the results from different methods, it is evident that BIAN not only demonstrates better accuracy but also achieves faster convergence compared to the other three proposed methods. More importantly, BIAN requires only boundary information to solve the problem, significantly simplifying the data acquisition process.

TABLE I
THE L_2 ERROR OF THE SOLUTION AND MEDIUM DISTRIBUTION FOR THE FOUR METHODS

L_2 errorMethod	PINN	BIAN
medium distribution	0.0313	0.0143
solution	0.3117	0.0121

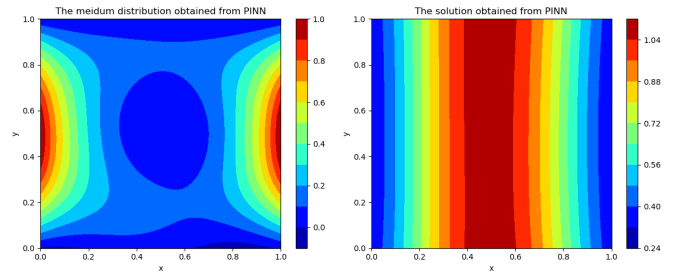


Fig. 2. The solution and the medium distribution of the Laplace problem obtained from PINN.

Fig. 2 illustrates the PINN-based reconstruction results. While the method successfully captures the general trend of both the solution field and medium distribution, there are visible discrepancies when compared to the analytical solution, the error may be due to the lack of internal information. In contrast, Fig. 3 presents the results and medium distribution

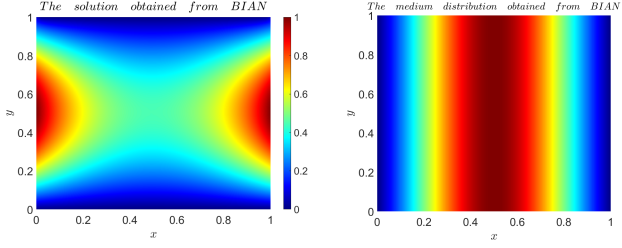


Fig. 3. The solution and the medium distribution of the Laplace problem obtained from BIAN.

obtained utilizing the BIAN approach, the error may be caused by the singular integral when training the neural network.

B. Performance of adaptive regularization

In this section, we investigate the performance of BIAN in solving problems involving piecewise uniform medium, with particular focus on the impact of adaptive regularization. Piecewise uniform medium represent a significant challenge in inverse problems due to the discontinuities at material interfaces, which are difficult to capture accurately using conventional methods.

Consider a 2-D Poisson problem with the following forms:

$$\begin{aligned} -\nabla \cdot (\varepsilon(\mathbf{x}) \nabla u(\mathbf{x})) &= f(\mathbf{x}), & \mathbf{x} \in \Omega \\ u(\mathbf{x}) &= 1, & \mathbf{x} \in \partial\Omega. \end{aligned} \quad (11)$$

where $\Omega = \{(x_1, x_2) | (x_1, x_2) \in (0, 1) \times (0, 1)\}$, $\Omega_1 = \{(x_1, x_2) | (x_1, x_2) \in (0, 0.5) \times (0, 1)\}$, $f(\mathbf{x}) = 10$, $\varepsilon(\mathbf{x}) = 1$ when $\mathbf{x} \in \Omega_1$ and $\varepsilon(\mathbf{x}) = 5$ when $\mathbf{x} \in \Omega \setminus \Omega_1$.

The solution of this piecewise uniform problem is shown as Fig. 4. We employ PINN, BIAN, and adaptive regularization BIAN to solve for the unknown internal medium distribution. The variation of the L_2 error of the output of PINN, BIAN and AR-BIAN with respect to training iterations is illustrated as follows.

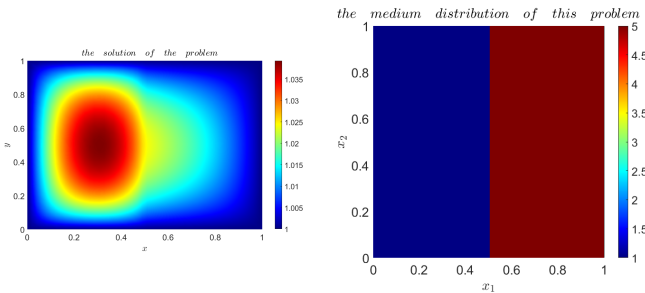


Fig. 4. The solution and the medium distribution of the piecewise uniform problem.

From Fig. 5, it can be observed that, compared to PINN, BIAN achieves better approximation performance under the same training data. Moreover, the introduction of the adaptive regularization term further improves the approximation accuracy of the neural network. However, the fluctuations observed during the iterative training process are primarily attributed to the stochastic sampling strategy employed in the training

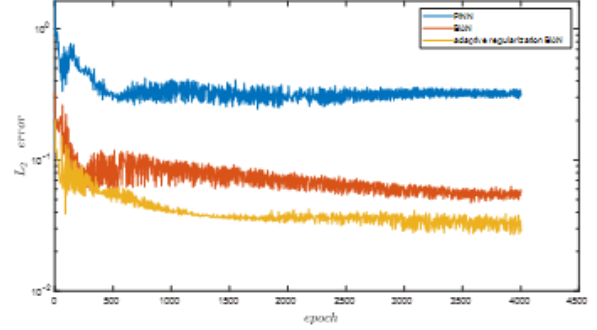


Fig. 5. . The variation of the L_2 error of the network output with respect to training iterations.

phase. Although stochastic sampling introduces variability in the training dynamics, this mechanism enhances the model's adaptability to the input data distribution, thereby contributing to improved training accuracy.

C. Conclusion

In summary, we proposed an adaptive total variation (TV)-regularized framework to enhance the boundary integral neural operator (BIAN) for solving inverse problems in piecewise heterogeneous media. The method dynamically adjusts regularization parameters to achieve a balance between two critical objectives, preserving solution smoothness within individual media subdomains and accurately capturing interfacial discontinuities at material boundaries. Experimental evaluations demonstrate that the proposed approach outperforms both PINN and BIAN in terms of reconstruction accuracy and convergence stability. Notably, compared to conventional inverse problem solvers, this method requires only boundary measurement data without the need for internal sensor deployment, significantly improving data efficiency and engineering feasibility. These advantages position the method as a promising candidate for non-invasive applications such as defect detection in industrial systems.

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